

**Problem 7.**

(Evans problems 11 and 12, page 427)

1. Let  $\{S(t)\}_{t \geq 0}$  be a contraction semi-group on a real Banach space  $X$ , with infinitesimal generator  $A : D(A) \subset X \rightarrow X$ . Show that the following identity holds:

$$A \int_0^\infty e^{-\lambda t} S(t) u dt = \int_0^\infty e^{-\lambda t} A S(t) u dt,$$

for all  $u \in D(A)$ .

2. Show that for  $\lambda, \mu \in \rho(A)$  it holds that

$$R_\lambda - R_\mu = (\mu - \lambda) R_\lambda R_\mu,$$

and

$$R_\lambda R_\mu = R_\mu R_\lambda,$$

where  $R_\lambda = (\lambda I - A)^{-1} : X \rightarrow X$  is the resolvent operator at  $\lambda$ .