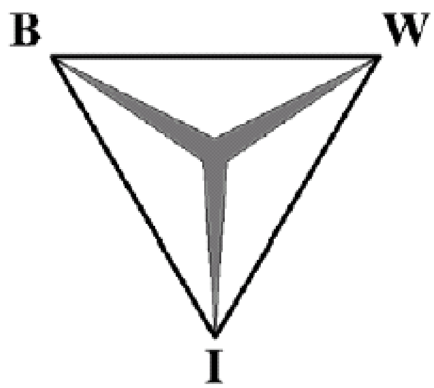
Contracts in outsourcing



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Preface

The BMI-paper is one of the final compulsory subjects of my study Business Mathematics and Informatics (BMI) at the *vrije* Universiteit in Amsterdam. The objective is to investigate the available literature in reference to a topic related to at least two out of the three fields (Mathematics, Economics and Computer Science) integrated in the study.

The subject for this paper was set in consultation with dr. S. Bhulai of the Stochastics group of the Faculty of Sciences. Hereby, I would like to thank my supervisor, dr. S. Bhulai. Despite his busy schedule he made time to guide me. I am thankful for his advice and comments.

Hima Chander
September 2006

Summary

Outsourcing is a business process term for hiring an external entity, an independent contractor (subcontractor), to do a specific task or tasks for an organization in which the organization either does not have the time or the expertise to do on their own.

Outsourcing consists of two parties, the user company and the subcontractor, who have conflicting interests. For example, in call center outsourcing, the user company wants to make profit and give optimal service to his customers and the subcontractor wants to make profit with minimal effort, resulting in possibly poor service quality. Taking into account the interests of both parties, coordination is necessary. The reason is that by coordination, the outsourcing supply chain can achieve the maximal profit possible. Then, with a proper contract, the total profit can be split between the user company and the subcontractor such that both parties are better off than when the outsourcing supply chain is not coordinated.

In the first part of this paper, contracts in call center outsourcing are discussed. The contracting issues in an outsourcing supply chain consisting of a user company and a call center, that does outsourcing work for the user company, is described. Here, the call center is modeled as an G/G/s queue with customer abandonments. Each call has a revenue potential, and the call center's service quality is modeled by the percentage of calls served and resolved. The call center makes two strategic decisions: how many agents to schedule and how much effort to exert to achieve the service quality. Of interest are the contracts which the user company can use to enforce the call center to both staff and exert effort at levels that are optimal for the outsourcing supply chain.

In a piece-meal (PM) contract, also called wholesale contract or a linear contract, the user company pays the call center a unit rate *b* for each call served. A piece-meal type of service contract can coordinate the staffing level of the call center. However, it is unable to coordinate the effort level to achieve system-optimality.

In a pay-per-call-resolved (PPCR) contract, the user company pays the call center for each call served and resolved. The PPCR contract induces the call center to staff enough people so that no calls are lost, because its revenue is directly tied to the volume of calls served. Furthermore, because the call center gets rewarded only when a call is served and resolved, it has an incentive to exert effort to increase the service quality and increase the volume of calls resolved. However, a quick comparison with the optimal effort level under the integrated system reveals that the effort level under the PPCR contract is still not sufficiently high to coordinate the call center outsourcing supply chain.

In the pay-per-call-resolved with cost sharing (PPCR+CS) contract, the user company shares the call center's staffing and effort costs, while the call center shares the user company's loss-of-goodwill cost. This coordination achieves the maximal profit in the outsourcing supply chain. Because an arbitrary split is possible, there is a way to share the total profit in such a way that both parties are better off than when the outsourcing supply chain is not coordinated.

In the partnership, contract the user company first needs to share the staffing costs of the call center. Moreover, in order to induce the call center to spend more effort to improve service quality to meet system-optimality, the call center's profit margin needs to be adjusted to match that of the whole outsourcing supply chain. This requires both parties to share all of their cost information.

In order to achieve coordination, the call center and the outsourcing company need to collaborate closely. These contracts suggest that close attention has to be paid to service quality and its contractibility in seeking call center outsourcing.

The second part of this paper discusses contracts in the inventory supply chain. Of the numerous supply chain models, the single location base stock model and the two location base stock model are discussed in the second part of this paper.

The single location base stock model is a stochastic demand model in which the retailer receives replenishments from a supplier after a constant lead time. Coordination requires that the retailer chooses a “higher action”, which in this model is a larger base stock level. The cost of this higher action is more inventory on average, but the supplier can verify the retailer’s inventory and therefore share the holding costs of carrying more inventory with the retailer.
In the single location model the supplier coordinates the supply chain with a contract that has linear transfer payments based on the retailer’s inventory and backorders. Coordination with the infinite horizon base stock model is qualitatively the same as coordination in the single period newsvendor model. In particular, coordination via a holding cost and backorder cost transfer payment is similar to coordination via a buy-back contract.

The two location base stock model makes the supplier hold inventory, although at lower holding costs than the retailer. Whereas the focus in the single location base stock model is primarily on coordinating the downstream actions, in this model the supplier’s action also requires coordination, and that coordination is non-trivial. To be more specific, in the single location model the only critical issue is the amount of inventory in the supply chain, but here the allocation of the supply chain’s inventory between the supplier and the retailer is important as well.
In the two location base stock model decentralized operations always leads to sub-optimal performance, but the extent of the performance deterioration (i.e., the competition penalty) depends on the supply chain parameters. If the firms’ backorder costs are similar, then the competition penalty is often reasonably small. The competition penalty can be small even if the supplier does not care about customer service because her operations role in the supply chain may not be too important. To coordinate the retailer’s actions, the firms can agree to a pair of linear transfer payments that function like the buy-back contract in the single period newsvendor model. To coordinate the supplier’s action a linear transfer payment based on the supplier’s backorders is used. With these contracts the optimal policy is the unique Nash equilibrium.

The contracts of the single and two location base stock model resemble most the pay-per-call-resolved with cost sharing (PPCR+CS) contract of the call center. Out of this it can be concluded that a good contract rewards and penalizes.

Table of contents

1 Introduction

This paper discusses coordination of contracts in outsourcing, in particular, contracts in call center outsourcing and contracts in inventory supply chain outsourcing. The term "outsourcing" became better known largely because of a growth in the number of high-tech companies in the early 1990s that were often not large enough to be able to easily maintain large customer service departments of their own [1]. Outsourcing or contracting out is a business process term for hiring an external entity, an independent contractor (subcontractor), to do a specific task or tasks for an organization in which the organization either does not have the time or the expertise to do on their own. Outsourcing involves transferring a significant amount of management control and decision-making to the outside supplier.

An increasing number of companies are transferring part of their work to other countries; this is called offshore outsourcing (offshoring). Philips had already outsourced part of their work to other countries in the eighties. Many companies have gained significant negative publicity for their decisions to use outsourced labor for customer service and technical support. In fact, for some companies, their outsourcing strategies backfired and they had to re-evaluate or even abort their outsourcing mission. One of the most prominent customer complaints is that the outsourced staff delivers a lower quality of service to customers.

Outsourcing consists of two parties, the user company and the subcontractor, who have conflicting interests. For example, in call center outsourcing, the user company wants to make profit and give optimal service to his customers and the subcontractor want to make profit with minimal effort, resulting in potentially poor service quality. Taking into account the interests of both parties, coordination is necessary. The reason is that by coordination the outsourcing supply chain can achieve the maximal profit possible. Then, with a proper contract the total profit can be split between the user company and the subcontractor such that both parties are better off than when the outsourcing supply chain is not coordinated. In other words, with a coordinating contract both parties can 'make a bigger pie', and share it in such a way that each gets a bigger piece than before. A contract determines the legal parameters of the service and the responsibilities of each party and must pay attention to the combined value. The definition of the problem in this paper can be written as:

How to coordinate the different players in the chain (the user company and the subcontractor) to achieve system optimality?

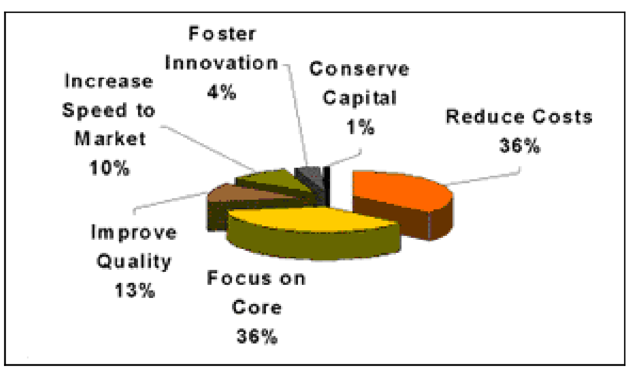

Figure 1: Top reasons for outsourcing [2]

In the first chapter of this paper, contracts in call center outsourcing are discussed. The contracting issues in an outsourcing supply chain consisting of a user company and a call center, that does outsourcing work for the user company, is described. Here, the call center is modeled as an G/G/s queue with customer abandonments. Each call has a revenue potential, and the call center's service quality is modeled by the percentage of calls served and resolved. The call center makes two strategic decisions: how many agents to schedule and how much effort to exert to achieve the service quality.

Of interest are the contracts which the user company can use to enforce the call center to both staff and exert effort at levels that are optimal for the outsourcing supply chain. Two commonly used contracts are analyzed first: the piece-meal and the pay-per-call-resolved contracts. Although they can coordinate the staffing level, they result in service quality that is below system optimal. Then, two contracts are proposed that can coordinate both.

The following chapter gives a description of contracts in inventory supply chain outsourcing. An optimal supply chain performance requires the execution of a precise set of actions. Unfortunately, those actions are not always in the best interest of the individual members in the supply chain, i.e., the supply chain members are primarily concerned with optimizing their own objectives, and that self-serving focus often results in poor overall performance. However, optimal performance is achievable if the firms coordinate by contracting on a set of transfer payments such that each firm's objective becomes aligned with the supply chain's objective. Of the numerous supply chain models, the single location base stock model and the two location base stock model are discussed. The single location base stock model is a stochastic demand model in which the retailer receives replenishments from a supplier after a constant lead time. Coordination requires that the retailer chooses a "higher action", which in this model is a larger base stock level. The cost of this higher action is more inventory on average, but the supplier can verify the retailer's inventory and therefore share the holding costs of carrying more inventory with the retailer. The two location base stock model makes the supplier hold inventory, although at lower holding costs than the retailer. Whereas the focus in the single location base stock model is primarily on coordinating the downstream actions, in this model the supplier's action also requires coordination, and that coordination is non-trivial. To be more specific, in the single location model the only critical issue is the amount of inventory in the supply chain, but here the allocation of the supply chain's inventory between the supplier and the retailer is important as well. This chapter is followed by contracts in general. Finally, Chapter 5 gives a summary and conclusions on the findings.

2 Contracts in call centers

2.1 Introduction

A growing number of companies are moving their call center operations offshore. The overhead costs of customer service are typically less when outsourcing is used, leading to many companies to close their in-house customer relations departments and outsourcing their customer service to third party call centers. The logical extension of these decisions was the outsourcing of labor overseas to countries with lower labor costs. Due to this demand, call centers have sprung up in Canada, China, Eastern Europe, India, Sri Lanka, Israel, Ireland, Pakistan, the Philippines, and even in the Caribbean.

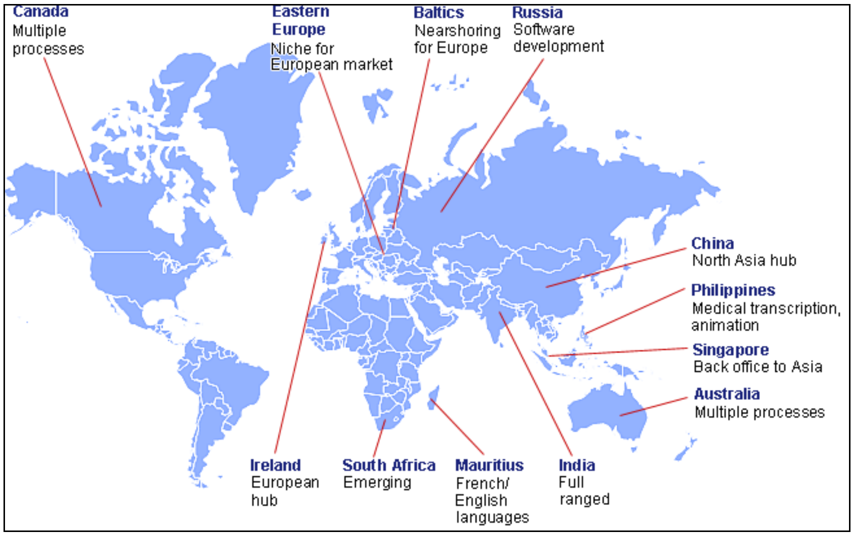

Figure 2: Offshoring destinations worldwide [3]

Outsourcing often brings immediate cost-savings. But companies should keep in mind that there can also be some 'hidden costs' in outsourcing. Service quality cost is one of them. The call center is largely invisible to the end customers. Therefore, when the call center under-performs, it is the user company that suffers customer backlash. For this reason, the service quality of call centers needs to be taken into account when making outsourcing decisions, and needs to be carefully managed by the user company. Service quality is measured by the percentage of calls that are served to the customers' satisfaction. Because each service encounter is unique, it may not be possible to contract directly on service quality. Therefore it is important to find a contract that can induce the call center to exert effort to provide high service quality.

This chapter will answer the following questions:
1. How effective are the call center outsourcing contracts commonly observed in practice?
A piece-meal contract, a pay-per-call-resolved contract, and a pay-per-call-resolved plus cost sharing contract are among the commonly observed contracts in call center outsourcing. Are they capable of inducing system-optimal staffing levels and efforts (to improve service quality) of the call center?
2. How to achieve coordination, i.e., system-optimal staffing levels and service quality, with a contracting mechanism? In practice, there have been different forms of observed call center outsourcing agreements. Some companies take a 'hands-off' approach, while other companies form partnerships with their outsourcers, sharing set-up and operating costs. What form of outsourcing should one choose?

2.2 Model parameters

Consider a call center outsourcing supply chain that consists of two separately managed companies: a call center (the outsourcer) and a user company. The call center considered here is typically large (employing hundreds of agents), and is modeled as a multi-server queuing system (G/G/s) with customer abandonments. Costumers arrive to the call center with rate λ, and the call center staffs *s* servers (i.e., agents) each with service rate μ. Customers who are not immediately served upon arrival enter a waiting queue. They are impatient, and leave the system after a random amount of time, which has a continuously differentiable Probability Density Function (PDF) *f* and a Cumulative Distribution Function (CDF) *F*. The waiting cost rate is c_w, and for each customer that abandons, there is a cost of c_a.

For the customers who are eventually served, there is a fraction *p* of them whose calls are satisfactorily resolved. For each call that is resolved, a revenue *r* is earned. For the rest, a fraction $1 - p$ of the calls (served but not resolved), there is a loss of goodwill c_g for each call.

We assume the call resolution probability *p* is a non-negative continuous random variable with support (0, 1) and CDF G (*p*). We assume that *p* is influenced by the call center's effort *e*, which may be unobservable or unverifiable by the user company. The expected value of the percentage calls resolved, denoted as $\overline{p}(e)$ for a given effort level *e,* is then

$$\overline{p}(e) = \int_0^1 \left[1 - G(p \mid e)\right] dp. \tag{1}$$

Effort is costly to the call center with a unit rate of c_e. Hiring staff also costs the call center money with a unit rate of c_s.

List of symbols:

λ	Arrival rate
μ	Service rate
p(e)	Percentage of calls resolved as a function of effort
s	Number of servers/agents
T(s)	Number of costumers served in steady state
W(s)	Waiting time in steady state
Q(s)	Number of costumers waiting (queue length)
L(s)	Number of abandonments in steady state
r	Unit revenue
c_s	Unit staffing cost
c_e	Unit effort cost
c_a	Abandonment cost
c_w	Unit waiting cost
c_g	Loss of goodwill from calls served but not resolved

2.3 An integrated outsourcing supply chain

We first look at the integrated outsourcing supply chain where the call center and the user company are owned by the same company, and makes the centralized decision on the optimal staffing level and the effort level. By staffing *s* agents and exerting an effort *e*, the system's total profit is:

$$\pi^I(s,e) = \underbrace{r\overline{p}(e)T(s)}_{revenue} - \underbrace{c_s\mu s - c_e e}_{staffing\ \&\ effort\ \cos t} - \underbrace{c_a L(s)}_{abandonment\ \cos t} - \underbrace{c_w \lambda W(s)}_{waiting\ \cos t} - \underbrace{c_g\left(1-\overline{p}(e)\right)T(s)}_{loss\ of\ goodwill\ \cos ts}. \tag{2}$$

We are assuming that the effort costs are not dependent on the number of servers hired (the staffing level). The integrated system solves the following profit maximization problem:

$$\max_{s,e} \pi^I(s,e). \tag{3}$$

It is hard to find exact solutions to G/G/s+M queuing systems, which prohibits one to analyze the optimization problem (3). Therefore, one can use fluid approximations for general G/G/s + G models. A fluid approximation allows for analytical tractability, from which one can gain important managerial insights. In the fluid approximation, the number of busy servers in steady state is the minimum of the arrival rate and the number of servers: $T(s) = \min(\lambda, \mu s)$.

The abandonment rate is

$$L(s) = (\lambda - \mu s)^+ = \lambda - T(s), \tag{4}$$

where $(x)^+ = \max\{0, x\}$.

Furthermore, by using a Taylor series approximation of the CDF F around t = 0, and with a mild assumption that f (0) ≠ 0, one can obtain a relatively simple relationship on steady state waiting time:

$$W(s) = \frac{L(s)}{f(0)\lambda}. \tag{5}$$

This suggests that in the fluid approximation the waiting time is proportional to the abandonment rate. Of course, these relationships do not all apply to the original stochastic model, but they do capture the first-order effects of the queuing system. Plugging (1), (4), and (5) into (2), and taking the expectation we have

$$\begin{aligned}\pi^I(s,e) &= r\overline{p}(e)(\lambda - L(s)) - c_s \mu s - c_e e - c_a L(s) - \frac{c_w}{f(0)} L(s) - c_g(1-\overline{p}(e))(\lambda - L(s)) \\ &= \left[r\overline{p}(e) - c_g(1-\overline{p}(e))\right]\lambda - \left[r\overline{p}(e) + c_a + \frac{c_w}{f(0)} - c_g(1-\overline{p}(e))\right]L(s) - c_s \mu s - c_e e\end{aligned} \tag{6}$$

To avoid the situation where the expected profit is negative, we assume that $r\overline{p}(0 >)c_s + (1-\overline{p}(0))c_g$.

That is, the unit revenue is sufficiently large compared to the costs. This way, the integrated system has a positive profit margin by operating its call center. Since L(s) is piece-wise linear and convex in *s*, the expected profit also behaves nicely with respect to *e*. By solving the first-order conditions of (6) (the proof of it can be seen in the Appendix) the profit-maximizing staffing level s^I and effort e^I for the integrated system are such that:

$$s^I = \lambda / \mu, \tag{7}$$

$$\overline{p}'(e^I) = \frac{c_e}{(r+c_g)\lambda},\ if\ \frac{c_e}{(r+c_g)\lambda} < p_0';\ e^I = 0\ otherwise. \tag{8}$$

The solution in (7) indicates that the integrated system would staff enough servers to meet demand. Therefore, in steady state there is no customer abandonment or waiting. On the other hand, the solution in (8) indicates that the system optimally balances the cost and benefit of increasing effort. For example, when the revenue rate r increases, it calls for an increase in effort because its expected benefit from more resolved calls becomes larger. This is true for the goodwill cost c_g and customer arrival rate λ as well. However, when the cost of effort c_e increases, the call center will exert less effort. If the ratio $\frac{c_e}{(r+c_g)\lambda}$ is over the threshold p_0' (recall that $p_0' = \overline{p}'(e=0)$), the call center would exert no effort.

At optimality, the total outsourcing supply chain profit is

$$\pi^I\left(s^I, e^I\right) = \left[r\overline{p} + c_g\,\overline{p} - c_g - c_s\right]\lambda - c_e e^I . \qquad (9)$$

With this integrated model as a benchmark, the decentralized case of call center outsourcing will now be investigated.

2.4 Decentralized system of outsourcing

In the decentralized outsourcing supply chain, the user company and the call center are two independent entities. The user company offers a contract to outsource its call center functions to the call center. If the call center accepts the contract, it chooses its staffing level as well as its effort level. The call center serves incoming calls from the customers of the user company, and gets paid by the user company. The user company, on the other hand, receives revenues from those calls served and resolved by the call center. Because the call center is invisible to the customers when they call, it is the user that bears the negative consequences of customer abandonments, customer dissatisfaction from waiting, and calls not being resolved. The amount the user company pays to the call center is denoted by Ψ. Ψ is specified in the contract and could be a function of a number of variables such as T, W, or L. The task of contract design is to determine what factors determine Ψ, and how. From (2), the total expected outsourcing supply chain profit is decomposed into two parts: $\pi^c(s,e)$ for the call center and $\pi^u(s,e)$ for the user company:

$$\pi^c(s,e) = E[\Psi] - c_s\ \mu s - c_e e,$$
$$\pi^u(s,e) = r\overline{p}(e)T(s) - c_a L(s) - c_w\ \lambda W(s) - c_g\left(1-\overline{p}(e)\right)T(s) - E[\Psi].$$

The goal is to identify those contracts that can achieve two objectives under the decentralized setting: (1) achieve a system-optimal staffing and effort level, and (2) achieve an arbitrary split of the system profit between the user company and the call center. The advantage of coordination is that the outsourcing supply chain can achieve the maximum profit possible. Then, with a proper contract, the total profit can be split between the user company and the call center such that both parties are better off than when the outsourcing supply chain is not coordinated.

The key here is to see how the call center makes staffing and effort decisions in the decentralized setting as compared to the integrated outsourcing supply chain. It obviously depends on the specific form of the contract Ψ. For example, the simplest form of Ψ is a fixed payment, i.e., Ψ = σ, where σ is a constant, although it is easy to see that a fixed payment will not induce the call center to staff or exert effort at the system-optimal level. In the next sections, four specific forms of contracts will be described: a piece-meal contract, a pay-per-call-resolved contract, a pay-per-call-resolved with cost sharing contract, and a partnership contract.

2.4.1 Piece-meal (PM) contract

A piece-meal contract, also called a wholesale contract or a linear contract, is the most commonly observed contract in the industry. The user company pays the call center a unit rate *b* for each call served:

$$\Psi = bT(s). \qquad (10)$$

In this contract, the user company only needs to decide *b* and the payment depends only on *T(s)*. In order for the contract to be acceptable to both parties, we have $c_s < b < r$. Under this contract, the profits for both the outsourcing supply chain parties are:

$$\pi^c_{PM}(s,e) = bT(s) - c_s\ \mu s - c_e e,$$
$$\pi^u_{PM}(s,e) = r\overline{p}(e)T(s) - c_a L(s) - c_w\ \lambda W(s) - c_g\left(1-\overline{p}(e)\right)T(s) - bT(s) \qquad (11)$$

A piece-meal type of service contract can coordinate the staffing level of the call center. However, it is unable to coordinate the effort level to achieve system-optimality.

When the contract links the call center's income to the number of calls served, the call center has an incentive to staff adequately. Unfortunately, the piece-meal contract does not link the call center's income to the volume of calls resolved. As a rational response, the call center has no incentive to exert any effort to increase its service quality. Consequently, the outsourcing supply chain reaches optimality only along one of the two important dimensions; it is not fully coordinated.
There are some managerial insights that one can draw from the result above. First, it indicates that if we look at call centers on a grand scale, where the deterministic aspects of a fluid approximation dominate the stochastic variations at the detailed queue level, then a linear piece-meal contract can work well in terms of coordinating the staffing level. Plus, it has a simple form and is easy to implement. This might explain the popularity of this contract in call center outsourcing.
However, such a contract does not address the service quality in outsourcing. Therefore, when service quality is uncertain and depends on the call center's private actions (i.e., efforts), the user company needs more than just a piece-meal contract to achieve the service quality it desires.

2.4.2 Pay-per-call-resolved (PPCR) contract

In order to take the call center's effort into account, a natural extension of the piece-meal contract is to pay the call center for each call served and resolved:

$$\Psi = bp(e)T(s). \tag{12}$$

Unlike in the peace-meal contract, here b is the unit rate the user pays the call center for each call served and resolved. The payment to the call center now depends on *p(e)T(s)*, which is determined by both *s* and *e*.

Under this contract, the profits for both outsourcing supply chain parties are:

$$\begin{aligned} \pi^{c}_{PPCR}(s,e) &= b\overline{p}(e)T(s) - c_s\,\mu s - c_e e, \\ \pi^{u}_{PPCR}(s,e) &= r\overline{p}(e)T(s) - c_a L(s) - c_w\,\lambda W(s) - c_g\left(1-\overline{p}(e)\right)T(s) - b\overline{p}(e)T(s). \end{aligned} \tag{13}$$

The pay-per-call-resolved contract can not only coordinate staffing level, but also motivate the call center to exert effort to improve service quality.

Similar to the peace-meal contract, the pay-per-call-resolved contract induces the call center to staff enough people so that no calls will be lost, because its revenue is directly tied to the volume of calls served. Furthermore, because the call center gets rewarded only when a call is served and resolved, it has an incentive to exert effort to increase the service quality and increase the volume of calls resolved. However, a quick comparison with the optimal effort level under the integrated system reveals that the effort level under the pay-per-call-resolved contract is still not high enough to coordinate the call center outsourcing supply chain.

Underlying this result is the difference in the profit margin of the integrated system and that of the call center. In the integrated system, by exerting an additional unit of effort (at the marginal cost of c_e), the benefit to the system is the increase in revenue from additionally resolved calls and the reduction in the loss-of-goodwill costs, $(r + c_g)\lambda$. For the call center under the PPCR contract, however, an additional unit of effort would only increase the revenue by $b\lambda$, which is less than $(r + c_g)\lambda$. Due to this difference in the profit margin, the call center will under-invest in the effort. This under-investment can be viewed as a 'double marginalization' problem analogous to that observed in an inventory supply chain. It is well known that in an inventory supply chain, 'double marginalization' causes the retailer to order less than the supply chain optimal (with a linear wholesale contract).

Double marginalization occurs when the upstream (manufacturers) and downstream (retailers) markets are not perfectly competitive, and the product is traded with a uniform wholesale price. If a monopoly manufacturer supplies a good to a monopoly retailer and charges the retailer a monopolistic price, the result is double marginalization pricing [6]. Figure 3 provides an illustration.

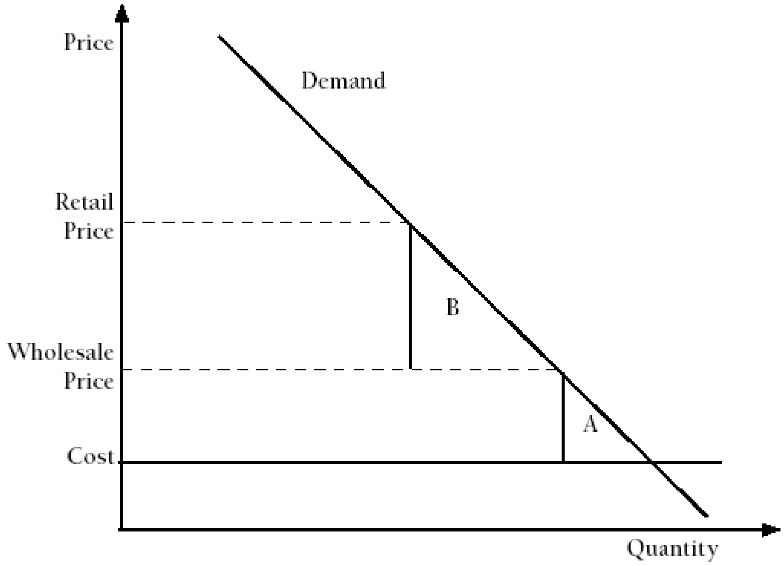

Figure 3: Double marginalization

Both the wholesaler and the retailer mark up the price above their full costs, causing efficiency or "deadweight" losses equal to the triangles A and B in Figure 3. If there is one thing worse than a monopolist, it is two successive monopolists. Monopolists charge their customers a markup above cost. In cases of double marginalization, we have markups on top of markups. The interesting thing is, not only is double marginalization bad for consumers, but actually the firms themselves may end up with lower profits. However, such a problem does not exist in the call center outsourcing supply chain, because the call center, which provides service to end customers, is paid by the user company, and not by the customer. When the profit margin of the call center does not match that of the integrated outsourcing supply chain, the call center would rationally exert less effort, resulting in an inferior service quality, compared to that in an integrated outsourcing supply chain. In other words, in terms of coordinating the call center's effort to improve service quality, a type of 'double marginalization' exists in the outsourcing supply chain as well.

Is there a contract that can fix the 'double marginalization' problem in the outsourcing supply chain? The answer is positive, but the form of this contract will depend on whether effort is contractible. We study both cases below.

2.4.3 Pay-per-call-resolved with cost sharing (PPCR+CS) contract

Many companies already use a 'cost-plus' type of contract, which calls for the sharing of cost information. In these cases, the user company can propose to share a proportion of the call center's costs in order to induce the call center to exert enough effort to coordinate the outsourcing supply chain. Specifically, consider the following contract. The user company modifies the pay-per-call-resolved contract by sharing a fraction $(1-\alpha)$ of the call center's costs, where $\alpha = b/r$. In other words, the call center now pays a proportion α of the total staffing and effort cost: $\alpha(c_s \mu s + c_e e)$. Moreover, the call center pays a penalty of αc_g for each call served but not resolved. In summary, the user company shares the call center's staffing and effort costs, while the call center shares the user company's loss-of-goodwill cost. Therefore, the contractual payment is

$$\Psi = bp(e)T(s) + (1-\alpha)(c_s\ \mu s + c_e e) - \alpha\left[c_g(1-p(e))T(s) + c_a\ L(s) + \frac{c_w}{f(0)}L(s)\right]. \qquad (14)$$

We call this contract a "pay-per-call-resolved plus cost sharing" contract (PPCR+CS). As in the pay-per-call-resolved contract, b is the unit rate for each call served and resolved, and it (or equivalently α) is the only decision variable the user company has to consider in designing the contract.
Under this contract, the outsourcing supply chain parties' profits are:

$$\begin{aligned}\pi^c_{PPCR+CS}(s,e) &= \alpha\left\{\left[r\overline{p}(e) - c_g(1-\overline{p}(e))\right]T(s) - c_s\mu s - c_e\, e - c_a\ L(s) - \frac{c_w}{f(0)}L(s)\right\},\\ \pi^u_{PPCR+CS}(s,e) &= (1-\alpha)\left\{\left[r\overline{p}(e) - c_g(1-\overline{p}(e))\right]T(s) - c_s\mu s - c_e\, e - c_a\ L(s) - \frac{c_w}{f(0)}L(s)\right\}.\end{aligned} \qquad (15)$$

Clearly, under the PPCR+CS contract, the call center's share of the total system profit is α , and the user's share is (1-α). Therefore the call center's incentive is completely aligned with that of the outsourcing supply chain. So it will take the system-optimal actions in staffing and effort, and the supply chain will be coordinated.

By coordination, the outsourcing supply chain can achieve the maximum profit. Because an arbitrary split is possible, there is a way to share the total profit in such a way that both parties are better off than when the outsourcing supply chain is not coordinated.
The PPCR+CS contract seems quite difficult because the two parties need to share information about many cost items. Note, however, that the abandonment and waiting costs in (14), $c_a\ L(s)$ and $\frac{c_w}{f(0)}L(s)$, serve the same purpose: to punish the call center for under-staffing. As long as one of them is in the contract, the contract will continue to coordinate the system. Therefore, by removing either one of them, we can simplify the PPCR+CS contract.
This is not the only cost-sharing contract that can coordinate the supply chain. For example, consider the following contract. The user company shares only the effort cost, $(1-\alpha)c_e e$, but not the staffing cost. It can be shown that if the user company also sets $\alpha = b/(r+c_g)$, then the call center would also be induced to staff and exert efforts at the system-optimal level. However, the drawback of this contract is that it cannot achieve an arbitrary split of profits, because the expected profit of the call center is no longer exactly α proportion of the total system profit.

2.4.4 Partnership contract

Oftentimes the call center's effort cannot be observed or verified. For example, it is often difficult to measure a call center manager's effort in supervising his staff and solving day-to-day operational problems. If a contract relies on the call center to truthfully report its managers' supervision and training effort, then the call center has an incentive to over-state its effort.
In face of this challenge, we propose the following contract, which consists of two parts.

• First, the call center has ro pay a fee to serve each call, excluding the abandoned calls, buts get to keep all the revenue from served and resolved calls. This fee is $(1-\alpha)\left[r\overline{p}(e^I) - c_g(1-\overline{p}(e^I))\right]$, where $\alpha \in [0,1]$ is the only contract parameter the user company needs to decide. The call center must pay a penalty of c_g to the user company for each call served but not resolved.

• Second, the user company shares the call center's staffing and effort costs by paying the call center $(1-\alpha)c_s\,\mu s+(1-\alpha)c_e e^I$. No costs will be shared when the call center staffs no servers (i.e., s = 0).

It is important to note that because the call center's effort can not be observed and verified, all the payments in the contract cannot be based on the call center's real effort level. Instead, they are based on the call center's real staffing level (which can be observed) and the chain-optimal effort level e^I (which can be calculated). Part of the user's payment to the call center, $(1-\alpha)c_e e^I$ is fixed. When α is too small, the fixed payment may be so high that it induces the call center to keep a minimal staffing level and exert no effort. In this case, the sole purpose of the call center's operation is to collect the fixed payment. This solution is clearly impractical. To avoid this trivial situation, we assume in the following discussion that the user will choose a sufficiently large α.
The contract has the flavor of that of a franchise, in the sense that it lets the call center earn revenue directly from the customer, and pays a 'franchise fee' based on call volume. But it is more than a common franchise contract, because the user company also shares part of the cost of the call center. Therefore it is more like a partnership. Under this partnership contract (which we denote as PART), the transfer payment is

$$\begin{aligned}\Psi &= rp(e)T(s)-(1-p(e))c_g T(s)-(1-\alpha)\left[rp(e^I)-c_g\left(1-p(e^I)\right)\right]T(s)\\ &\quad+(1-\alpha)c_s\mu s+(1-\alpha)c_e e^I-\alpha\left(c_a+\frac{c_w}{f(0)}\right)L(s),\end{aligned} \tag{16}$$

and the profits for both parties in the outsourcing supply chain are:

$$\begin{aligned}\pi^c_{PART}(s,e) &= \left[r\overline{p}(e)-c_g(1-p(e))\right]T(s)-(1-\alpha)\left[r\overline{p}(e^I)-c_g\left(1-\overline{p}(e^I)\right)\right]T(s)\\ &\quad-\alpha c_s\mu s+(1-\alpha)c_e e^I-c_e e-\alpha\left(c_a+\frac{c_w}{f(0)}\right)L(s),\\ \pi^u_{PART}(s,e) &= (1-\alpha)\left[r\overline{p}(e^I)-c_g\left(1-\overline{p}(e^I)\right)\right]T(s)-(1-\alpha)\left(c_a+\frac{c_w}{f(0)}\right)L(s)\\ &\quad-(1-\alpha)c_s\mu s-(1-\alpha)c_e e^I.\end{aligned} \tag{17}$$

This partnership contract can also use the parameter α to divide the profit.

In order to achieve the full efficiency in call center outsourcing, a close relationship between the user company and the call center has to be created. It is interesting to note that there seems to be a direct correlation between the information content of a contract and its effectiveness in terms of achieving system efficiency. In the piece-meal contract, all the user company needs to know is the call center's throughput *T* in order to calculate its contractual payment.
In the pay-per-call-resolved contract, the user company needs to have information on *pT*, the call center's resolved calls, or equivalently, the revenue generated by the call center. In the pay-per-call-resolved plus cost sharing contract, the user company also needs to know the cost information of the call center so that he could share a part of it. Moreover, the user company has to share his own cost with the call center so that she could hold the call center responsible for suboptimal actions.

Collaboration and information sharing is especially important when the service quality is not directly contractible. As is demonstrated in the partnership contract, the user company first needs to share the staffing costs of the call center. Moreover, in order to induce the call center to spend more effort to improve service quality to meet system-optimality, the call center's profit margin needs to be adjusted to match that of the whole outsourcing supply chain. This requires both parties to share all of their cost information. This approach demands the close collaboration between the two parties. Also in practice, from an accounting point of view, the transactions are more complicated, and each party needs to keep close track of their bookkeeping. In short, coordination in outsourcing cannot be done in a simple 'hands-off' fashion by some simple-term contract. The higher efficiency an outsourcing supply chain wants to achieve, the closer collaboration between outsourcing supply chain parties is needed.

3 Contracts in inventory supply chains

3.1 Introduction

Optimal supply chain performance requires the execution of a precise set of actions. Unfortunately, those actions are not always in the best interest of the members in the supply chain, i.e., the supply chain members are primarily concerned with optimizing their own objectives, and that self-serving focus often results in poor performance. However, optimal performance is achievable if the firms coordinate by contracting on a set of transfer payments such that each firm's objective becomes aligned with the supply chain's objective.

There are numerous supply chain models, but in this chapter the single location base stock model and the two location base stock model are discussed. In each model the supply chain's optimal actions are identified. In each case, the firms could implement those actions, i.e., each firm has access to the information needed to determine the optimal actions and the optimal actions are feasible for each firm. However, firms lack the incentive to implement those actions. To create that incentive, the firms can adjust their terms of trade via a contract that establishes a transfer payment scheme. A number of different contract types are identified and their benefits and drawbacks are illustrated.

This chapter will answer the following questions:
1. Which contracts coordinate the supply chain?
A contract is said to coordinate the supply chain if the set of supply chain optimal actions is a Nash equilibrium, i.e., no firm has a profitable unilateral deviation from the set of supply chain optimal actions. Ideally, the optimal actions should also be a unique Nash equilibrium; otherwise the firms may "coordinate" on a sub-optimal set of actions.
2. Which contracts have sufficient flexibility (by adjusting parameters) to allow for any division of the supply chain's profit among the firms?
If a coordinating contract can allocate profit arbitrarily, then there always exists a contract that Pareto dominates a non-coordinating contract, i.e., each firm's profit is no worse off and at least one firm is strictly better off with the coordinating contract.
3. Which contracts are worth adopting?
Although coordination and flexible rent allocation are desirable features, contracts with those properties tend to be costly to administer. As a result, the contract designer may actually prefer to offer a simple contract even if that contract does not optimize the supply chain's performance. A simple contract is particularly desirable if the contract's efficiency is high (the ratio of supply chain profit with the contract to the supply chain's optimal profit) and if the contract designer captures the lion's share of supply chain profit.

3.2 Coordination in the single location base stock model

This section considers a model with perpetual demand and many replenishment opportunities.
The base stock inventory policy is optimal: with a base stock policy a firm maintains its inventory position (on-order plus in-transit plus on-hand inventory minus backorders) at a constant base stock level. It is assumed, for tractability, that demand is backordered, i.e., there are no lost sales. As a result, expected demand is constant (i.e., it does not depend on the retailer's base stock level). Optimal performance is now achieved by minimizing total supply chain costs: the holding cost of inventory and the backorder penalty costs. In this model the supplier incurs no holding costs, but the supplier does care about the availability of her product at the retail level. To model that preference, it is assumed that a backorder at the retailer incurs a cost at the supplier. Since the retailer does not consider that cost when choosing a base stock level, one can show that the retailer chooses a base stock level that is lower than optimal for the supply chain, which means that the retailer carries too little inventory. Coordination is achieved and costs are arbitrarily allocated by providing incentives to the retailer to carry more inventory.

3.2.1 Model and analysis

Consider a supplier that sells a single product to a single retailer. Let L_r be the lead time to replenish an order from the retailer. The supplier has infinite capacity, so the supplier keeps no inventory and the retailer's replenishment lead time is always L_r, no matter what the retailer's order quantity is. (There are two firms, but only the retailer keeps inventory, which is why this is considered a single location model.) Let $\mu_r = E[D_r]$, where D_r is the demand. Let F_r and f_r be the distribution and density functions of D_r, respectively: assume that F_r is strictly increasing, differentiable, and $F_r(0) = 0$ which rules out the possibility that it is optimal to carry no inventory.

The retailer incurs inventory holding costs at rate $h_r > 0$ per unit of inventory. For analytical tractability, demand is backordered if stock is not available. The retailer incurs backorder penalty costs at rate $\beta_r > 0$ per unit backordered. The supplier has unlimited capacity, so the supplier does not need to carry inventory. However, the supplier incurs backorder penalty costs at rate $\beta_s > 0$ per unit backordered at the retailer. In other words, the supplier incurs a cost whenever a customer wants to purchase the supplier's product from the retailer but the retailer does not have inventory. This cost reflects the supplier's preference for maintaining sufficient availability of her product at the retail level in the supply chain. Let $\beta = \beta_r + \beta_s$, so β is the total backorder cost rate incurred by the total supply chain.

Sales occur at a constant rate μ_r due to the backorder assumption, no matter how the firms manage their inventory. As a result, the firms are only concerned with their costs. Both firms are risk neutral. The retailer's objective is to minimize his average inventory holding and backorder cost per unit time. The supplier's objective is to minimize her average backorder cost per unit time. Define the retailer's inventory level to be equal to the inventory in-transit to the retailer plus the retailer's on-hand inventory minus the retailer's backorders. (This has also been called the effective inventory position.) The retailer's inventory position equals his inventory level plus on-order inventory (inventory ordered, but not yet shipped). Since the supplier immediately ships all orders, the retailer's inventory level and position are identical in this setting.

Let $I_r(y)$ be the retailer's expected inventory at time $t + L_r$ when the retailer's inventory level is y at time t:

$$I_r(y) = \int_0^y (y - x) f_r(x) dx = \int_0^y F_r(x) dx. \tag{18}$$

$$\int_0^y (y - x) f_r(x) dx = \int_0^y \int_x^y 1 f_r(x)\, dk\, dx = \int_0^y \int_0^x f_r(k)\, dk\, dx = \int_0^y F_r(x) dx$$

Let $B_r(y)$ be the analogous function that provides the retailer's expected backorders:

$$B_r(y) = \int_y^\infty (x - y) f_r(x) dx = \mu_r - y + I_r(y). \tag{19}$$

Inventory is monitored continuously, so the retailer can maintain a constant inventory position. In this environment it can be shown that a base stock policy is optimal. With that policy the retailer continuously orders inventory so that his inventory position always equals his chosen base stock level, s_r.

Let $c_r(s_r)$ be the retailer's average cost per unit time when the retailer implements the base stock policy s_r:

$$\begin{aligned} c_r(s_r) &= h_r I_r(s_r) + \beta_r B_r(s_r) \\ &= \beta_r(\mu_r - s_r) + (h_r + \beta_r) I_r(s_r). \end{aligned}$$

Given the retailer's base stock policy, the supplier's expected cost function is

$$c_s(s_r) = \beta_s B_r(s_r)$$
$$= \beta_s(\mu_r - s_r + I_r(s_r)).$$

Let $c(s_r)$ be the supply chain's expected cost per unit time,

$$c(s_r) = c_r(s_r) + c_s(s_r)$$
$$= \beta(\mu_r - s_r) + (h_r + \beta)I_r(s_r). \tag{20}$$

$c(s_r)$ is strictly convex, so there is a unique supply chain optimal base stock level, s_r^o. It satisfies the following critical ratio equation

$$I_r'(s_r^o) = F_r(s_r^o) = \frac{\beta}{h_r + \beta}. \tag{21}$$

Let s_r^* be the retailer's optimal base stock level. The retailer's cost function is also strictly convex, so s_r^* satisfies

$$F_r(s_r^*) = \frac{\beta_r}{h_r + \beta_r}.$$

Given $\beta_r < \beta$, it follows from the above two expressions that $s_r^* < s_r^o$, i.e., the retailer chooses a base stock level that is less than optimal. Hence, channel coordination requires the supplier to provide the retailer with an incentive to raise his base stock level.

Suppose the firms agree to a contract that transfers from the supplier to the retailer at every time t

$$t_I I_r(y) + t_B B_r(y),$$

where y is the retailer's inventory level at time t and t_I and t_B are constants. Furthermore, consider the following set of contracts parameterized by $\lambda \in (0,1]$,

$$t_I = (1-\lambda)h_r$$
$$t_B = \beta_r - \lambda\beta.$$

(Here we choose to rule out $\lambda = 0$, since then any base stock level is optimal.) Given one of those contracts, the retailer's expected cost function is now

$$c_r(s_r) = (\beta_r - t_B)(\mu_r - s_r) + (h_r + \beta_r - t_I - t_B)I_r(s_r). \tag{22}$$

The contract parameters have been chosen so that

$$\beta_r - t_B = \lambda\beta > 0,$$

and

$$h_r + \beta_r - t_I - t_B = \lambda(h_r + \beta) > 0.$$

It follows from (20) and (22) that with these contracts

$$c_r(s_r) = \lambda c(s_r). \tag{23}$$

Hence, s_r^o minimizes the retailer's cost, i.e., those contracts coordinate the supply chain.

In addition, those contracts arbitrarily allocate costs between the firms, with the retailer's share of the cost increasing in the parameter λ. Note, the λ parameter is not explicitly incorporated into the contract, i.e., it is merely used for expositional clarity.

Now, consider the sign of the t_I and the t_B parameters. Since the contract must induce the retailer to choose a higher base stock level, it is natural to conjecture $t_I > 0$, i.e., the supplier subsidizes the retailer's inventory holding cost. In fact, that conjecture is valid when $\lambda \in (0,1]$. It is also natural to suppose $t_B < 0$, i.e., the supplier penalizes the retailer for backorders. But $\lambda \in (0,1]$ implies $t_B \in [-\beta_s, \beta_r)$, i.e., with some contracts the supplier subsidizes the retailer's backorders $(t_B > 0)$: in those situations the supplier encourages backorders by setting $t_B > 0$, because without that encouragement the large inventory subsidy leads the retailer to $s_r^* > s_r^o$.

The above analysis is reminiscence of the analysis with the newsvendor model and buy-back contracts. This is not a coincidence, because this model is qualitatively identical to the newsvendor model. To explain, begin with the retailer's profit function in the newsvendor model (assuming $c_r = g_r = g_s = \upsilon = 0$):

$$\begin{aligned}\pi_r(q) &= pS(q) - wq \\ &= (p-w)q - pI(q).\end{aligned}$$

The retailer's profit has two terms, one that increases linearly in q, and the other that depends on the demand distribution. Now let $p = h_r + \beta_r$ and $w = h_r$. In that case,

$$\pi_r(q) = \beta_r q - (h_r + \beta_r)I_r(q) = -c_r(q) + \beta_r \mu_r.$$

Hence, there is no difference between the maximization of $\pi_r(q)$ and the minimization of $c_r(s_r)$.

Now recall that the transfer payment with a buy-back contract is $wq - bI(q)$, i.e., there is a parameter (i.e., w) that affects the payment linearly in the retailer's action (i.e., q), and a parameter that influences the transfer payment through a function (i.e., $I(q)$) that depends on the retailer's action and the demand distribution. In this model

$$t_I I_r(y) + t_B B_r(y) = (t_I + t_B)I_r(y) + t_B(\mu_r - y),$$

so t_B is the linear parameter and $t_I + t_B$ is the other parameter. In the buy-back contract the parameters work independently. To get the same effect in the base stock model the supplier could adopt a transfer payment that depends on the retailer's inventory position s_r, and the retailer's inventory. That contract would yield the same results.

3.3 Coordination in the two location base stock model

The two location base stock model builds upon the single location base stock model discussed in the previous section. Now the supplier no longer has infinite capacity. Instead, she must order replenishments from her source and those replenishment always are filled within L_s time (i.e., her source has infinite capacity). So in this model the supplier enjoys reliable replenishments but the lead time of the retailer's replenishment depends on how the supplier manages her inventory. Only if the supplier has enough inventory to fill an order, then the retailer receives that order in L_r time. Otherwise, the retailer must wait longer than L_r to receive the unfilled portion. That delay could lead to additional backorders at the retailer, which are costly to both the retailer and the supplier, or it could lead to lower inventory at the retailer, which helps the retailer.

In the single location model the only critical issue is the amount of inventory in the supply chain. In this model the allocation of the supply chain's inventory between the supplier and the retailer is important as well. For a fixed amount of supply chain inventory the supplier always prefers that more is allocated to the retailer, because that lowers both her inventory and backorder costs. (Recall that the supplier is charged for retail backorders.) On the other hand, the retailer's preference is not so clear: less retail inventory means lower holding costs, but also higher backorder costs. There are also subtle interactions with respect to the total amount of inventory in the supply chain. The retailer is biased to carry too little inventory: the retailer bears the full cost of his inventory but only receives a portion of the benefits (i.e., he does not benefit from the reduction in the supplier's backorder costs). On the other hand, there is no clear bias for the supplier because of two effects. First, the supplier bears the cost of his inventory and does not benefit from the reduction in the retailer's backorder costs, which biases the supplier to carry too little inventory. Second, the supplier does not bear the cost of the retailer's inventory (which increases along with the supplier's inventory), which biases the supplier to carry too much inventory. Either bias can dominate, depending on the parameters of the model.

Even though it is not clear whether the decentralized supply chain will carry too much or too little inventory (however, it generally carries too little inventory), it is shown that the optimal policy is never a Nash equilibrium of the decentralized game, i.e., decentralized operation is never optimal. However, the competition penalty (the percentage of loss in supply chain performance due to decentralized decision making) varies considerably: in some cases the competition penalty is relatively small, e.g., less than 5%, whereas in other cases it is considerable, e.g., more than 40%. Therefore, the need for coordinating contracts is not universal.

3.3.1 Model

Let h_s, $0 < h_s < h_r$, be the supplier's per unit holding cost rate incurred with on-hand inventory. (When $h_s \geq h_r$ the optimal policy does not carry inventory at the supplier and when $h_s \leq 0$ the optimal policy has unlimited supplier inventory. Neither case is interesting.).

The firms' operating decisions have no impact on the amount of in-transit inventory, so no holding cost is charged for either the supplier's or the retailer's pipeline inventory. Let $D_s > 0$ be demand during an interval of time with length L_s . (As in the single location model, $D_s > 0$ ensures that the supplier carries some inventory in the optimal policy.) Let F_s and f_s be the distribution and density functions of that demand. As with the retailer, assume F_s is increasing and differentiable. Let $\mu_s = E[D_s]$. Retail orders are backordered at the supplier but there is no explicit charge for those backorders. The supplier still incurs per unit backorder costs at rate β_s for backorders at the retailer. The comparable cost for the retailer is still β_r. Even though there are no direct consequences to a supplier's backorder, there are indirect consequences: lower retailer inventory and higher retailer backorders.

Both firms use base stock policies to manage inventory. With a base stock policy, firm $i \in \{r, s\}$ orders inventory so that its inventory position remains equal to its base stock level, s_i. (Recall that a firm's inventory level equals on-hand inventory minus backorders plus in-transit inventory and a firm's inventory position equals the inventory level plus on-order inventory.) These base stock policies operate only with local information, so neither firm needs to know the other firm's inventory position. The firms choose their base stock levels once and simultaneously. The firms attempt to minimize their average cost per unit time. (Given that one firm uses a base stock policy, it is optimal for the other firm to use a base stock policy as well.) They are both risk neutral. There exists a pair of base stock levels, $\{s_r^o, s_s^o\}$, that minimize the supply chain's cost. Hence, it is feasible for the firms to optimize the supply chain, but incentive conflicts may prevent them from doing so.

The first step in the analysis of this model is to evaluate each firm's average cost. The next step evaluates the Nash equilibrium base stock levels. The third step identifies the optimal base stock levels and compares them to the Nash equilibrium ones. The final step explores incentive structures to coordinate the supply chain.

3.3.2 Cost functions

As in the single location model, $c_r(y), c_s(y)$, and $c(y)$ are the firms' and the supply chain's expected costs incurred at time $t + L_r$ at the retail level when the retailer's inventory level is y at time t. However, in the two location model the retailer's inventory level does not always equal the retailer's inventory position, s_r, because the supplier may be out of stock. Let $c_i(s_r, s_s)$ be the average rate at which firm i incurs costs at the retail level and $c(s_r, s_s) = c_r(s_r, s_s) + c_s(s_r, s_s)$. To evaluate c_i, note that at any given time t the supplier's inventory position is s_s (because the supplier uses a base stock policy). At time $t + L_s$ either the supplier's on-hand inventory is $(s_s - D_s)^+$ or the supplier's backorder equals $\left(D_s - s_s\right)^+$. Therefore, the retailer's inventory level at time $t + L_s$ is $s_r - \left(D_s - s_s\right)^+$. So

$$c_i\left(s_r, s_s\right) = F_s\left(s_s\right)c_i\left(s_r\right) + \int_{s_s}^{\infty} c_i\left(s_r + s_s - x\right)f_s\left(x\right)dx:$$

at time $t + L_s$ the supplier can raise the retailer's inventory level to s_r with probability $F_s\left(s_s\right)$, otherwise the retailer's inventory level equals $s_r + s_s - D_s$. Based on analogous reasoning, let $I_r\left(s_r, s_s\right)$ and $B_r\left(s_r, s_s\right)$ be the retailer's average inventory and backorders given the base stock levels:

$$I_r\left(s_r, s_s\right) = F_s\left(s_s\right)I_r\left(s_r\right) + \int_{s_s}^{\infty} I_r\left(s_r + s_s - x\right)f_s\left(x\right)dx,$$

$$B_r\left(s_r, s_s\right) = F_s\left(s_s\right)B_r\left(s_r\right) + \int_{s_s}^{\infty} B_r\left(s_r + s_s - x\right)f_s\left(x\right)dx.$$

Let $\pi_i\left(s_r, s_s\right)$ be firm $i's$ total average cost rate. Since the retailer only incurs costs at the retail level,

$$\pi_r\left(s_r, s_s\right) = c_r\left(s_r, s_s\right).$$

Let $I_s\left(s_s\right)$ be the supplier's average inventory. Analogous to the retailer's functions (defined in the previous section)

$$I_s\left(y\right) = \int_0^y F_s\left(x\right)dx.$$

The supplier's average cost is

$$\pi_s\left(s_r, s_s\right) = h_s I_s\left(s_s\right) + c_s\left(s_r, s_s\right).$$

Let $\prod\left(s_r, s_s\right)$ be the supply chain's total cost, $\prod\left(s_r, s_s\right) = \pi_r\left(s_r, s_s\right) + \pi_s\left(s_r, s_s\right)$.

3.3.3 Behavior in the decentralized game

Let $s_i\left(s_j\right)$ be an optimal base stock level for firm i given the base stock level chosen by firm j, i.e., $s_i\left(s_j\right)$ is firm $i's$ best response to firm $j's$ strategy. Differentiation of each firm's cost function demonstrates that each firm's cost is strictly convex in its base stock level, so each firm has a unique best response. With a Nash equilibrium pair of base stocks, $\left\{s_r^*, s_s^*\right\}$, neither firm has a profitable unilateral deviation, i.e.,

$$s_r^* = s_r\left(s_s^*\right) \text{ and } s_s^* = s_s\left(s_r^*\right).$$

Existence of a Nash equilibrium is not assured, but in this game existence of a Nash equilibrium follows from the convexity of the firm's cost functions. (Technically it is also required that the firms' strategy spaces have an upper bound. Imposing that bound has no impact on the analysis.) In fact, there exists a unique Nash equilibrium. To demonstrate uniqueness, start by bounding each player's feasible strategy space, i.e., the set of strategies a player may choose. For the retailer it is not difficult to show that $s_r\left(s_s\right) > \hat{s}_r > 0$, where $\hat{s}_r$ minimizes $c_r\left(y\right)$, i.e.,

$$F_r\left(\hat{s}_r\right) = \frac{\beta}{h_r + \beta}.$$

In other words, if the retailer were to receive perfectly reliable replenishments, the retailer would choose $\hat{s}_r$, so the retailer certainly does not choose $s_r < \hat{s}_r$ if replenishments are unreliable. (In other words, $\hat{s}_r$ is optimal for the retailer in the single location model discussed in the previous section.) For the supplier, $s_s(s_r) > 0$, because $\partial \pi_s(s_r, s_s)/\partial s_s < 0$ given $F_s(s_s) = 0$ and $c_s^{'}(y) < 0$. Uniqueness of the Nash equilibrium holds if for the feasible strategies, $s_r > \hat{s}_r$ and $s_s > 0$, the best reply functions are contraction mappings, i.e.,

$$\left|s_i^{'}(s_j)\right| < 1. \tag{24}$$

From the implicit function theorem

$$s_r^{'}(s_s) = \frac{\int_{s_s}^{\infty} c_r^{''}(s_r + s_s - x) f_s(x)\,dx}{F_s(s_s) c_r^{''}(s_r) + \int_{s_s}^{\infty} c_r^{''}(s_r + s_s - x) f_s(x)\,dx},$$

and

$$s_s^{'}(s_r) = \frac{\int_{s_s}^{\infty} c_s^{''}(s_r + s_s - x) f_s(x)\,dx}{\left[h_s - c_s^{'}(s_r)\right] f_s(s_s) + \int_{s_s}^{\infty} c_s^{''}(s_r + s_s - x) f_s(x)\,dx}.$$

Given $s_r > \hat{s}_r$ and $s_s > 0$, it follows that $c_r^{''}(x) > 0,\ c_s^{'}(y) < 0,\ c_s^{''}(y) > 0$, and $F_s(s_s) > 0$. Hence, (24) holds for both the supplier and the retailer.

A unique Nash equilibrium is quite convenient, since that equilibrium is then a reasonable prediction for the outcome of the decentralized game. (With multiple equilibria it is not clear that the outcome of the game would even be an equilibrium, since the players may choose strategies from different equilibria.) Hence, the competition penalty is an appropriate measure of the gap between optimal performance and decentralized performance, where the competition penalty is defined to be

$$\frac{\Pi(s_s^*, s_r^*) - \Pi(s_s^o, s_r^o)}{\Pi(s_s^o, s_r^o)}.$$

In fact, there always exists a positive competition penalty, i.e., decentralized operations always lead to suboptimal performance in this game. To explain, note that the retailer's marginal cost is always greater than the supply chain's

$$\frac{\partial c_r(s_r, s_s)}{\partial s_r} > \frac{\partial c(s_r, s_s)}{\partial s_r},$$

because $c_r^{'}(s_r) > c^{'}(s_r)$. Since both $c_r(s_r, s_s)$ and $c(s_r, s_s)$ are strictly convex, it follows that, for any s_s the retailer's optimal base stock is always lower than the supply chain's optimal base stock. Hence, even if the supplier chooses s_s^o, the retailer does not choose s_r^o, i.e., $s_s(s_s^o) < s_r^o$.

Although the Nash equilibrium is not optimal, the magnitude of the competition penalty depends on the parameters of the model. When the firms' backorder penalties are the same (i.e., $\beta_r/\beta_s = 1$) the median competition penalty for their sample is 5% and the competition penalty is no greater than 8% in 95% of their observations. However, very large competition penalties are observed when either $\beta_r/\beta_s < 1/9$ or $\beta_r/\beta_s > 9$. The retailer does not have a strong concern for customer service when $\beta_r/\beta_s < 1/9$, and so the retailer tends to carry far less inventory than optimal. Since the supplier does not have direct access to customers, the supplier can do little to prevent backorders in that situation, and so the supply chain cost is substantially higher than need be. In the other extreme, $\beta_r/\beta_s > 9$, the supplier cares little about customer service, and thus does not carry enough inventory. In that situation the retailer can still prevent backorders, but to do so requires a substantial amount of inventory at the retailer to account for the supplier's long lead time. The supply chain's cost is substantially higher than the optimal one if the optimal policy has the supplier carry inventory to

provide reliable replenishments to the retailer. However, there are situations in which the optimal policy does not require the supplier to carry much inventory: either the supplier's holding cost is nearly as high as the retailer's (in which case keeping inventory at the supplier gives little holding cost advantage) or if the supplier's lead time is short (in which case the delay due to a lack of inventory at the supplier is negligible). In those cases the competition penalty is relatively minor.
In the single location model decentralization always leads to less inventory than optimal for the supply chain. In this setting the interactions between the firms are more complex, and so decentralization generally leads to too little inventory, but not always. Since the retailer's backorder cost rate is lower than the supplier's backorder cost rate, for a fixed s_s the retailer always carries too little inventory, which certainly contributes to a less than optimal amount of inventory in the system. However, the retailer is only part of the supply chain.
In fact, the supplier's inventory may be so large that even though the retailer carries too little inventory, the total amount of inventory in the decentralized supply chain may exceed the supply chain's optimal quantity. Suppose β_r is quite small and β_s is quite large. In that case, the retailer carries very little inventory. To attempt to mitigate the build up of backorders at the retail level the supplier provides the retailer with very reliable replenishments, which requires a large amount of inventory, an amount that may lead to more inventory in the supply chain than optimal.
The main conclusion from the analysis of the decentralized game is that the competition penalty is always positive, but only in some circumstances it is very large. It is precisely in those circumstances that the firms could benefit from an incentive scheme to coordinate their actions.

3.3.4 Coordination with linear transfer payments

Supply chain coordination in this setting is achieved when $\{s_r^o, s_s^o\}$ is a Nash equilibrium.
In the single location model the supplier coordinates the supply chain with a contract that has linear transfer payments based on the retailer's inventory and backorders. Suppose the supplier offers the same arrangement in this model with the addition of a transfer payment based on the supplier's backorders:

$$t_I I_r(s_r, s_s) + t_B^r B_r(s_r, s_s) + t_B^s B_s(s_s),$$

where t_I, t_B^r, and t_B^s are constants and $B_s(s_s)$ is the supplier's average backorder:

$$B_s(y) = \mu_s - y + I_s(y).$$

Recall that a positive value for the above expression represents a payment from the supplier to the retailer and a negative value represents a payment from the retailer to the supplier.
While both firms can easily observe $B_s(s_s)$, an information system is needed for the supplier to verify the retailer's inventory and backorder.
The first step in the analysis provides some results for the optimal solution. The second step defines a set of contracts and confirms that those contracts coordinate the supply chain.
Then, the allocation of costs is considered. Finally, it is shown that the optimal solution is a unique Nash equilibrium.

The traditional approach to obtain the optimal solution involves reallocating costs so that all costs are preserved. Base stock policies are then optimal and easily evaluated. However, to facilitate the comparison of the optimal policy to the Nash equilibrium of the decentralized game, it is useful to evaluate the optimal base stock policy without that traditional cost reallocation.
Given that $\Pi(s_r, s_s)$ is continuous, any optimal policy with $s_s > 0$ must set the following two marginals to zero

$$\frac{\partial \Pi(s_r, s_s)}{\partial s_r} = F_s(s_s) c'(s_r) + \int_{s_s}^{\infty} c'(s_r + s_s - x) f_s(x) dx, \qquad (25)$$

and

$$\frac{\partial \prod(s_r, s_s)}{\partial s_s} = F_s(s_s)h_s + \int_{s_s}^{\infty} c'(s_r + s_s - x)f_s(x)dx. \qquad (26)$$

Since $F_s(s_s) > 0,$ there is only one possible optimal policy with $s_s > 0,$ $\{\tilde{s}_r^1, \tilde{s}_s^1\}$, where $\tilde{s}_r^1$ satisfies

$$c'(\tilde{s}_r^1) = h_s, \qquad (27)$$

and $\tilde{s}_s^1$ satisfies $\partial \prod(\tilde{s}_r^1, \tilde{s}_s^1)/\partial s_s = 0$. (27) simplifies to $F_r(\tilde{s}_r^1) = \frac{h_s + \beta}{h_r + \beta},$

so it is apparent that $\tilde{s}_r^1$ exists and is unique. Since $\prod(\tilde{s}_r^1, s_s)$ is strictly convex in $s_s, \tilde{s}_s^1$ exists and is unique. There may also exist an optimal policy with $s_s \le 0$. In that case, the candidate policies are $\{\tilde{s}_r^2, \tilde{s}_s^2\}$ where $\tilde{s}_s^2 \le 0,$ $\tilde{s}_r^2 + \tilde{s}_s^2 = \bar{s}$, and $\bar{s}$ satisfies

$$\int_0^{\infty} c'(\bar{s} - x)f_s(x)dx = 0. \qquad (28)$$

The above simplifies to $\Pr(D_r + D_s \le \bar{s}) = \frac{\beta}{h_r + \beta},$

so $\bar{s}$ exists and is unique.

Given that $\prod(\tilde{s}_r^1, s_s)$ is strictly convex in s_s, $\prod(\tilde{s}_r^1, \tilde{s}_s^1) < \prod(\tilde{s}_r^2, \tilde{s}_s^2)$ whenever $\partial \prod(\tilde{s}_r^1, 0)/\partial s_s < 0.$ Since $\partial \prod(s_r, 0)/\partial s_s$ is increasing in s_r from (11), that condition holds when $\tilde{s}_r^1 < \bar{s}$, otherwise it does not. Therefore, $\{\tilde{s}_r^1, \tilde{s}_s^1\}$ is the unique optimal policy when $\tilde{s}_r^1 < \bar{s},$ otherwise any $\{\tilde{s}_r^2, \tilde{s}_s^2\}$ is optimal:

$$\{s_r^o, s_s^o\} = \begin{cases} \{\tilde{s}_r^1, \tilde{s}_s^1\} & \tilde{s}_r^1 < \tilde{s}_r^2 \\ \{\tilde{s}_r^2, \tilde{s}_s^2\} & \tilde{s}_r^1 \ge \tilde{s}_r^2 \end{cases}.$$

Now consider the firms' behavior with the following set of contracts parameterized by $\lambda \in (0,1]$,

$$t_I = (1 - \lambda)h_r, \qquad (29)$$

$$t_B^r = \beta_r - \lambda\beta, \qquad (30)$$

$$t_B^s = \lambda h_s \left(\frac{F_s(s_s^o)}{1 - F_s(s_s^o)} \right). \qquad (31)$$

The retailer's cost function, adjusted for the above contracts is

$$\begin{aligned} c_r(y) &= (h_r - t_I)I_r(y) + (\beta_r - t_B^r)B_r(y) - t_B^s B_s(s_s) \\ &= \lambda c(y) - t_B^s B_s(s_s) \end{aligned}$$

and so

$$\pi_r(s_r, s_s) = \lambda \prod(s_r, s_s) - t_B^s B_s(s_s). \qquad (32)$$

Recall $c(s_r, s_s) = c_r(s_r, s_s) + c_s(s_r, s_s),$ so

$$\begin{aligned} \pi_s(s_r, s_s) &= h_s I_s(s_s) + (1 - \lambda)c(s_r, s_s) + t_B^s B_s(s_s) \\ &= (h_s + t_B^s)I_s(s_s) + (1 - \lambda)c(s_r, s_s) + t_B^s(\mu_s - s_s). \end{aligned} \qquad (33)$$

There are two cases to consider: either $s_s^o > 0$ or $s_s^o = 0$. Take the first case. If $s_s^o > 0$, then (27) implies

$$\frac{\partial \pi_r(s_r^o, s_s^o)}{\partial s_r} = \left(\frac{\lambda}{1 - \lambda} \right) \frac{\partial \pi_s(s_r^o, s_s^o)}{\partial s_s} = \lambda \frac{\partial \prod(s_r^o, s_s^o)}{\partial s_s} = 0.$$

Further, $\pi_r(s_r, s_s)$ is strictly convex in s_r and $\pi_s(s_r^o, s_s)$ is strictly convex in s_s, so $\{s_r^o, s_s^o\}$ is indeed a Nash equilibrium. In fact, it is the unique Nash equilibrium. From the implicit function theorem, $s_r(s_s)$ is decreasing, whereas in the decentralized game without the contract, $s_i(s_j)$ is firm $i's$ best response to firm $j's$ strategy,

$$\frac{\partial s_r(s_s)}{ds_s} = -\frac{\int_{s_s}^{\infty} c''(s_r + s_s - x) f_s(x)dx}{f_s(s_s)c''(s_r) + \int_{s_s}^{\infty} c''(s_r + s_s - x) f_s(x)dx} \leq 0.$$

Hence, for $s_r = s_r(s_s)$ and $\lambda \leq 1$, the supplier's marginal cost is increasing:

$$\frac{\partial \pi_s(s_r(s_s), s_s)}{\partial s_s} = F_s(s_s)(h_s - (1-\lambda)c'(s_r(s_s)) + t_B^s) - t_B^s.$$

Thus, there is a unique s_s that satisfies $s_s(s_r(s_s)) = s_s$, i.e., there is a unique Nash equilibrium.
Now suppose $s_s^0 \leq 0$. It is straightforward to confirm that all of the $\{\tilde{s}_r^2, \tilde{s}_s^2\}$ pairs satisfy the firms' first and second order conditions. Hence, they are all Nash equilibria. Even though there is not a unique Nash equilibrium, the firms' costs are identical across the equilibria.
These contracts do allow the firms to arbitrarily allocate the retail level costs in the system, but they do not allow the firms to arbitrarily allocate all of the supply chain's costs.
This limitation is due to the $\lambda \leq 1$ restriction, i.e., it is not possible with these contracts to allocate to the retailer more than the optimal retail level costs: while the retailer's cost function is well behaved even if $\lambda > 1$, the supplier's is not; with $\lambda > 1$ the supplier has a strong incentive to increase the retail level costs. Of course, fixed payments could be used to achieve those allocations if necessary. But since it is unlikely a retailer would agree to such a burden, this limitation is not too restrictive.
An interesting feature of these contracts is that the t_I and t_B^r transfer payments are identical to the ones used in the single location model. This is remarkable because the retailer's critical ratio differs across the models: in the single location model the retailer picks s_r such that

$$F_r(s_r) = \frac{\beta}{\beta + h_r},$$

whereas in the two location model the retailer picks such that

$$F_r(s_r) = \frac{\beta + h_s}{\beta + h_r}.$$

In the two location base stock model decentralized operations always leads to sub-optimal performance, but the extent of the performance deterioration (i.e., the competition penalty) depends on the supply chain parameters. If the firms' backorder costs are similar, then the competition penalty is often reasonably small. The competition penalty can be small even if the supplier does not care about customer service because her operations role in the supply chain may not be too important (e.g., if h_s is large or if L_s is small). To coordinate the retailer's actions, the firms can agree to a pair of linear transfer payments that function like the buy-back contract in the single period newsvendor model. To coordinate the supplier's action a linear transfer payment based on the supplier's backorders is used. With these contracts the optimal policy is the unique Nash equilibrium.

4 Contracts in general

In this chapter contracts in general will be discussed through contracts in the call center and in the supply chain.

The contracts of the call center are listed below:

- Piece-meal (PM) contract: $\Psi = bT(s)$.
- Pay-per-call-resolved (PPCR) contract: $\Psi = bp(e)T(s)$.
- Pay-per-call-resolved with cost sharing (PPCR+CS) contract:
  $$\Psi = bp(e)T(s) + (1-\alpha)(c_s\ \mu s + c_e e) - \alpha\left[c_g(1-p(e))T(s) + c_a\ L(s) + \frac{c_w}{f(0)}L(s)\right].$$
- Partnership contract:
  $$\Psi = rp(e)T(s) - (1-p(e))c_g T(s) - (1-\alpha)\left[rp(e^I) - c_g(1-p(e^I))\right]T(s)$$
  $$+ (1-\alpha)c_s \mu s + (1-\alpha)c_e e^I - \alpha\left(c_a + \frac{c_w}{f(0)}\right)L(s).$$

If the contracts of the supply chain are rewritten in the form of the contracts of the call center they will have the following form:

- The single location base stock model contract: $\Psi = t_I I_r(y) + t_B B_r(y)$, where y is the retailer's inventory level at time t and t_I and t_B are constants. $I_r(y)$ is the retailer's expected inventory and $B_r(y)$ is the retailer's expected backorders.
- The two location base stock model contract: $\Psi = t_I I_r(s_r, s_s) + t_B^r B_r(s_r, s_s) + t_B^s B_s(s_s)$, where t_I, t_B^r, and t_B^s are constants and $B_s(s_s)$ is the supplier's average backorder.

The contracts of the single and two location base stock model resemble most the pay-per-call-resolved with cost sharing (PPCR+CS) contract of the call center.

Out of this we can conclude that a good contract rewards and penalizes.

- In the PPCR+CS contract the user company subsidizes the call center staffing and effort costs and the call center has to pay abandonment and waiting costs to the user company, a punishment for under-staffing.
- In the single location base stock model contract the supplier incurs a cost whenever a customer wants to purchase the supplier's product from the retailer but the retailer does not have inventory. This cost reflects the supplier's preference for maintaining sufficient availability of her product at the retail level in the supply chain.
- In the two location base stock model a payment from the supplier to the retailer has to be made if $B_s(s_s)$ is positive and a negative value represents a payment from the retailer to the supplier.

A general contract will have the following form:
$\Psi_{2\to 1}(s_1, s_2) = h_1 c_1(s_1, s_2) + h_2 c_2(s_1, s_2)$, where $h_1 > 0, h_2 < 0$ if $c_i(s_1, s_2)$ increases in s_1 and s_2.
$c_i(s_1, s_2)$ is the cost function of the inventory model.

5 Conclusions

When a company outsources its call center function to an external call center, the centralized system changes into an outsourcing supply chain. In this paper the coordination of this outsourcing supply chain with contracts has been studied. Specifically, from the user's point of view, it is interesting to study how to induce the call center to staff enough agents and exert enough effort to achieve a high service quality.

The call center is modeled as a multi-server queue with customer abandonments. It is shown that various contracts between the call center and the user company result in different staffing and effort levels.
In a piece-meal (PM) contract, the user company pays the call center a unit rate *b* for each call served. A piece-meal type of service contract can coordinate the staffing level of the call center. However, it is unable to coordinate the effort level to achieve system-optimality.
In a pay-per-call-resolved (PPCR) contract, the user company pays the call center for each call served and resolved. The PPCR contract induces the call center to staff enough people so that no calls will be lost, because its revenue is directly tied to the volume of calls served. Furthermore, because the call center gets rewarded only when a call is served and resolved, it has an incentive to exert effort to increase the service quality and increase the volume of calls resolved.
In the pay-per-call-resolved with cost sharing (PPCR+CS) contract, the user company shares the call center's staffing and effort costs, while the call center shares the user company's loss-of-goodwill costs. By coordination, the outsourcing supply chain can achieve the maximum profit.
In the partnership contract, the user company first needs to share the staffing costs of the call center. Moreover, in order to induce the call center to spend more effort to improve service quality to meet system-optimality, the call center's profit margin needs to be adjusted to match that of the whole outsourcing supply chain. This requires both parties to share all of their cost information.

In order to achieve coordination in this setting, the call center and the outsourcing company need to collaborate closely. These contracts suggest that close attention has to be paid to service quality and its contractibility in seeking call center outsourcing.

Of the numerous supply chain models, the single location base stock model and the two location base stock model are discussed in this paper.
The single location base stock model is a stochastic demand model in which the retailer receives replenishments from a supplier after a constant lead time. Coordination requires that the retailer chooses a "higher action", which in this model is a larger base stock level. The cost of this higher action is more inventory on average, but the supplier can verify the retailer's inventory and therefore share the holding costs of carrying more inventory with the retailer.
The two location base stock model makes the supplier hold inventory, although at lower holding costs than the retailer. Whereas the focus in the single location base stock model is primarily on coordinating the downstream actions, in this model the supplier's action also requires coordination, and that coordination is non-trivial. To be more specific, in the single location model the only critical issue is the amount of inventory in the supply chain, but here the allocation of the supply chain's inventory between the supplier and the retailer is important as well.

The contracts of the single and two location base stock model resemble most the pay-per-call-resolved with cost sharing (PPCR+CS) contract of the call center.
In the PPCR+CS contract the user company subsidizes the call center staffing and effort costs and the call center has to pay abandonment and waiting costs to the user company, a punishment for under-staffing.
In the single location base stock model contract the supplier incurs a cost whenever a customer wants to purchase the supplier's product from the retailer but the retailer does not have inventory. This cost

reflects the supplier's preference for maintaining sufficient availability of her product at the retail level in the supply chain.
In the two location base stock model a payment from the supplier to the retailer has to be made if $B_s(s_s)$ is positive and a negative value represents a payment from the retailer to the supplier.
Out of this we can conclude that a good contract rewards and penalizes.

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Appendix

In this appendix, we proof the first-order conditions of (6). It derives the call center's optimal staffing and effort levels, s^I and e^I, in the integrated system.

Recall that the profit function (6) is

$$\pi^I(s,e) = r\bar{p}(e)(\lambda - L(s)) - c_s \mu s - c_e e - c_a L(s) - \frac{c_w}{f(0)} L(s) - c_g (1 - \bar{p}(e))(\lambda - L(s))$$

$$= (r\bar{p}(e) + c_g \bar{p}(e) - c_g)\lambda - \left(r\bar{p}(e) + c_a + \frac{c_w}{f(0)} - c_g (1 - \bar{p}(e)) \right) L(s) - c_s \mu s - c_e e.$$

Which can be simplified to

$$\pi^I(s,e) = \left[r\bar{p}(e) - c_g (1 - \bar{p}(e)) \right]\lambda - \left[r\bar{p}(e) + c_a + \frac{c_w}{f(0)} - c_g (1 - \bar{p}(e)) \right] L(s) - c_s \mu s - c_e e.$$

- When $s \geq \lambda/\mu$, there are no abandonments, thus $L(s) = 0$, and

$$\pi^I(s,e) = \left[r\bar{p}(e) - c_g (1 - \bar{p}(e)) \right]\lambda - c_s \mu s - c_e e.$$

Note that this is decreasing in *s*.

- When $s \leq \lambda/\mu$, $L(s) = \lambda - s\mu \geq 0$, and

$$\pi^I(s,e) = -\left[c_a + \frac{c_w}{f(0)} \right]\lambda + \left[r\bar{p}(e) + c_a + \frac{c_w}{f(0)} - c_g (1 - \bar{p}(e)) - c_s \right]\mu s - c_e e.$$

Note that this is increasing in *s*.

- When $s^I = \lambda/\mu$ and the profit reduces to

$$\pi^I(s^I,e) = \left[r\bar{p}(e) - c_g (1 - \bar{p}(e)) - c_s \right]\lambda - c_e e = (r + c_g)\lambda\bar{p}(e) - c_e e - (c_g + c_s)\lambda. \tag{34}$$

Note that one necessary condition for the profit to be positive is that

$r\bar{p}(e) + c_a + \frac{c_w}{f(0)} - c_g (1 - \bar{p}(e)) - c_s > 0$, which is satisfied by the assumption $r\bar{p}(0) > c_g (1 - \bar{p}(0)) + c_s$. Because $\bar{p}(e)$ is concave in *e*, it is easy to see from (34) that $\pi(s^I, e)$ is concave in *e*. The first order derivative of (34) with respect to *e* is:

$$(r + c_g)\lambda\bar{p}'(e) - c_e. \tag{35}$$

Therefore, when $\frac{c_e}{(r + c_g)\lambda} \geq \bar{p}_0'$, the marginal cost is too big relative to the marginal revenue to be worth spending any effort. Otherwise, the optimal effort can be found by $\bar{p}'(e) = \frac{c_e}{(r + c_g)\lambda}$. That is,

$$\bar{p}'(e^I) = \frac{c_e}{(r + c_g)\lambda},\ if\ \frac{c_e}{(r + c_g)\lambda} < p_0';\ e^I = 0\ otherwise.$$