MATH GAME(S) – AN ALTERNATIVE (APPROACH) TO TEACHING MATH?

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KEYWORDS
games, mathematics, learning

ABSTRACT

Getting students to read, digest and practice material is difficult in any discipline, but even more so for math, since many students have to cope with motivational problems and feelings of inadequacy, often due to prior unsuccesful training and teaching methods. In this paper we look at the opportunities offered by computer graphics, visual programming and game design as an alternative for traditional methods of teaching mathemathics. In particular, games may be deployed both as intruments to drill concepts and skills, but in addition as a way to identify challenges and possible strategies for solving problems in the mathematical domain. Our, perhaps somewhat optimistic, message is that, when coupled to instruction of visual programming, web technology and new media deployment, students gain insight by constructive explorations in a wide variety of mathematical problems. Moreover, by creating their own game worlds, reflection on the structure and complexity of mathematics is encouraged taking each student's capabilities and limitations into account, thus avoiding the fear of mathematics that haunts so many students.

INTRODUCTION

In Eliens & Ruttkay (2009) we observed that *educators continuously face the problem of motivating their students to learn, to consult textbooks, do exercises, reflect on the material, and repeat this cycle over and over again.* For mathematics this problem is even aggravated by the lack of motivation of students, or in many cases even fear and feelings of inadequacy due to prior unsuccesful training and teaching methods that do not align well with the needs of the students. That is, however, not to say that, as many seem to do, the mathematical skills of undergraduate students are declining, which we believe not to be the case, but that a proper context is needed to deploy these skills effectively. In this paper, we will report on our experiences, and in all honesty, our intentions, to make learning mathematics appealing to students of a variety of flavors, that is across both the technical as well as the more society and arts oriented profiles of students. Our approach is strongly motivated by the need expressed by designers as well as artist to become *familiar with mathematics* in order to better understand the visual effects of complex phenomena, e.g. Lundgren (2006). Another motivation and the actual background to our work is the need to bring mathematics education to artists and creative engineers, in both the Netherlands and Hungary.
Inspiration and support for our attempts to bring mathematics and computation to the attention of artists and creative engineers, and enrich their set of skills, comes among others from the *processing*[1] community, that has arisen out of a long-standing tradition of what may be characterized as *aesthetics by numbers*, pioneered at MIT, Reas & Fry (2008). Computation and mathematics have been adopted by new media artists, using *processing*, *flash* and other graphical frameworks with stunning works, available in many online galleries all over the web, see for example Jared Tarbell's Computation Gallery[2]. For (future) game designers and developers, knowledge of mathematics needs no argument, as exemplefied by the many sites about game physics and math for game designers[3].

structure The structure of this paper is as follows. First we will sketch our motivations and the background for this work, that is the educational context in which our developments take place, then we will present our first attempts of defining a taxonomy and overview of the use of computer graphics and games related to mathematics education. After that we will present our insights and (limited) experiences with using visual programming and game design in teaching mathematical concepts. With an overview of available tools and technologies, focused, in particular on the presentation of math and graphics on the web, we conclude this paper by outlining how we think to deal with the motivational issues that provided the inspiration of our work, as well as issues for future research and development.

[1] www.processing.org
[2] www.complexification.net/gallery
[3] gamemath.wikicomplete.info

MATH IN THE CREATIVE TECHNOLOGY CURRICULUM

At our university, we are currently developing the new BsC curriculum Creative Technology[4], with a paradigm shift compared to traditional academic education, advocating exploratory learning of maths and programming. See our blog at creativetechnology.eu

We are not the only ones to observe that the attitude of our pupils and students is changing. The young generations of our days are:

student(s)

- highly visually oriented, they see and learn by trying out things rather than consulting traditional textbooks;
- proficient with using computers (games) at early age,
- less interested in science, and particularly, in mathematics, and do not perform well in these respects in many Western societies

This trend is to be noticed by the decay of interest in programming and mathematics academic educations in many countries, and the attempts of using interactive games and computer graphics as attractive vehicle towards these disciplines, Duchowski & Davis (2007).

We observe that the power of visual representation and active involvement (including manipulative tasks) was identified as key to education from the earliest times. Nowadays videogames hold a strong promise to that effect, Gee (2003). Also in maths education the 'manipulative' power, albeit without computer, has been proven valuable decades ago. Interactive games and computer graphics open new ways for mathematical investigations exploiting visual, manipulative and explorative aspects.

TOWARDS A TAXONOMY OF MATH GAMES

To determine how games and interactive graphics can be used in math education, we present a (preliminary) classification of games related or involving math. For a more extensive discussion, with illustrations, see Ruttkay & Eliens (2009).

taxonomy of math games

1. games to practice routine tasks
2. manipulative virtual environments for *try-out*
3. simulations for making conjectures
4. strategic games – manipulative & combinatorial
5. visualization of structure(s) – patterns of nature
6. explorations in 2D and 3D geometry
7. generative (visual) art

[4] create.ewi.utwente.nl

Simple computer games (1) may be seen as a wrapping to practice some repetitive routine task, like mental arithmetic. Classical puzzles (2) include the 3 jars problem requiring to measure some k liter of water having empty jars of m and n liters and a full jar of n+m liters[5]; or the puzzle of 3 cannibals and 3 missionaries who all need to cross a river and remain alive, using a 2 persons boat[6] as well as the wolf-goat-cabbage puzzle[7]. For many mathematical questions (3) it is useful to check empirically many concrete cases, in order to gain some insight and guess the answer (which will then have to be proven). The design and interpretation of the visualization is straightforward for problems of geometrical nature (or ones which have a natural geometrical analog), as well as for guessing probabilistic distributions. But e.g. visualizing number distributions may provide surprising beauty as well as some mathematical insight[8]. Combinatorial graph games (4) involve the finding of some characteristic subgraph (Hamilton path, Euler path), or changing the graph in an iterative way (against an opponent) to reach some goal. For young kids, the graph may be presented in a real-life context, as roads in a town[9]. Natural forms (5) lend themselves as excellent topics for experimentation with parametric generation and mathematical analysis of these forms, and the underlying generation mechanisms. Similarly, Perlin noise is an excellent vehicle to learn about probability, and shown as basic principle for computer-generated natural phenomena, ranging from mountain highlights to flocking behaviour. For these phenomena, there seems to be ample teaching material[10]. In interactive geometry applications (6), a drawing usually illustrating some theorem in planar geometry can be modified by dragging some point on it, resulting in adjustment of the other, related points and lines. We developed an interactive environment to reproduce historical drawing machines[11] that allows children to explore various geometrical transformations, Ruttkay (2008). As an interesting example of generative art (7) we mention Vera Molnar[12].

LEARNING MATH BY VISUAL PROGRAMMING AND GAME DESIGN

In the previous section we have indicated how games and interactive CG applications can enhance maths

[5] www.cut-the-knot.org/water.shtml
[6] zap.psy.utwente.nl/zaps/zaps/zaps/kannibalen.res/frames.html
[7] www.mathcats.com/explore/river/crossing.html
[8] www.numberspiral.com
[9] www.coolmath-games.com/0-loopthecity
[10] www.shiffman.net/teaching/nature
[11] wwwhome.cs.utwente.nl/∼zsofi/machines
[12] www.atariarchives.org/artist/sec11.php

education. These, however, may easily lead to a passive attitude and no more than a scattered understanding of the mathematical principles involved. As the value of the construction of proofs by pupils themselves is well-recognized in traditional approaches to teaching mathematics, we also must consider the value of visual programming and game design assignments. Such assignments could range over creating some of the previously discussed examples, e.g. developing generative grammars for producing visual patterns, Terzidis (2008), programming simple physical systems with collisions, Peters (2006), or even the implementation of casual games. Such assignments would require the understanding of certain mathematical principles (e.g. Newtonian motion) while adopting them for a goal, or even would raise genuine mathematical challenges (e.g. getting insight into an invented strategic game). But the domain of the game itself may be mathematical as well one could think of inventing a game for graph exploration exercises, or devote a game to the life and work of some famous mathematician.

In developing our curriculum for the Creative Technology bachelor, we have chosen to adopt this more constructive approach to teaching and we have set ourselves the goal of assembling a coherent set of assignments, complementing our lectures in math, that allow the students to become intimately familiar with the mathematical principles involved in their (creative) engineering practice, by hands-on programming experience, with a clear focus on visual programming and game development.

To enable the actual incorporation in our curriculum, we have planned, apart from a limited number of traditional math courses, a new course creative explorations, that will introduce a number of mathematical topics, which we consider to the distress of some mathematicians the *heart of mathematics*, some directly related to mathematical aspects of design, such as generative algorithms, geometry, perspective and noise, but others of a more distinct abstract nature, to make our students familiar with the reasoning underlying mathematics.

As we will argue in the next section in more detail, such an approach nowadays is made feasible by a variety of tools and techniques.

To promote a coherent application context of the fragments, to be developed by students, we formulate the outline(s) of a mathematics adventure game, centered around a (not entirely fictitious) figure, Mr. E:

Mr. E's knapsack

> Mathematician Mr. E is traveling the world with his knapsack full of (mathematical) knowledge. Exploring the world, he meets famous colleagues like Euclid, Euler, Gauss, Newton, Einstein, Gödel, Leibniz, etc. He has to collaborate - learning as well as teaching - to proceed, get goods, get a regenerating cup of coffee, and sometimes a place to stay. He uses his problem solving skills to discover the world, and to unravel it's Big Mystery. Mr. E's journey is taking him around the globe and through the history of science and mathematics.

Mr. E's knapsack is meant to be an adventure game in historic and scientific setting, which combines the attractiveness of a fantasy world as in games like Zelda and Star Wars, with education, by providing a context that is on the one hand fictional, but on the other hand provides scientific knowledge and historic information. It lets the character learn 'real' uses of mathematical items, since the player has to actively use such items somewhere else in the game, e.g. pick up a theorem at Euler and use it to solve a problem posed by Gauss.

This brief scenario stems from a group of mathematics (master) students of the University of Amsterdam, and originated in the first place from the wish to get a more clear idea on how all the mathematics concepts learned fit together, and in the second place to create an environment to teach mathematics in an integrated way, supplementing ordinary textbooks, ranging from high-school students to advanced master-level students. It is also meant to play a role in their work for De Praktijk[13], a company that develops new concepts for natural science education, primarily targeting high-school students.

An adventure game, that we take as a convenient metaphor for an online mathematics textbook, provides flexible access to a variety of illustrations and exercises dependent on the skill level of the students, and allows for extensions by student-created content both visual (e.g. generated plants) and intellectual (e.g. a question raised for the future players). In addition interactive video may be used to provide the necessary historical and societal context, Eliens et al. (2008).

MATHS ON THE WEB – FRAMEWORKS AND TOOLS

When Java applets were introduced about a decade ago, there was a general enthusiasm among educators, at least in CS departments, that their topics - mathematics, programming languages - could be presented in an interactive way. However, although Java has been adopted as the programming language of choice at many universities, the early vision(s) of interactive education has not been realized, despite the increasingly large collection of interactive examples of mathematics, physics, and related disciplines on the Web. When we look at how, for example, Wikipedia has become a trusted source of information on mathematics, we may attribute this to the lack of a coherent framework for

[13] www.praktijk.nu

incorporating interactive math examples. Such a framework, apparently, is more easily provided for (hyper) textual information, or images and video, as testified by the large amounts of user-contributed content in sites such as flickr and youtube.

Is it possible to create a similar repository for samples of interactive mathematics, and, on a more modest scale, to formulate guidelines to enforce a coherent approach in both (visual) style and (mathematical) content?

In developing our Creative Technology curriculum, we discussed computer programming platform options ranging over C++, flash/actionscript and, of course, Java. Finally, as indicated before, we have chosen for processing, as a language to teach programming, but foremost as a platform for exploring both mathematical and visual ideas in an exploratory fashion using computational means, suited for educating creative engineers. Although, clearly, we strongly endorse the visually-oriented programming-by-example paradigm supported by the processing environment, we will in a later phase of the curriculum also introduce other programming environments, in particular javascript/php, flex/as3, C++ with suitable libraries, as well as one or more of the many game engines and game development platforms such as Unity3D and the Half Life 2 SDK. Where the choice for processing was motivated by both ease-of-use and ease-of-learning, the other platforms and environments are primarily motivated by reasons of deployment, that is the means available by our (target) audience. In our efforts to decide on a first programming language, we found that processing libraries are available for both javascript[14] and (Open Frameworks[15]) C++, enabling a direct transfer of skills and knowledge. when using *processing* to teach (introductory) programming. Adobe flex/as3 would provide a strong alternative, not only with regard to popularity of the flash player, but also looking at issues of efficiency and support for graphics. For example, the effects that can be obtained using pixelbender[16] shaders, using relatively simple mathematical formulae, are simply stunning. However, we found that the flash CS3/4 tool would rather confound our methods of teaching. Also, the many examples made by code-artists, as they are called in the *processing* community, seem to fit the targets we set our students better than the bewildering amount and variety of flash examples.

MOTIVATIONAL ISSUES – THE HEART(S) OF MATHEMATICS

Mathematics, by nature, is an abstract and mental discipline. The formal notations and the among mathematicians agreed rules of deriving proofs form the basis of the traditional pure mathematical textbooks, reminiscent e.g. of the style of the Bourbaki school, Aubin (1997). While reading such a pure and formal work may be considered as the way of coding and transferring mathematical knowledge, and provide high intellectual (even, aesthetical) pleasure for the professional, it does not raise the interest or provide an entry to grasp the very idea and the essence of the topic, neither the (exploratory) road of discovery to it, in an individual or cultural-historical context. However, accounts on mathematical discoveries are rich in visual representations, enlightening analogical stories, inspirations gained by observation of (natural) patterns, Chalmers & Cunningham (2002), and as such an excellent topic for a visually appealing adventure game.

guidelines for a maths adventure game A loosely coupled collection of game components, organized as an adventure game, is a excellent means to avoid the linear format of traditional textbooks, and by allowing students to contribute content, a unique facility to learn mathematics by exploration, as well as, actively, constructing (computational) samples themselves. A detailed description of the architecture of such a system, however, is outside the scope of this paper, and, clearly, an issue for further research. Perhaps more important than the possible technical solutions is to indicate what type of interaction we wish to achieve, and how guidelines with respect to visual style may be enforced.

As for interaction, interesting examples are provided by a collection of physics-driven construction game(s)[17], where vehicles must be assembled that will then either roll to their destination or crash halfway, dependent on the skill of the player, and, roughly based on the same principles, the Magic Pen[18] game where the player can simply draw shapes, that then get a life dependent on the physical laws at work. Both games, in our mind, however, are lacking in (aesthetic) visual style and, frankly, too childish. An interesting example of a game where game play is determined by drawing is MijnNaamIsHaas[19], meant to teach children vocabulary (in Dutch). Dependent on the vocabulary learnt so far, the game reacts on the childs drawing, intelligently, by generating additional content. Using a simple line style, with simple visual effects mimicking child-like drawing, this game may act as a reference for the aesthetics of our mathematics adventure game, where intelligent drawing tools obviously may play an important role in interaction, and applying well-established techniques of code-generation, be used to store code-fragments in a database of game components.

[14] processingjs.org
[15] www.openframeworks.cc
[16] labs.adobe.com/technologies/pixelbender
[17] funnygames.nl/spel/constructie_bouwer.html
[18] magic.pen.fizzlebot.com
[19] mijnnaamishaas.nl

CONCLUSIONS AND FUTURE WORK

It is apparent, when browsing through resources, how many applications lack aesthetic appeal and design wit, usually because the author is a computer scientists or mathematician not equipped with the skills, or the environment he/she used to create the content does not provide good support for the visual design.
From an educational, or if you will, methodological point of view, games and interactive CG may be used:

methodology

- as a means to provide illustrations or trigger interest
- in assignments to exercise routine tasks
- to enhance the curriculum (additional explorations)
- as the major theme and medium to teach mathematics
- (and) to learn algorithms and programming

In our assessment, the above list also indicates the big jumps in challenges to develop the kind of application - starting from stand-alone puzzles via explorative environments to full-fledged interactive course materials. The challenges encompass both technological/design and mathematical/pedagogical aspects.
By setting our ourselves the goal of a comprehensive mathematics adventure game, we not only hope to improve our teaching of mathematics, but also find appropriate styles and patterns that may guide the development of math (related) games and applications, Björk & Holopainen (2005).
In conclusion, answering the somewhat provocative question in the title of our paper, *no*, (math) *games will not replace* (traditional) *math education*, but (may) augment it with challenges and explorations, supported by the new developments in technology.

ACKNOWLEDGEMENT(S) The MrE's Knapsack game concept was developed by Sebastiaan Eliëns and Jacobien Carstens[20], students from the University of Amsterdam.

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[20] www.jacobiencarstens.com/projects.htm