Monotonicity results for call center models

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Relevance

- Staffing multiple intervals
- For given # of agents and AHT, which schedule maximizes SL?

AHT = av. handling times, TTA = time to answer,

SL = service level

	A	B	C	D	E	F	G
1		Forecast	# of agents				
2	8:00-8:30	60			AHT	4	min
3	8:30-9:00	90			TTA	20	sec
4	9:00-9:30	90			SL	0%	
5	9:30-10:00	120			# of agents	55	

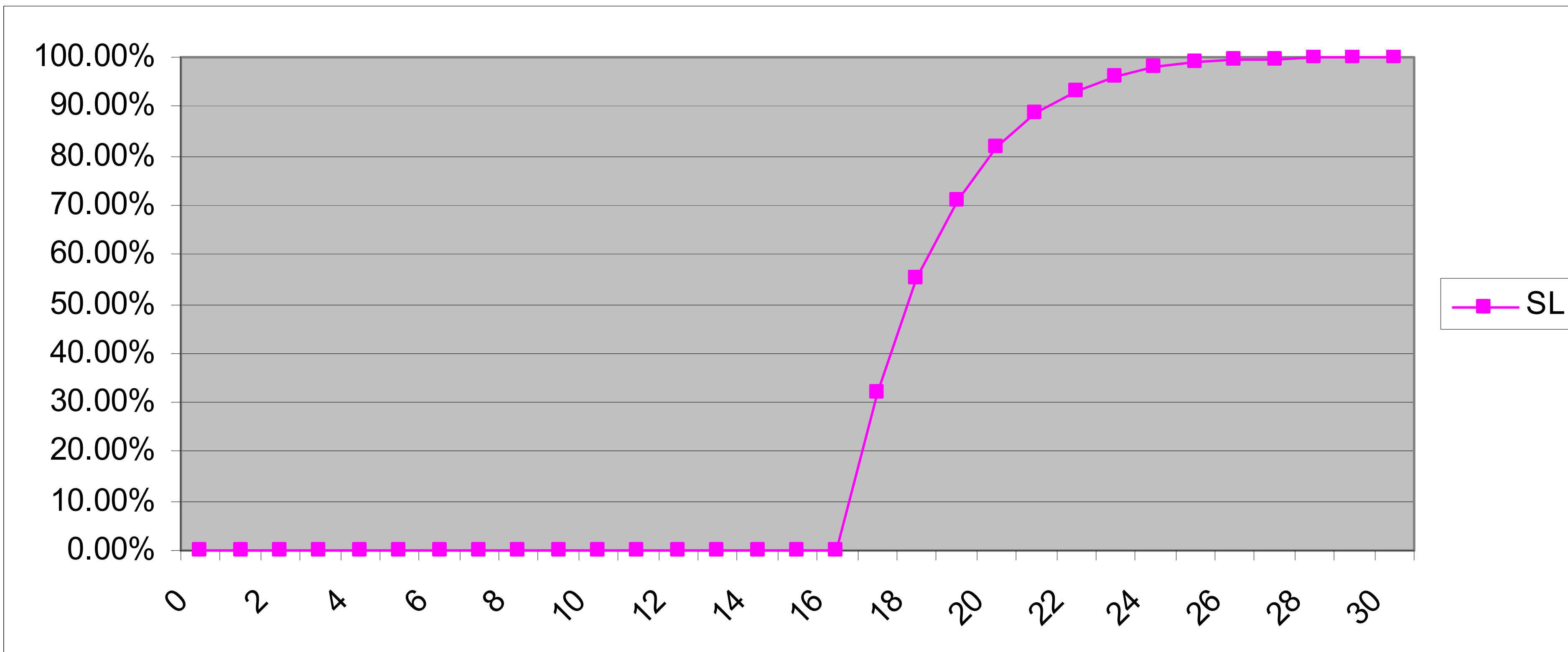
Relevance

- Solve problem with Excel solver and CCmath Erlang add-in
- Performance model: Erlang C
- Solution:

	A	B	C	D	E	F	G
1		Forecast	# of agents				
2	8:00-8:30	60	13		AHT	4	min
3	8:30-9:00	90	18		TTA	20	sec
4	9:00-9:30	90	0		SL	72%	
5	9:30-10:00	120	24		# of agents	55	

Relevance

- What is “wrong”?
- SL Erlang C not concave in # of agents



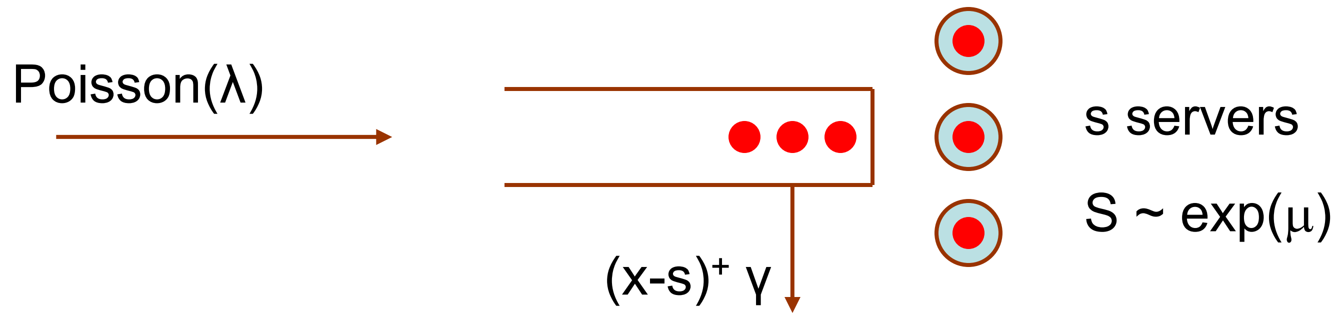
Relevance

Consequences

- A greedy algorithm will not always find optimal solution
- Optimal solution not always “balanced”

- For which models do monotonicity results hold?
- And which objective functions?

Erlang A



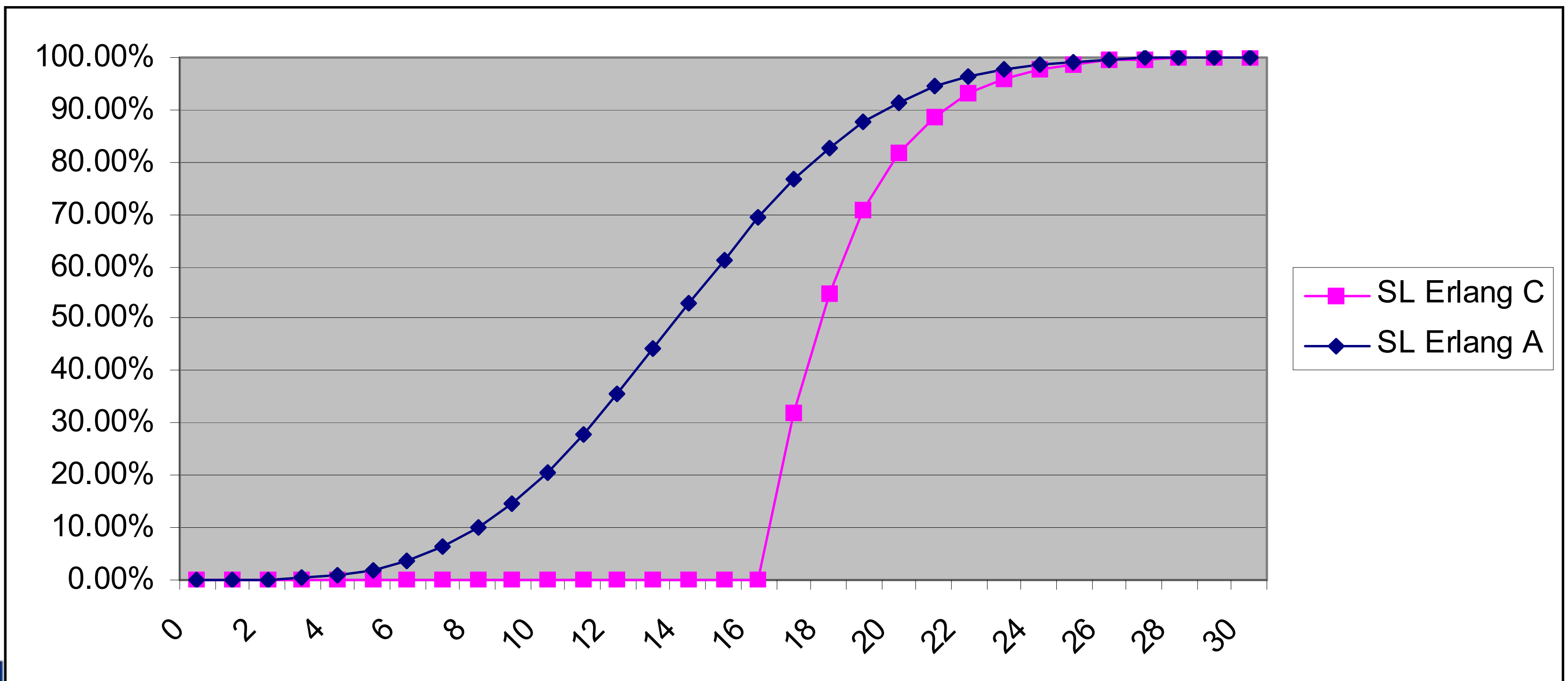
- Every waiting customer leaves the queue with rate γ
- $\Leftrightarrow$ Every customer has *patience* $P \sim \exp(\gamma)$
- Time in queue = $\min \{ P , W_q \}$
- Can also add *balking* (finite # of lines)

Erlang C/A, SL in s

- Erlang C: SL concave in s for ρ<1

(Jagers & v Doorn 91, using analytic arguments)

- Erlang A: not concave nor convex



About the SL

Theory

- Service level =

  Σ_n I{waiting time of call n $\leq$ t} / # of calls
- Costs 0 or 1 for each call: not cv or cx in waiting time
- Average cost = SL cannot be expected to be cv or cx (unless very special waiting time dist)

Practice

- SL (aka TSF) lousy “SL” definition

ASA & AE

- ASA = average speed of answer = average waiting time
- AE = av. excess = (waiting time – TTA)$^+$
- Both cx in waiting time for each call
- Also cx in number of servers?

Dynamic programming

- $V_s(x)$ = dp value function with s servers and currently x calls in system
- $2V_{s+1}(x) \leq V_s(x) + V_{s+2}(x)$
- Required for proof:
  - Direct costs are convex is s
  - $V_s(x)$ is convex in x

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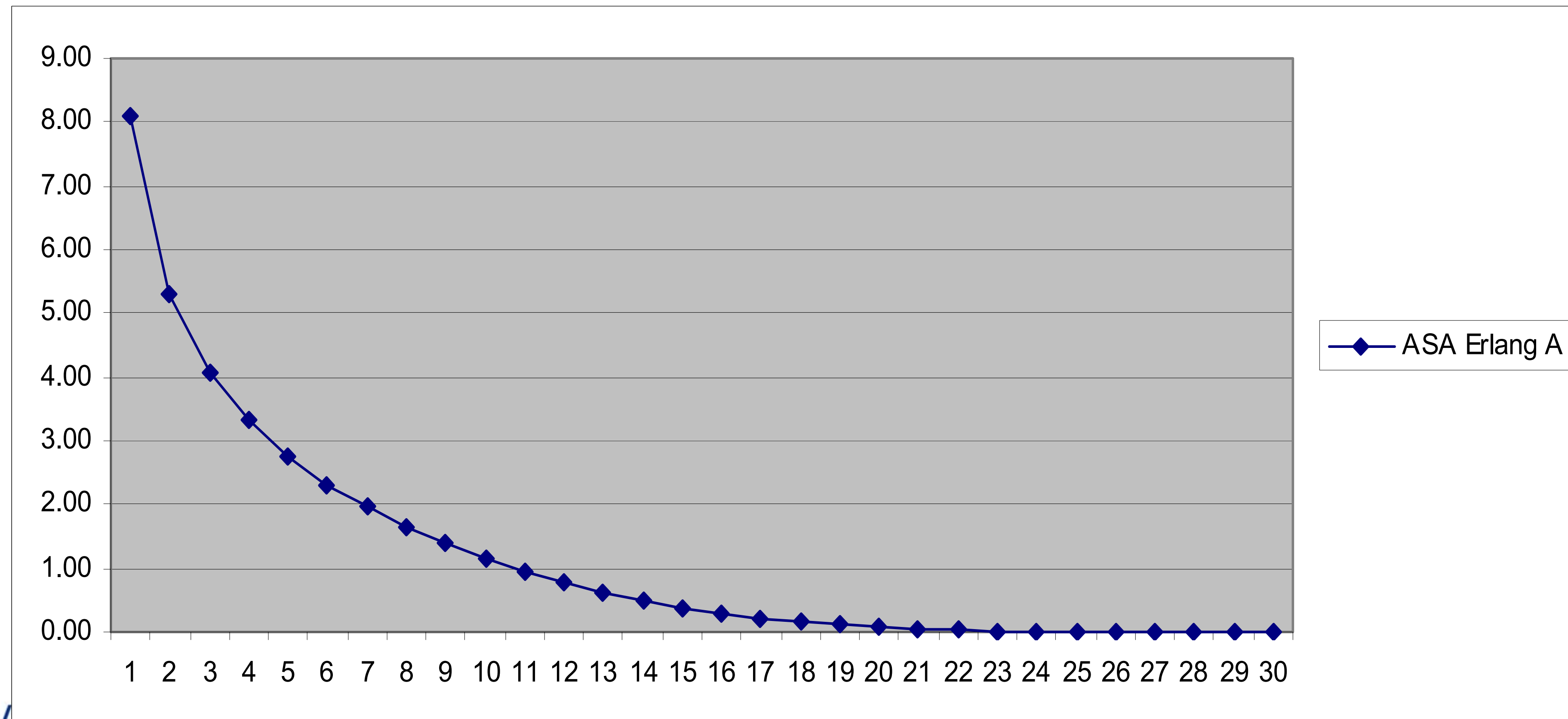
Convexity in x

- $2V_s(x+1) \leq V_s(x) + V_s(x+2)$
- Required:
  – Direct costs are convex is x
  – Service rate concave in x
  $\Rightarrow$ Erlang A: $\gamma \leq \mu$

Convexity ASA

- ASA in s of *virtual* waiting time
- Greedy algorithm works



Admission control

- Admission control = finite # of lines = dissuasion (call-backs)
- Two-dim problem: choose s and n
- Fixed s: performance unimodal in n (costs: idleness + waiting)
- No convexity nor unimodality in s
- Admission control destroys the convexity

Example profit model

of agents → s; profit → g^s; optimal # of waiting places → n_s

s	g^s	n_s	Abandonments (in %)	Blocked (in %)	SL (in %)
0	0.0000	0	0.00	100.00	100.00
1	0.0594	0	0.00	93.75	100.00
2	0.1105	0	0.00	87.55	100.00
3	0.1521	0	0.00	81.40	100.00
4	0.1825	0	0.00	75.32	100.00
5	0.2396	1	4.47	66.05	86.91
6	0.3147	1	3.45	59.99	91.42
7	0.3665	1	2.70	54.05	94.41
8	0.3907	1	2.15	48.28	96.39
9	0.3855	2	4.02	39.79	90.53
10	0.3993	2	3.26	34.36	93.56
11	0.3636	2	2.64	29.22	95.71
12	0.2951	3	3.54	21.94	92.46
13	0.1771	3	2.81	17.62	94.97
14	0.0033	4	3.13	11.86	93.21
15	−0.2561	5	3.09	7.29	92.49

Table 1: Values of g^s for various s for $\lambda = 15$, $\mu = 1$, $c = 0.39$, $r = 1.52$, and $\gamma = 1/2.9$

More intriguing results

- Convexity of Erlang A holds also in arrival rate
- Holds also for fixed finite # of lines
- Again: Does not hold in case optimal admission control

- Also relevant: increase s and λ simultaneously
- Erlang B not yet completely solved
- Erlang C/A?