Call center capacity allocation with random workload

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ABSTRACT

We consider a call center staffing problem with two types of costumers of which the arrival rates are allowed to be a random and non-stationary. In order to efficiently cope with such random workload fluctuations, the workforce presents some flexibility: the agents can be, in real-time, affected to each type of customers according to the instantaneously observed workload and the associated/relative cost criteria. We model this staffing problem as a cost optimization-based newsboy-type model. We then show how to numerically solve this model. In order to deal with the randomness characterizing the workloads of the call processes, we consider several solution tracks. First, we solve the model under the assumption that the workloads are deterministic and equal to their average values. In the second approach, we explicitly formulate in the optimization model the stochastic nature of the workloads. As a third approach, we develop a robust-type solution. Via several numerical analyzes we show the impact of the arrival rates randomness on the optimal staffing policy and on the operating costs.
Keywords: Call centers, staffing, non-stationary arrival rates, newsboy-type model, stochastic approach, robust approach.

1. Introduction

1.1. The general setting and literature review

Telephone call centers have become more and more important for many large organizations to communicate with their customers. For example, it is estimated that in 2002 more than 70% of all customer-business interactions were handled in call centers, and the U.S call center industry employed more than 3.5 million people, or 2.6% of the workforce (see[3]). Managing call center operations more efficiently is of significant economic interest.

Up to now, the focus on call center research has been mainly on forecasting, the translation of forecasts into staffing levels (using the Erlang C or some other queuing formula), and agent scheduling (see [1, 2]). A third planning activity, after forecasting and agent planning, concerns intra-day performance management. This has rarely been studied in the literature, except for a few papers that study intra-day forecasting where the consequences of having the realization of the first part of the day on the forecast for the rest of the day are studied (see [5, 6, 14, 13, 12]).

Workforce management is a critical component of call center operations. Since labor costs are a major component of the total cost of operation, efficient staff scheduling is critical. But when uncertainty plagues the arrival rates, efficient staffing is difficult. The variability has several origins. A cause that has attracted some attention in the literature is the forecasting error. It is not uncommon that forecasts are regularly off by 5% to 10%. The realization of the volume on a day can be modelled as a random variable with expectation equal to the forecast. This is not all the randomness related to the call demand: for a given average workload, the arrival rates as well as the handling times are random. For fixed staffing levels, this leads to a considerable variability in the service level, even for extended periods such as an entire day. A final cause of variability, that we do not consider in this paper, is absence of agents due to illness or other reasons. Most call center models in the literature focus on the stochastic variability of inter-arrival times for a given known arrival rate and ignore the issue of arrival rate uncertainty. Only a few papers have addressed this issue. In the what follows, we highlight some of these works.

T. Robbins (see [6, 7, 8, 9]) focused several research papers on this specific topic. Its research considers the issue of arrival rate uncertainty and its impact on scheduling. He reviews empirical data from call centers that demonstrates the level of uncertainty present in many applications, and proposes heuristic scheduling and staffing models that consider arrival rate uncertainty. In ([7, 8]), the authors develop a solution method consisting of applying a stochastic programming approach (without recourse) based on a set of scenarios for the arrival process. In ([6, 9, 10, 11]), the authors evaluate by simulation the impact of fluctuating arrival rates on the call center performance indices, for different types of call centers.

In [14], W. Whitt proposes simple methods for staffing a single-class call center with uncertain arrival rate and uncertain staffing due to employee absenteeism. The basic model is a multi-server queue with customer abandonment, allowing non-exponential service-time and time-to-abandon distributions. The goal is to maximize the expected net return. Two approximations are simultaneously used for the conditional performance measures: a deterministic fluid approximation and a Markovian birth-and-death model, having state-dependent death rates. In [15], this author considers the staffing problem for staffing a single-class call center with uncertain arrival rates. The solution method relies on exploiting detailed knowledge of system state in order to obtain reliable estimates of the mean and variance of the demand in the near future. The necessary staff is then accordingly adapted.

1.2. The considered problem

In this paper, we focus on a relatively simple model in this random arrival rates setting. We consider a call center staffing problem with two types of costumers and with

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random and non-stationary arrival rates.

In order to efficiently cope with such random workload fluctuations, the workforce presents some flexibility: the agents can be, in real-time, affected to each type of customers according to the instantaneously observed workload and the associated/relative cost criteria. Namely, we consider a call center with inbound calls and emails. The inbound calls should be handled (almost) immediately, using a standard service level measure (for example 80% answered within 20 seconds). Emails are storable and they should be answered within the same day. A crucial point is how the trafficer deals with the randomness in the arrival process. We assume that the trafficer can react to the current service level, by adding or taking away agents from the inbound channel, in such a way that at the end of the day the service level is as required. Emails are treated by the remaining agents. It is furthermore assumed that switching from one task type to the other is instantaneous. Thus, at the end of the day, given the number of agent hours that have been required for inbound, the number of hours that have been scheduled, and the time required to deal with emails (which is possibly random as well), the number of hours overtime can be determined. Overtime is assumed to be more expensive than regular hours. The decision variable is the number of agents and is assumed to be fixed for the whole day.

We model this staffing problem as a cost optimization-based newsboy-type model. We then show how to numerically solve this model. In order to deal with the randomness characterizing the workload of the call processes, we considered several solution tracks. First, we solve the model under the assumption that the workload are deterministic and equal to their average values. In the second approach, we explicitly formulate in the optimization model the stochastic nature of the workload. As a third approach, we develop a robust-type solution. Via several numerical analysis we show the impact of the workload randomness on the optimal staffing policy and on the costs.

2. The newsboy reformulation

In our model, the random workloads associated to the call process are assumed to be directly expressed as the random numbers of agents required in order to guarantee a given Qos such as the standard service level measure. Clearly this number depends on the value of the arrival rate.

The whole day is divided into n fixed periods. The randomness characterizing the global workload level is modelled through a single random parameter, $\tilde{D}$, with a discrete probability distribution function $G(.)$ over the interval $[\tilde{D}_{min}, \tilde{D}_{max}]$, with the explicit distribution $(p_k, \tilde{D}_k)$, for $k = 1, ..., K$.

For a given realization of the parameter $\tilde{D}$, the required periodic number of agents are denoted as D_i with the probabilities distributions $f_{D_i}(\cdot)(i = 1, ..., n)$. Several papers (see [5, 19]) have recently identified significant correlation between arrival rates in different time intervals within the same day. As a consequence, we assume that D_i in each time interval follows a probability distribution with a mean

$$\bar{D}_i = \alpha_i \tilde{D} \tag{1}$$

where α_i is a constant value associated to period i.

The required number of agents for handling to emails, corresponding to the daily workload, is described by the random variable N, with probability distribution $f_N(\cdot)$. The email workload is assumed to be independent of the parameter $\tilde{D}$ and it arrives right after the beginning of the day.

At the beginning of the day, before $\tilde{D}$ and N are observed, the decision maker plans the capacity y, namely the number of agents available for the day. These agents are assumed to be flexible, i.e. able to handle both type of calls and emails. The cost structure of the problem is as follows. Each agent gets a salary c per period. If at the end of the day, when the regular shift finishes, there is still a backlog of emails, they have to be handled on overtime. The overtime salary is p per agent per period. In case of call under staffing, a penalty b is paid per difference unit between the capacity y and each periodic requirement D_i, for $i = 1, ..., n$.

Under this economic framework, the objective consists of choosing the optimal y value which minimizes daily cost.

The corresponding notations are the following.

2.1. Notations

Decision variables

- y: the number of flexible agents available for the whole day.

Deterministic Parameters

- c: the regular salary per agent per period,
- p: the overtime salary per agent per period,
- b: penalty cost unit per difference unit between the effective under capacity y and each periodic requirement.

Random Parameters

- n : number of periods in a day,
- $\tilde{D}$: the random global workload level,
- G : the discrete probability distribution of $\tilde{D}$ (with the explicit distribution p_k, (for $k = 1, ..., K$),
- D_i : the number of agents necessary for dealing with the calls, $i = 1...n$,
- f_{D_i}: the probability distribution of D_i with a mean $\bar{D}_i = \alpha_i \tilde{D}$, $i = 1...n$,
- N : the number of agents necessary for dealing with the emails,
- f_N : the probability distribution function of the random variable N.

2.2. Assumption

The assumptions $c < p < b$ is necessary to guarantee coherence of the problem. This assumption ensures that it is not optimal to have agents working overtime if it can be avoided. This assumption furthermore ensures that it is not profitable to choose a solution with totally under staffing (namely $y = 0$).

2.3. The fundamental cost function model

Let us denote by $C(y)$ the daily expected cost associated with the workload level $\tilde{D}$ and the staffing level y. This expected cost can be expressed as

$$C(y) = n\,c\,y + p\,E[N - E[\Sigma_{i=1}^n (y - D_i)^+]\,]^+ \\ +b\,E[\Sigma_{i=1}^n (D_i - y)^+]. \quad (2)$$

In this expression, the term $n\,c\,y$ is the salary for the staff working on a regular time basis, the term $p\,E[N - E[\Sigma_{i=1}^n (y - D_i)^+]\,]^+$ is the expected salary for overtime necessary in order to deal with emails and the term $b\,E[\Sigma_{i=1}^n (D_i - y)^+]$ is the expected penalty cost for calls answered with an unsatisfactory quality level during n periods.

Proposition:
The cost function is convex in y.

proof

Let us denote by: $M(y) = E[\Sigma_{i=1}^n (y - D_i)^+] = \Sigma_{i=1}^n \int_0^y (y-m) f_{D_i}(m) dm$, and rewrite the cost expression under the form

$$C(y) = n\,c\,y + p\int_{M(y)}^{\infty} [x - M(y)] f_N(x)\,dx \\ +b\,\Sigma_{i=1}^n \int_y^{\infty} (m-y) f_{D_i}(m)\,dm. \quad (3)$$

By applying Leibniz formula, we find

$$\begin{aligned} \frac{d}{dy}M(y) &= \frac{d}{dy}\Sigma_{i=1}^n \int_0^y (y-m) f_{D_i}(m)\,dm \\ &= \Sigma_{i=1}^n \int_0^y f_{D_i}(m) dm \\ &= \Sigma_{i=1}^n F_{D_i}(y). \end{aligned} \quad (4)$$

The two integral terms in (2) can be differentiated as follows,

$$\begin{aligned} &\frac{d}{dy}\int_{M(y)}^{\infty} \Big[x - M(y)\Big] f_N(x)\,dx \\ &= \int_{M(y)}^{\infty} \frac{\partial}{\partial y}\Big[x - M(y)\Big]\, f_N(x) dx \\ &= \int_{M(y)}^{\infty} \Big[-\frac{\partial}{\partial y} M(y)\Big]\, f_N(x) dx \\ &= -\int_{M(y)}^{\infty} \Sigma_{i=1}^n F_{D_i}(y)\, f_N(x) dx \end{aligned} \quad (5)$$

and

$$\begin{aligned} &\frac{d}{dy}\Sigma_{i=1}^n \int_y^{\infty} (m-y) f_{D_i}(m)\,dm \\ &= -\Sigma_{i=1}^n \int_y^{\infty} f_{D_i}(m) dm \\ &= \Sigma_{i=1}^n F_{D_i}(y) - n. \end{aligned} \quad (6)$$

As a consequence, we find

$$\begin{aligned} &\frac{d}{dy}C(y) \\ &= n(c-b) + \Big[-p\int_{M(y)}^{\infty} f_N(x) dx + b\Big]\, \Sigma_{i=1}^n F_{D_i}(y) \\ &= n(c-b) + \Big(b - p\,(1 - F_N(M(y)))\Big)\, \Sigma_{i=1}^n F_{D_i}(y). \end{aligned}$$

The second derivative satisfies

$$\begin{aligned} &\frac{d^2C(y)}{dy^2} \\ &= \frac{d}{dy}\Big[\Big(b - p(1 - F_N(M(y))\Big)\, \Sigma_{i=1}^n F_{D_i}(y)\Big] \geq 0 \end{aligned} \quad (7)$$

as the functions $b - p(1 - F_N(M(y))$ and $\Sigma_{i=1}^n F_{D_i}(y)$ are positive and increasing.

This shows that, for a given $\tilde{D}$ value, as in standard newsboy formulations, the cost function $C(y)$ is differentiable and convex with respect to y and the optimal solution $y^* \in R$ satisfies

$$\Big(b - p\int_{M(y^*)}^{\infty} f_N(x) dx\Big)\, \Sigma_{i=1}^n F_{D_i}(y^*) = n(b-c). \quad (8)$$

If the solution has to be an integer, one has to consider the optimization problem

$$\begin{aligned} \min\, C(y) \;\; &= \;\; n\,c\,y + p\int_{M(y)}^{\infty} [x - M(y)] f_N(x)\,dx \\ &\quad +b\,\Sigma_{i=1}^n \int_y^{\infty} (m-y) f_{D_i}(m)\,dm \\ &\quad s.t. \quad y \in N. \end{aligned} \quad (9)$$

Clearly, in this simple case the corresponding optimal integer solution is either given by $\lfloor y^* \rfloor$ or by $\lceil y^* \rceil$, with y^* satisfying (8).

3. Numerical Approaches

The objective of this paper is to experiment different solution procedures for the model with a random global workload $\tilde{D}$,

- in section 3.1, we propose a deterministic-type approximation, based on the average workload $E(\tilde{D})$,
- in section 3.2, we develop a stochastic programming type approach, based on the $\tilde{D}$ discrete probability distribution $G(\cdot)$ from interval $[\tilde{D}_{min}, \tilde{D}_{max},]$,
- in section 3.3, we consider a robust approach, which consists of optimizing the capacity with respect to the worst case in the uncertainty set of $\tilde{D}$.

3.1. Average-based approximation

In this simple case, we approximate $\tilde{D}$ by its expected value, $\overline{\tilde{D}} = E(\tilde{D}) = \Sigma_{k=1}^K p_k \tilde{D}_k$. In order to numerically solve the model, we discretize the probability distributions of the required agents number for replying calls. We consider a discrete probability distribution with K_{D_i} states, denoted D_{ij} with the associated probability masses $p_{D_{ij}}$. Along the same lines, the continuous probability distribution characterizing the required agents number for dealing with emails is discretized into a discrete probability distribution with K_N states, denoted N_s, with the associated probability masses p_{Ns}.

The optimization problem can then be reformulated as

$$\min C_1(y) = n\,c\,y + \tag{10}$$
$$p \sum_{s=1}^{K_N} p_{Ns} \left(N_s - (\sum_{i=1}^{n} \sum_{j=1}^{K_{Di}} p_{Dij}\, M_{ij}^{+}) \right)^{+}$$
$$+b \sum_{i=1}^{n} \sum_{j=1}^{K_{Di}} p_{Dij}\, M_{ij}^{-}$$

s.t.

$$M_{ij} = y - D_{ij},$$
$$M_{ij} = M_{ij}^{+} - M_{ij}^{-},$$
$$M_{ij}^{+}, M_{ij}^{-} \geq 0, \forall i, j.$$
$$y \in N.$$

3.2. A direct stochastic approach

We recall that it is assumed that $\tilde{D}$ follows the discrete probability distribution $(p_k, \tilde{D}_k)$, for $k = 1, ..., K$. For each $\tilde{D}_k$ value, the required agent number for replying calls, D_{ik}, follows a probability distribution with mean $\alpha_i\, \tilde{D}_k$. If we discretize the distributions of the variables D_{ik} and N_s as described in the previous section, the optimization problem with a random $\tilde{D}$ parameter can reformulated as

$$\min C_2(y) = \sum_{k=1}^{K} p_k\, C_{\tilde{D}_k}(y) \tag{11}$$

with

$$C_{\tilde{D}_k}(y) = nc\,y$$
$$+p \sum_{s=1}^{K_N} p_{N_s} \left(N_s - (\sum_{i=1}^{n} \sum_{j=1}^{K_{Di}} p_{Dikj}\, M_{ikj}^{+}) \right)^{+}$$
$$+b \sum_{i=1}^{n} \sum_{j=1}^{K_{Di}} p_{Dikj}\, M_{ikj}^{-}$$

s.t.

$$M_{ikj} = y - D_{ikj},$$
$$M_{ikj} = M_{ikj}^{+} - M_{ikj}^{-},$$
$$M_{ikj}^{+}, M_{ikj}^{-} \geq 0, \forall i, j, k.$$
$$y \in N.$$

3.3. A robust approach

In real-life applications, stochastic programming can suffer from dimensionality problems or from the assumption of full knowledge of the demand distribution (see [18, 20]). Robust optimization is an attractive alternative choice. It takes into account uncertainty without assuming a specific distribution, while remaining highly tractable and providing insight into the corresponding optimal policy. It also allows adjustment of the level of robustness of the solution to trade off performance and protection against uncertainty (see [16, 17]).

In our particular case, the robust approach is taken with respect to the $\tilde{D}$ parameter and amounts to

$$\min_{y} \max_{\tilde{D} \in \{\tilde{D}_1, ..., \tilde{D}_K\}} C_3(y) = n\,c\,y +$$
$$p \sum_{s=1}^{K_N} p_{Ns}\, (N_s - (\sum_{i=1}^{n} \sum_{j=1}^{K_{Di}} p_{Dij}\, M_{ij}^{+}))^{+}$$
$$+b \sum_{i=1}^{n} \sum_{j=1}^{K_{Di}} p_{Dij}\, M_{ij}^{-} \tag{12}$$

s.t.

$$M_{ij} = y - D_{ij},$$
$$M_{ij} = M_{ij}^{+} - M_{ij}^{-},$$
$$M_{ij}^{+}, M_{ij}^{-} \geq 0, \forall i, j.$$
$$y \in N.$$

From the cost function structure, it is easily seen that the variables D_i are increasing functions of $\tilde{D}$. As a consequence, the cost function is an increasing function of $\tilde{D}$, irrespective of the y value. It is then easy to see that the worst case always corresponds to $\tilde{D} = \tilde{D}_K = \tilde{D}_{max}$.

4. Numerical analysis of the three solutions

In order to compare the relative performances of the three solution procedures, we consider a simulation approach.

The horizon is $n = 8$ periods with the unit regular salary $c = 10$ and an unit overtime salary $p = 15$. $\tilde{D}$ follows an uniform distribution in the interval $[16, 24]$. Given the value of $\tilde{D}$ and the constant value α_i associated to period i, it is easy to get $\bar{D}_i$ which is the mean of the Gaussian distribution of D_i. The variances of the probability distributions of $D_i (i = 1...n)$ are fixed to 49. N, the required number of the agents for dealing with emails is assumed to follow a Gaussian probability distribution (with mean equal to 80 and variance equal to 400).

Firstly we get the optimal policy y^* and its objective value for each of the three solution procedures.

Then, we randomly generate $l = 10000$ scenarios $\tilde{D}_l$ for $\tilde{D}$, picked from the interval $[16, 24]$. After that, we randomly generate $K_{D_i} = 100$ scenarios, for the values $D_{ij,l} (j = 1, ..., K_{D_i})$ of the required agents number for replying to calls in the successive time intervals. These random values are generated according to Gaussian probability distributions with means evaluated by (1) and variance fixed to 49. Similarly, we generate $K_N = 100$ scenarios for the values $\hat{N}_s (s = 1, ..., K_N)$ of the required number of agents for dealing with emails. These random values are generated according to a Gaussian probability distribution (with mean equal to 80 and variance equal to 400).

For each simulated $\tilde{D}$ value, the three optimal solutions are put into the following cost evaluation model and we get the average of their numeric results

$$C'(y^*) = n\,c\,y^* +$$

$$p\sum_{s=1}^{K_N} 1/K_N(\hat{N}_s - (\sum_{i=1}^{n}\sum_{j=1}^{K_{D_i}} 1/K_{D_i}\, M_{ij}^{+})^{+}$$

$$+b\sum_{i=1}^{n}\sum_{j=1}^{K_{D_i}} 1/K_{D_i}\, M_{ij}^{-} \quad (13)$$

s.t.

$$M_{ij} = y^* - D_{ij},$$
$$M_{ij} = M_{ij}^{+} - M_{ij}^{-},$$
$$M_{ij}^{+}, M_{ij}^{-} \geq 0.$$
$$y \in N.$$

As a benchmark solution, we also have compared these numerical results with a perfect information model, i.e. a model where the value of the workload $\tilde{D}$ is known before the variable y is optimized. In the simulation setting, for each $\tilde{D}_l$ scenario the following problem has thus to be solved and an average of the costs for these perfect information solutions are calculated

$$\min C_l(y) = n\,c\,y_l +$$
$$p\sum_{s=1}^{K_N} p_{Ns}\left(N_s - (\sum_{i=1}^{n}\sum_{j=1}^{K_{Di}} p_{Dij}\, M_{ij,l}^{+})\right)^{+}$$
$$+b\sum_{i=1}^{n}\sum_{j=1}^{K_{Di}} p_{Dij}\, M_{ij}^{-} \quad (14)$$

s.t.

$$M_{ij,l} = y - D_{ij,l},$$
$$M_{ij,l} = M_{ij,l}^{+} - M_{ij,l}^{-},$$
$$M_{ij,l}^{+}, M_{ij,l}^{-} \geq 0, \forall i, j, l.$$

The simulation results are displayed in the following tables, which displays, for different values of b and different width of the interval, the optimal solutions (OPS) y^*, the numeric expected cost and the standard derivation of the costs estimated by simulation for the different approaches: Average-based approximation (A), stochastic approach(S) and robust approach(R).

case 1: α_i=1,i=1..n

In the medium-sized interval $[16, 24]$, when the penalty rate b takes reasonable values, since the stochastic approach performances the best between all the three approaches. When b increases, stochastic approach has the tendency to protect the bad cases, the shortfall penalty decreases but the staffing salary cost increases. Robust approach keeps to be conservative with a high average cost and a small standard derivation.

In the small sized interval the average-based approximation and stochastic approach have the same performance and robust approach keeps to be conservative. When the interval size is big, both stochastic and robust approaches tend to add staffing and average-based approximation has the lowest average cost. Robust approach works better when the interval is big and the penalty rate increases.

Tab. 1: table(interval[16,24],α_i =1)

b	approaches	OPS	Simulated average cost	std
b=30	PI	29.10	2458.7	184.80
	A	30	2530.0	144.64
	S	29	2517.6	188.09
	R	33	2659.3	34.99
b=50	PI	29.37	2467.7	186.33
	A	30	2539	153.76
	S	30	2539	153.76
	R	34	2727.6	16.73
b=500	PI	29.40	2479.7	183.42
	A	35	2801.4	4.51
	S	36	2880	0.21
	R	39	3120	0

Tab. 2: table(interval[19,21],b=50

approches	OPS	Simulated average cost	std
PI	29.78	2506.4	70.25
A	30	2516.2	64.15
S	30	2516.2	64.15
R	34	2536.2	42.57

case 2: α_i ≠1, i=1..n

Since the arrival rate during the whole day is no-stationary, some papers (see [4]) provide the statistic analysis of the call volumes for different periods during the whole day and we can then define α_i as $[0.75, 1.40, 1.2, 1.25, 1.1, 0.9, 0.75, 0.65]$ for example. As described in the precedent sections, we firstly find out the optimal solution by different approaches then test their effect.

It's shown in table 4 that the average-based approach works better than the others no matter b takes which value. The reason is that D_i floats a lot in different periods. All the approaches try to protect the higher D_i in order to avoid overtime cost and under staffing penalty. This leads to over staffing for all the approaches and the average-based approach has the least staff thus it costs the least.

This indicates that if we consider the no-stationary workload during the day ,we need to employee the staff by several shifts instead of by a single shift y.

5. Conclusion

In this paper, we have developed a call center staffing problem simultaneously characterized by a significant workload uncertainty and agent flexibility. This problem has been formulated as a newsboy- type model. The convexity property of this model has been analyzed. Three different ap-

Tab. 3: table(interval[10,30],b=50)

approches	OPS	Simulated average cost	std
PI	28.89	2483.6	488.14
A	30	2771.6	497.57
S	32	2785.2	43.27
R	40	3203.1	11.22

Tab. 4: table(interval[16,24],$\alpha_i \neq 1$)

b	approaches	optimal y*	simulated average cost	std
b=30	PI	29.74	2498.2	194.54
	A	30	2570.6	181.88
	S	30	2570.6	181.88
	R	34	2736.0	28.10
b=50	PI	30.0	2523.6	202.04
	A	31	2614.9	164.54
	S	31	31 2614.9	164.54
	R	35	2811.1	20.42
b=500	PI	32.54	2756.3	300.68
	A	38	3049.6	25.72
	S	39	3122.3	8.29
	R	43	3440	0

proaches have been proposed to solve this problem. The relative performances of these solution techniques have been numerically illustrated.

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