PICA PATIENT FLOW IMPROVEMENT CENTER AMSTERDAM

Bed assignments in shared wards

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Organization

- Erlang loss model
- Merging wards:
  - Drawbacks of merging
  - Alternatives
  - Results: examples revisited

Ward dimensioning: Erlang B

$$\pi(i) = \frac{(\lambda \mathbb{E} S)^i / i!}{\sum_{j=0}^{s} (\lambda \mathbb{E} S)^j / j!};$$

$$\mathbb{E} L = (1 - \pi(s)) \lambda \mathbb{E} S.$$

- Load = a = λ ES = λ β
  (compare: ρ = a / s)
- Blocking prob. = B(s,a) = π(s)

Calculating Erlang B

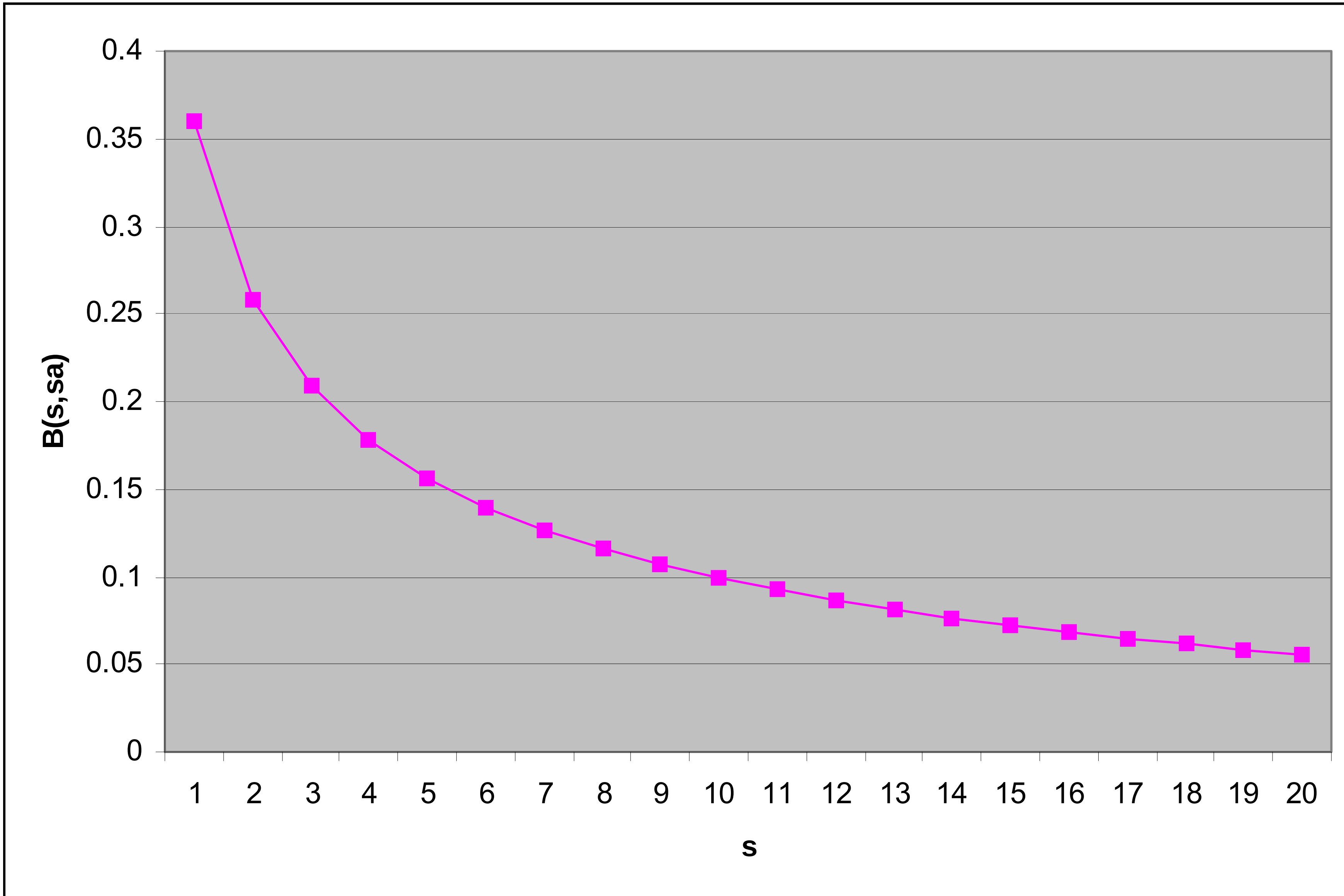
- X~exp(a): P(X=k) = (a^k / k!) exp(-a)
- Thus: B(s,a) = P(X=s) / P(X≤s)
- Excel: = POISSON(s,a,FALSE) / POISSON(s,a,TRUE)

Economies of scale

- B(s , sa) decreasing in s = ec of scale
- Thus merging is beneficial (?)
- Proof: Smith & Whitt 81, analytical properties of B(.,.)

- B(s , sa) convex?
- Diminishing returns
- No proof, numerical evidence

B(s,sa) plot, a=0.8



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Merging

- $\lambda\,\beta\,(1\text{-}B(s,\lambda\,\beta))$ = occupation, av # occupied beds
- Smith & Whitt 81:

$$(\lambda_1\beta_1+\lambda_2\beta_2)B(s_1+s_2,\lambda_1\beta_1+\lambda_2\beta_2)\le\lambda_1\beta_1B(s_1,\lambda_1\beta_1)+\lambda_2\beta_2B(s_2,\lambda_2\beta_2)$$

- Compares occupation
- Thus: merging wards leads to higher occupancy
- Merge all wards in case fixed revenue per night

Equal ALOS

- Divide formula by $\beta_1 = \beta_2 = \beta$:

$$(\lambda_1+\lambda_2)B(s_1+s_2,(\lambda_1+\lambda_2)\beta) \leq \lambda_1 B(s_1,\lambda_1\beta)+\lambda_2 B(s_2,\lambda_2\beta)$$

- Thus merging minimizes overall blocking rate
- Also if unequal ALOS?
- And how about rates per ward?

Merging (Ex. 1)

Ward 1
- Arrivals: 5 p/d
- ALOS = β = 4 days
- Offered load: 20
- Number of beds: 20
- Loss rate: 15.9%

Ward 2
- Arrivals: 2 p/d
- ALOS = β = 4 days
- Offered load: 8
- Number of beds: 12
- Loss rate: 5.1%

Total (weighted) loss rate: 12.8%

→ Loss rate after merging: 6.6%

Merging (Ex. 2)

Ward 1

- Arrivals: 20 p/d
- ALOS = 1 day
- Offered load: 20
- Number of beds: 27
- Loss rate: 2.7%

Ward 2

- Arrivals: 2 p/d
- ALOS = 10 days
- Offered load: 20
- Number of beds: 17
- Loss rate: 25.6%

Total (weighted) loss rate: 4.8%

→ Loss rate after merging: 6.5%

(unrealistic example)

Merging: pros and cons

<u>Separate wards</u>

Cons

- Inefficient

Pros

- Single-skilled staff
- Guarantee for beds
  - Prioritize patient types
- Small scale leads to motivated staff

<u>Merging</u>

Pros

- Efficient

Cons

- Multi-skilled staff
- Variation available beds
  - No prioritization
- Large scale leads to lack of motivation & organizational issues

Alternatives for merging

- Simple merging
  - Staff can treat both patient types
- Earmarking
  - Two separate wards with joint overflow
- Threshold
  - From threshold value admit only high-priority patients
- Optimal policy
  - Takes relative value of patients into account

Model

- *J* types of patients
  - Different wards
  - Different patient types (*clinical pathways*)
  - Elective vs. emergency
- Arrivals: Poisson with rate λ_j
- LOS: Exponential with rate μ_j
- Number of beds: *N*

Earmarking

- Number of beds type *i*: N_j
- Joint ward (overflow): $N - \sum_{j=1}^{J} N_j$

Combination between merging and separate wards

- Separate wards: $\sum_{j=1}^{J} N_j = N$
- Simple merging: $N_1 = \ldots = N_J = 0$

Model solution: product form

Threshold

- Merging with priorities
- Threshold type *i*: T_i
  - Refuse type *i* when less than $N - T_i$ beds free

Remark:

- Effective in prioritizing patients
- Drawbacks of simple merging

Optimal policy

- Depends on exact combination of patients present
- Goal: $$\min \sum_{j=1}^{J} c_j b_j$$ with b_j = loss fraction type *j*

Examples:

- Weighted combination: $c_j = \alpha_j \lambda_j / \lambda$
- Restrictions: $$\min \sum_{j=1}^{J} c_j b_j \quad \text{s.t.} \quad b_j \leq b_j^{\max}$$

Model solution: dynamic programming

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- Offered load: 8
- Number of beds: 12
- Loss rate: 5.1%

Total (weighted) loss rate: 12.8%

Earmarking:
- Number of beds: 0
- Loss rate: 8.4%

- Number of beds: 8
- Loss rate: 5.1%

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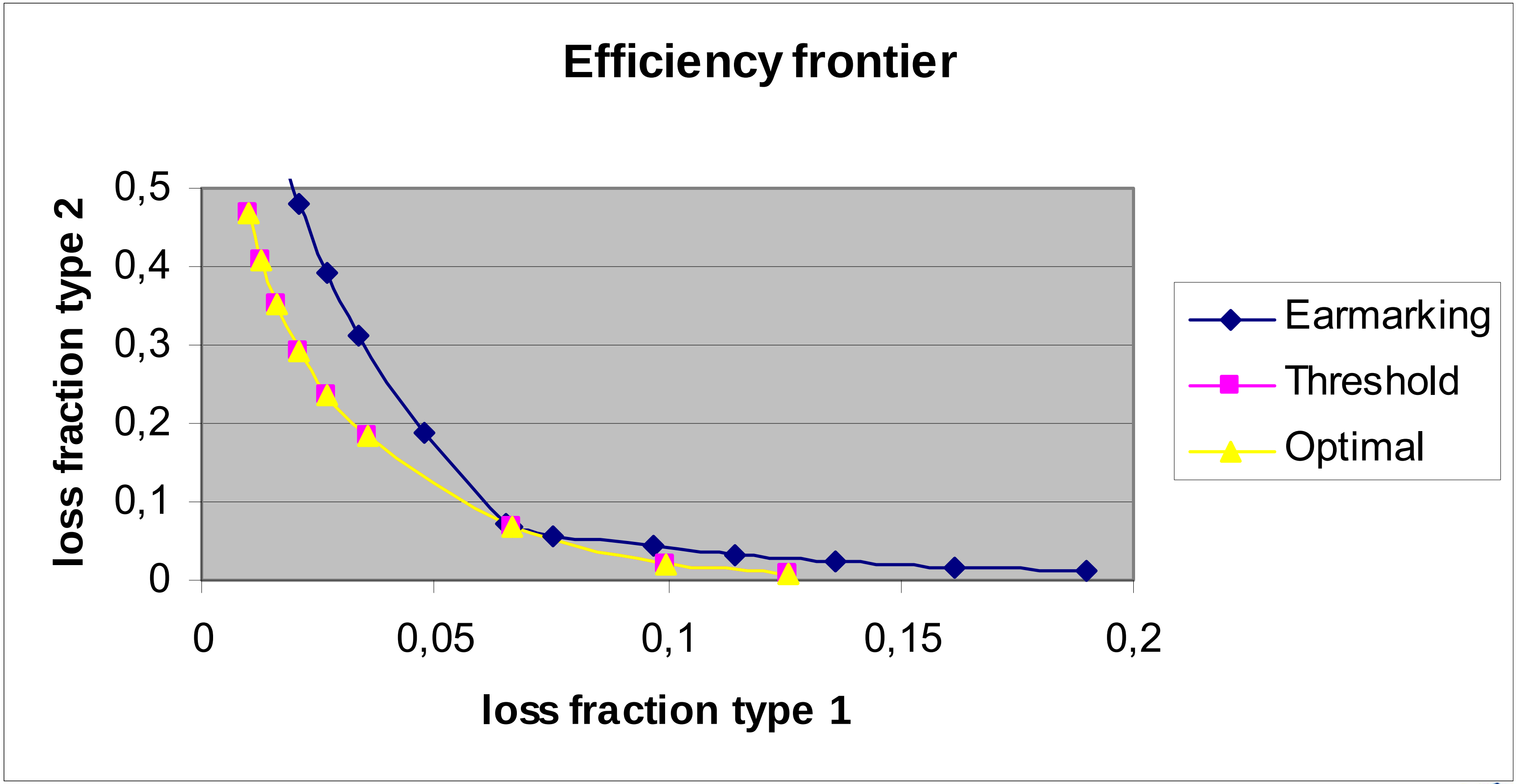
- ALOS = 4 days
- Offered load: 8
- Number of beds: 12
- Loss rate: 5.1%

Total (weighted) loss rate: 12.8%

Earmarking:
- Number of beds: **16**
- Loss rate: 8.4%

- Number of beds: 8
- Loss rate: 5.2%

Efficiency frontier (Ex. 1)



Merging (Ex. 2)

Ward 1

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- Loss rate: 25.6%

Total (weighted) loss rate: 4.8%

Earmarking:

- Number of beds: 24
- Loss rate: 3.2%

- Number of beds: 14
- Loss rate: 16.5%

Total (weighted) loss rate: 4.4%

Merging (Ex. 2)

	Type 1	Type 2	Weighted
Separate wards	2.7	25.6	4.8
Simple merging	6.5	6.5	6.5
Earmarking (24,14)	3.2	16.5	4.4

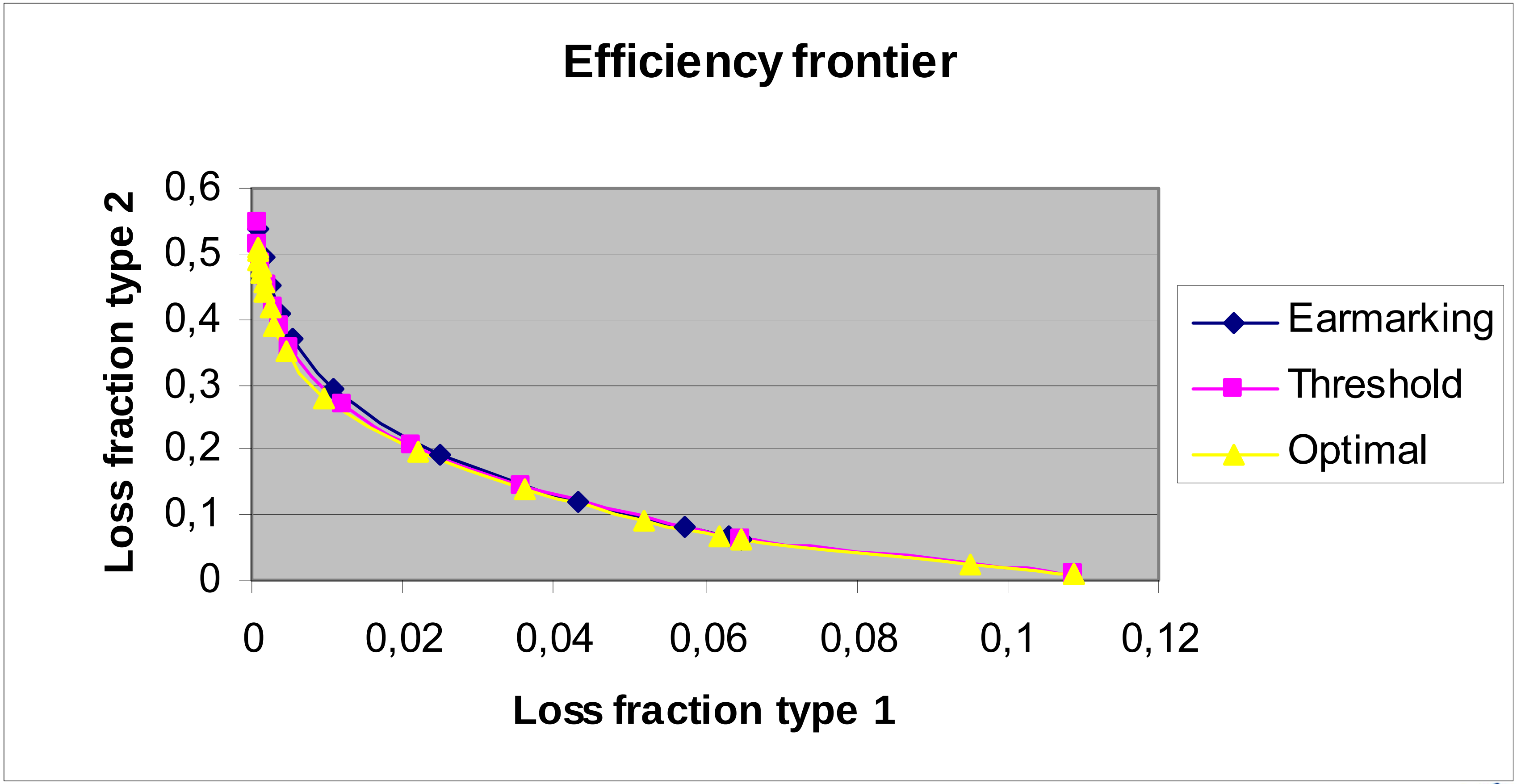
Loss rates in %

Merging (Ex. 2)

	Type 1	Type 2	Weighted
Separate wards	2.7	25.6	4.8
Simple merging	6.5	6.5	6.5
Earmarking (28,0)	1.1	29.3	3.7
Earmarking (24,14)	3.2	16.5	4.4
Optimal	1.0	28.0	3.4

Loss rates in %

Efficiency frontier (Ex. 2)



Conclusion

- Economies of scale possible

  `smart' merging

- Earmarking

  single-skilled staff & prioritization