A Mathematical Model for the Temporal Covariance of the Background Noise in MEG/EEG-measurements

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Abstract

The general covariance matrix of the background noise in MEG and EEG signals is huge. To reduce the dimensionality of this matrix it is modelled as a Kronecker product of a spatial and a temporal covariance matrix. The iterative Maximum Likelihood estimation of these two matrices is still too time consuming to be useful on a routine basis. Therefore further parameterisation of the temporal covariance matrix is investigated. Alpha oscillations in the temporal (co)variance give rise to a model for the temporal noise where alpha activity and additional noise are separated. The modulation of the alpha waves is described by a distorted Poisson process. The covariance of the rest of the noise is modelled as an exponentially decreasing function of time lag. This theoretical model is a stationary model however in practise the stationarity of the matrix is highly influenced by the baseline correction.
It appears that very good agreement between data and parametric model can be obtained when this correction window is taken into account properly. Depending on the moment of interest (e.g. the SEF N20 response) one can take advantage of this non-stationarity by optimising the baseline correction.

1 Introduction

Background noise in MEG/EEG-measurements is correlated both in space and in time. When localising equivalent current dipoles (sources) taking into account the spatial covariance improves the accuracy of the estimated source parameters [1]. Models for the spatial noise covariance have been developed [2]. Recently also for the temporal noise covariance it has been demonstrated that incorporating this correlations leads in general to more precise source parameters [3], [4]. To reduce the dimensionality of the huge spatio-temporal noise covariance matrix Σ to a feasible size Σ is modelled as a Kronecker product of a spatial (X) and a temporal covariance matrix (T), $\Sigma = X \otimes T$, in both approaches [3] and [4]. This means that the covariance between two measurements at channels i resp. i' and time samples j resp. j' is the product of a spatial factor $X_{i,i'}$ and a temporal factor $T_{j,j'}$. Furthermore trials are assumed to be independent.
In [3] a parametric model for both X and T is assumed, where the covariances only depend on sensor distance resp. time lag. In [4] Maximum Likelihood (ML) estimates $\hat{X}$ and $\hat{T}$ are derived for X and T. The iterative ML-estimation procedure is still too time consuming to be useful on a routine basis (typically 45 hours for 1000 time samples and 150 channels). Therefore further parameterisation on physiological grounds beyond the Kronecker product is desirable.
In general the elements of $\hat{X}$ can be interpreted as a function of sensor distance mainly (cf. [2]). For $\hat{T}$ the analogues property is temporal stationarity, which means that the matrix is constant along sub-diagonals (Toeplitz) and temporal covariance only depends on time lag (cf. [3]). In **fig. 1** an example of an ML-estimated $\hat{T}$ is visualised by plotting the average over sub-diagonals (i.e. the average covariance for certain time lag) as function of time lag. The standard deviation (SD) of this average shows how stationary the matrix is. Moreover the diagonal of $\hat{T}$ (i.e. the variance) is plotted as function of time.
Another image of the same matrix is plotted in **fig. 2** where all matrix elements are plotted in a bitmap. In

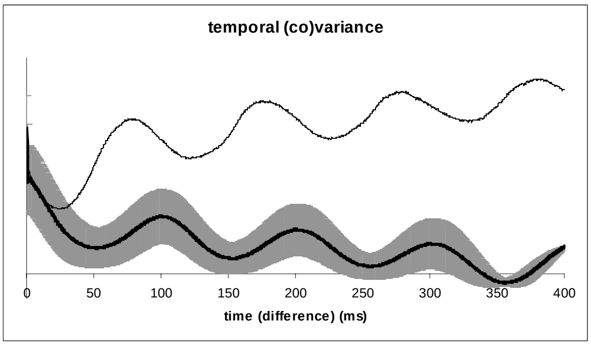

Fig. 1 *Average temporal covariance (± SD) as function of time lag in ms (bottom line) together with temporal variance (diagonal of* $\hat{T}$ *) as function of time in ms*

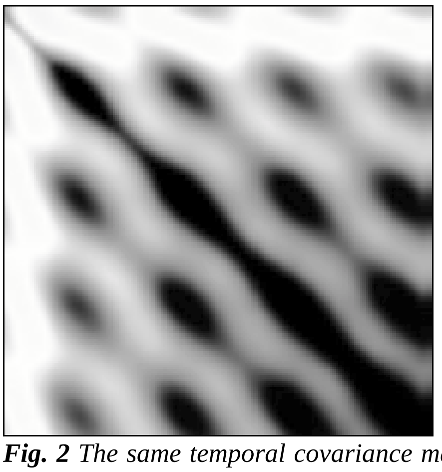

Fig. 2 *The same temporal covariance matrix as in fig. 1 now plotted as bitmap in greyscale (dark is higher value)*

fig. 1 an oscillation occurs not only in the average covariance but also in the variance. This is more clear in fig. 2 where oscillations in both directions result in dark spots. These two visualisations show that this sample ML-estimated matrix is not stationary. Because of the prominently present alpha oscillations the temporal noise is modelled as the sum of alpha activity and additional noise.

2 Model

2.1 Noise Model

In the simplified model the temporal noise is described as the sum of one ongoing α-wave and additional random noise. The wave and the rest of the noise are assumed to be independent. In the k^{th} trial the wave has random phase τ_k, uniform in $[-\pi, \pi]$, amplitude Ω and frequency ω. The noise for the k^{th} trial is then given by

$$\varepsilon_k(t) = \Omega \sin(\omega t + \tau_k) + n_k(t) \qquad (1)$$

where n_k the additional noise. The noise n_k is assumed to be temporally stationary and its covariance exponentially decreasing as function of time lag:

$$E[n_k(t_1)\ n_k(t_2)] = \sigma^2 e^{-\kappa|t_1-t_2|} \qquad (2)$$

where σ^2 is the variance and $\kappa>0$.

Using this simple model we get for the temporal covariance

$$\begin{aligned} Cov(t_1,t_2) &= E[\varepsilon_k(t_1)\,\varepsilon_k(t_2)] \\ &= \frac{1}{2}\Omega^2 \cos(\omega(t_1-t_2)) + \sigma^2 e^{-\kappa|t_1-t_2|} \end{aligned} \qquad (3)$$

This is a stationary model because $Cov(t_1, t_2)$ only depends on time difference. However it does not vanish for large time lag, which is due to the continuous alpha wave. For this reason the more realistic **Parametric Poisson Model** (PPM) is introduced [5].

In the PPM the alpha waves are assumed to occur randomly, one at a time and have fixed duration T_α. The onsets of the waves are modelled as the events in a distorted Poisson(λ) process. That is a statistical process generating events at random with mean intermediate time λ^{-1} [6]. After each event (onset), the process is disrupted for T_α, the following wave, after which it resumes to generate the next onset. The intermediate time between two successive events has the exponential(λ) distribution. For simplicity we assume the amplitude to be constant during the wave, equal to Ω. For the PPM $Cov(t_1, t_2)$ becomes

$$\begin{aligned} Cov(t_1,t_2) &= \Psi(t_1,t_2,T_\alpha,\lambda)\frac{1}{2}\Omega^2\cos(\omega(t_1-t_2)) \\ &+ \sigma^2 e^{-\kappa|t_1-t_2|} \end{aligned} \qquad (4)$$

where

$$\begin{aligned} &\Psi(t_1,t_2,T_\alpha,\lambda) = \\ &\lambda T_\alpha e^{\lambda T_\alpha}\Gamma(0,\lambda T_\alpha)\frac{T_\alpha - |t_1-t_2|}{T_\alpha} 1_{[-T_\alpha,T_\alpha]}(t_1-t_2) \end{aligned}$$

and

$$1_{[a,b]}(t) = \begin{cases} 1 & t \in [a,b] \\ 0 & t \notin [a,b] \end{cases}$$

$$\Gamma(0,a) = \int_a^\infty \frac{1}{\theta} e^{-\theta} d\theta \qquad a > 0$$

From (4) it is clear that the Parametric Poisson Model is still a stationary model but vanishes for large time differences.

2.2 Baseline correction

Due to external influences the baseline of single channel signals is usually shifted over an unknown offset. To correct for this an offset removal is carried out. One standard way of performing this correction is subtracting the average over a prestimulus interval, $[t_0-T_c, t_0]$. The formula for the corrected error in the k^{th} trial is

$$\varepsilon_k^c(t) = \varepsilon_k(t) - \frac{1}{T_c}\int_{t_0-T_c}^{t_0} \varepsilon_k(t')dt' \qquad (5)$$

and the corrected covariance $Cov^c(t_1, t_2)$ is

$$\begin{aligned} &Cov^c(t_1,t_2) = Cov(t_1,t_2) \\ &-\frac{1}{T_c}\int_{t_0-T_c}^{t_0} Cov(t_1,t') + Cov(t_2,t')dt' \\ &+\frac{1}{T_c^2}\int_{t_0-T_c}^{t_0}\int_{t_0-T_c}^{t_0} Cov(t',t'')dt'\,dt''. \end{aligned} \qquad (6)$$

The first term in (6) is the stationary covariance of the uncorrected model and the last term is a constant. The second term though is in general not stationary (both in the simplified model and the PPM). For the simplified model this term is equal to

$$\begin{aligned} &\frac{-2}{\omega T_c}\sin(\frac{\omega T_c}{2})\cos(\frac{\omega(t_1-t_2)}{2})\cos(\omega(\frac{t_1+t_2}{2}+t_0+\frac{T_c}{2})) \\ &-\frac{\sigma^2}{T_c}\int_{t_0-T_c}^{t_0} e^{-\kappa|t_1-t'|} + e^{-\kappa|t_2-t'|}dt'. \end{aligned} \qquad (7)$$

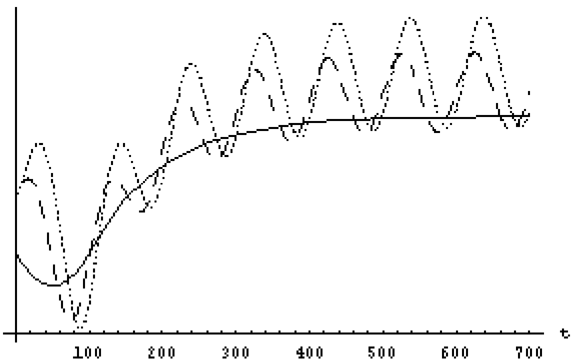

Fig. 3 *Corrected temporal variance ((6) with $t_1=t_2$) for the simplified model ($\omega=2\pi/100$, $\kappa=0.01$, $t_0=100$) as function of time t in ms. The dotted line corresponds to T_c = 25 ms, the dashed line to T_c = 50 ms and the solid line to T_c = 100 ms.*

In **fig. 3** three different temporal variances are plotted obtained from the same uncorrected simplified noise model using different baseline corrections. This figure shows that temporal stationarity of the corrected noise is highly dependent on the baseline correction. If T_c is equal to one alpha period, i.e. $\omega T_c = 2\pi$, the oscillation vanishes and the noise due to the alpha activity is stationary, although still a drop over the correction window is present due to the additional noise (second term in (7)).
The stationarity of the PPM depends on T_c in the same way as (7). Therefore one can regard the formulas for the simplified model to investigate the stationarity.

3 Results

To obtain a stationary matrix a baseline correction window of one alpha period was applied. The matrix thus obtained is shown in **fig. 4** as bitmap.
The Parametric Poisson Model was fitted to ML-estimated temporal covariance matrices of five different subjects. Data sets were acquired from Somatosensory Evoked Field experiments where the left median nerve was stimulated at 1 Hz. An additional parameter was added to model high frequency noise on the diagonal of T. In all this yields a model matrix that is described by six parameters.
This model was fitted to the five empirical ML-estimates in a Least Squares sense, using the same baseline correction as was used for the ML-estimations. The best fitting values for some of the parameters are stated in table 1.

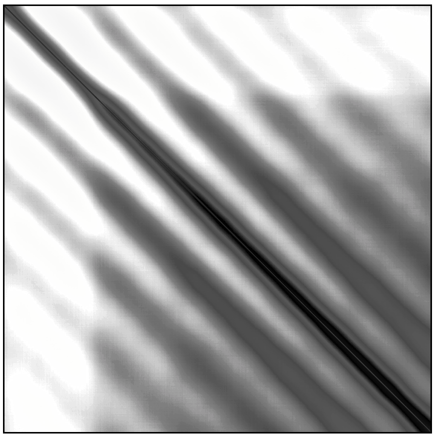

Fig. 4 *A bitmap image (as in fig. 2) of an ML-estimated temporal covariance matrix where the baseline correction interval was set to one alpha period*

subject	$\omega/(2\pi)$	κ	T_α	error
1	10.7	0.0437	287	0.4 %
2	9.9	0.0136	359	0.5 %
3	11.0	0.0613	289	0.7 %
4	8.9	0.0681	536	0.6 %
5	11.9	0.0117	61	0.4 %
unit	Hz	ms^{-1}	ms	matrix power

Table 1 *Best fitting values for the Parametric Poisson Model for five different subjects. The number of time samples is 500.*

4 Discussion

Using only six parameters the temporal covariance structure can be described accurately and in a physiological way by the Parametric Poisson Model. Table 1 shows that very good agreement between data and model is obtained (error < 1 % matrix power in all cases).
Temporal stationarity highly depends on the choice of baseline correction. Therefore it is of great significance that within one analysis one consistent baseline correction is used.
If T_c is set equal to one alpha period, the covariance due to the alpha activity will be stationary. Otherwise the sub-diagonals will contain an α-oscillation (see (7)).
The stationary structure does not always yield the minimum (co)variance (see **fig. 3**). Depending on the moment of interest one can therefore optimise the correction by minimising (6) with respect to T_c after substitution of the time and all other parameter values. For overall optimisation one should take T_c equal to one alpha period.

5 Literature

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