Program of IWOTA 2014

Amsterdam

VU university

Organizing committee:
Tanja Eisner
Birgit Jacob
André Ran (chair)
Hans Zwart

Thanks

The organizers gratefully acknowledge support from:

The Netherlands Organisation for Scientific Research (NWO)

VU University Amsterdam, Faculty of Sciences, Department of Mathematics

Stichting Advancement of Mathematics

National Research Cluster for Nonlinear Dynamics in Natural Systems (NDNS+)

Foundation Compositio

IWOTA Steering Committee: T. Ando (Sapporo), J.A. Ball (Blacksburg), H. Bart (Rotterdam), T. Bhattacharyya (Bangalore), J. Behrndt (Berlin), H. Bercovici (Bloomington), V. Bolotnikov (Williamsburg), R.E. Curto (Iowa City), A.F. dos Santos (Lisbon), A. Dijksma (Groningen), M. Dritschel (Newcastle), H. Dym (Rehovot), T. ter Elst (Auckland), K.-H. Foerster (Berlin), C. Foias (Bloomington), J.J. Grobler (Potchefstroom), G.J. Groenewald (Potchefstroom), J.W. Helton (La Jolla), I.B. Jung (Seoul), M.A. Kaashoek (Amsterdam), M. Klaus (Blacksburg), H. Langer (Vienna), W.Y. Lee (Seoul), C.V.M. van der Mee (Cagliari), R. Mennicken (Regensburg), A. Montes Rodriguez (Seville), N.K. Nikolskii (Bordeaux), V. Olshevsky (Storrs), P. Portal (Lille, temp. Canberra), D. Potapov (Sydney), L. Rodman (Williamsburg), S. Seatzu (Cagliari), I. Spitkovsky (Williamsburg), G. Stampfli (Bloomington), C. Trunk (Ilmenau), N. Vasilevski (Mexico City), V. Vinnikov (Beer-Sheva), N. Young (Leeds).

Previous IWOTA meetings. Santa Monica (CA, USA) [1981], Rehovot (Israel) [1983], Amsterdam (Netherlands) [1985], Mesa (AZ, USA) [1987], Rotterdam (Netherlands) [1989], Sapporo (Japan) [1991], Vienna (Austria) [1993], Regensburg (Germany) [1995], Bloomington (IN, USA) [1996], Groningen (Netherlands) [1998], Bordeaux (France) [2000], Faro (Portugal) [2000], Blacksburg (VA, USA) [2002], Cagliari (Italia) [2003], Newcastle (UK) [2004], Storrs (CT, USA) [2005], Seoul (Korea) [2006], Potchefstroom (South Africa) [2007], Williamsburg (VA, USA) [2008], Guanajuato (México) [2009], Berlin (Germany) [2010], Seville (Spain) [2011], Sydney (Australia) [2012], Bangalore (India) [2013] .

Three special occasions

The year 2014 marks three special occasions: it would have been the 100th anniversary of Vladimir Petrovich Potapov, it is the 80th birthday of Damir Arov, and the 65th birthday of Leiba Rodman. The latter two events will be celebrated at the conference on Tuesday and Thursday, respectively.

Vladimir Petrovich Potapov was born on January 24, 1914 in Odessa in the family of a privat-docent in the historical-filological department of the Newrussian University. He was taught by his parents, and didn't go to school up to class 10. After three years of music study (piano) in the Odessa conservatorium, Potapov became a student of the math-physics department of the Odessa University. He studied there from 1934 to 1939, attending lectures from, among others, M.G. Kreĭn and B. Ya. Levin.

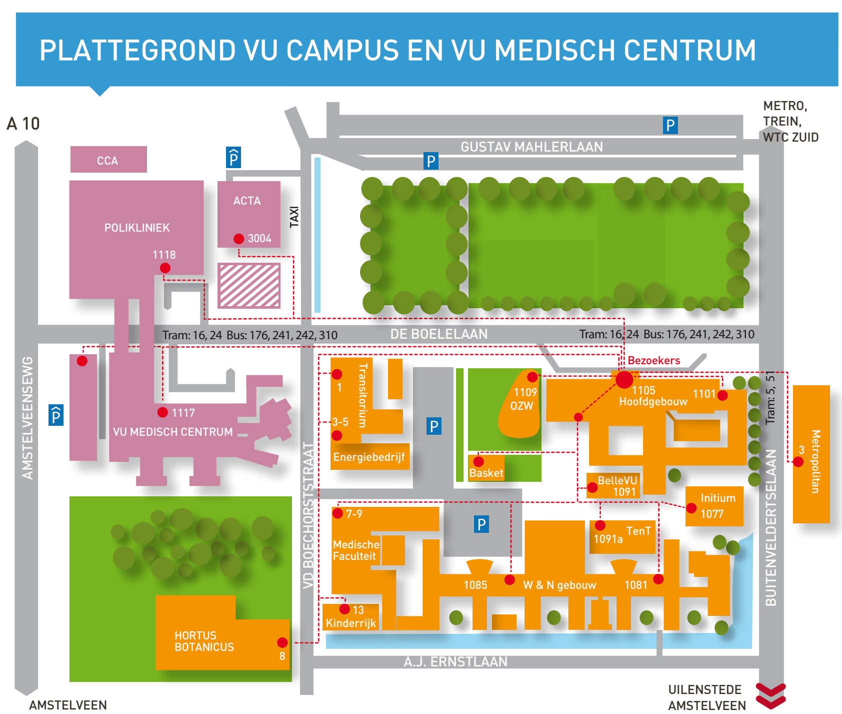
V.P. Potapov was chief of mathematical division and later dean of the department of physics and mathematics of the Odessa Pedagogical Institute (1948–1956). Thanks to him many mathematicians from the Kreĭn school, who couldn't work at Odessa University in view of the so-called fifth paragraph, were employed at Odessa Pedagogical Institute. Among them were M.S. Livshic, D.P. Milman, M.A. Rutman, and M.S. Brodskii. At that time also L.A. Sahnovich, E.Ya. Melamud, Yu.P. Ginzburg, and D.Z. Arov were graduate students in this institute. At the end of his life Potapov was the chief of Applied mathematical department of the Institute of lower temperature in Kharkov (1976–1980). His seminars in Odessa and than in Kharkov on the theory of J-contractive matrix functions and related problems attracted many mathematicians who are active in this area, and important contributions in this area were frequently presented first at this seminar.

A more extensive biography and a photo of V.P. Potapov can be found in Operator Theory Advances and Applications volume 95.

(The organizers are grateful to Dima Arov for providing the short biography above.)

Venue: Department of Mathematics, Faculty of Sciences, Vrije Universiteit, De Boelelaan 1085, Amsterdam

To reach the Vrije Universiteit from the city centre, use the GVB ticket that is provided in the conference material. This ticket will give you free public transport within the city of Amsterdam for six days after the first activation.
Take either tram line 5 (from city centre) or the bus line 55 (e.g., from Amstel Station when you are staying at hotel Casa 400) in the direction of Amstelveen. Get off at the stop called VU (immediately after the stop Station Zuid/WTC).
When exiting the tram 5 turn left on the platform and walk to the south side of it. Bus 55 stops directly at this point. There, turn right, crossing the street (careful please!!!!). In front of you is now a footpath and bike-path. Follow it, passing underneath a grey building (marked "Initium" on the map below). In front of you is a blue, low building (marked "TenT" on the map below). Continue straight, keeping that building on your left hand side. Keep on going straight, untill a parking lot opens up to your left. There, turn left, and enter the long building that is now in front of you at the entrance marked 1085. Enter into the building, and follow the signs to get to the registration desk.
There are also other ways of reaching the VU campus if your hotel is not directly in reach of lines 5 or 55. Tram lines 16 and 24 stop in front of the main building (marked "Hoofdgebouw"). Many buses also stop within easy walking distance of the conference location. Metro line 50 has a stop at station Amsterdam Zuid/WTC.
To reach the VU from Station Zuid/WTC you can take tram line 5 or bus line 55 in the direction of Amstelveen (one stop), or, if you feel like a short walk, it is possible to walk it (count on ten to fifteen minutes if you don't know the way).

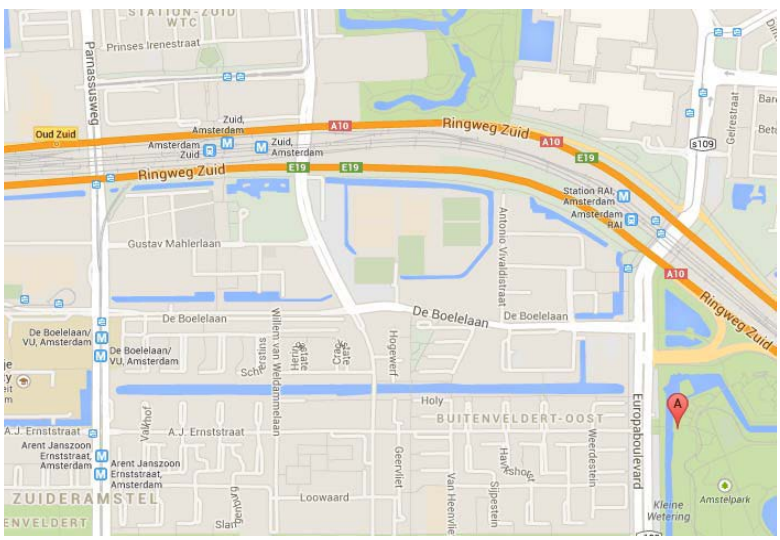


Social Program

There are several social events planned during the conference.
On Wednesday afternoon there will be a guided tour through the recently renovated Rijksmuseum. The Rijksmuseum can be reached from the VU by tram line 5.
On Thursday the conference dinner is taking place, starting at 19.15 in restaurant Rosarium. The conference dinner is included in the regsitration fee for participants of the conference.

How to get to the restaurant?

The walk takes about 20-25 minutes (see the map below). Find the road that runs behind the Sciences building (the A.J. Ernststraat), and turn onto it in an easterly direction. Walk this road all the way to the end, cross the (double) main road at the end and walk across the bridge into the park in front of you. The restaurant is now immediately on your right hand side. Halfway down the A.J. Ernststraat there is the possibility to take bus line 62 in the direction of Amstelstation. Get off at the next stop for the restaurant.
Alternatively, take bus 55 to station Rai and switch to bus 62 in the direction of Station Lelylaan. Again, it is only one stop.



How to get from the restaurant to your hotel?

To get back from the restaurant to hotel Casa 400 take bus 62 in the direction Amstelstation. To get back to city centre, either take bus 62 to Rai station, and switch to tram line 4, or get back to the VU and take one of the tram lines there.

Lunches

Lunch is not included in the registration fee. There are several possibilities to have lunch on campus. There is a restaurant in the sciences building, just behind the KC rooms on the ground floor. There is a small supermarket, "Spar", on campus, located in the sciences building. However, it can only be entered from outside the building. On and around the campus there are also other possibilities.

General outline of the program of
IWOTA 2014 conference
Amsterdam, 2014

The program has the following structure:

- Every morning from 9:00 to 10:00 there is a plenary lecture in KC 137.
- Coffee break from 10:00 to 10:30.
- Parallel semi-plenary lectures from 10:30 to 12:00 in KC 137 and KC 159.
- Lunch break 12:00-13:30.
- On Monday, Tuesday, Thursday and Friday afternoon from 13:30 to 17:30: sessions in the lecture rooms on the sixth floor.
- Coffee break from 15:15 to 15:45.
- Wednesday afternoon is free.

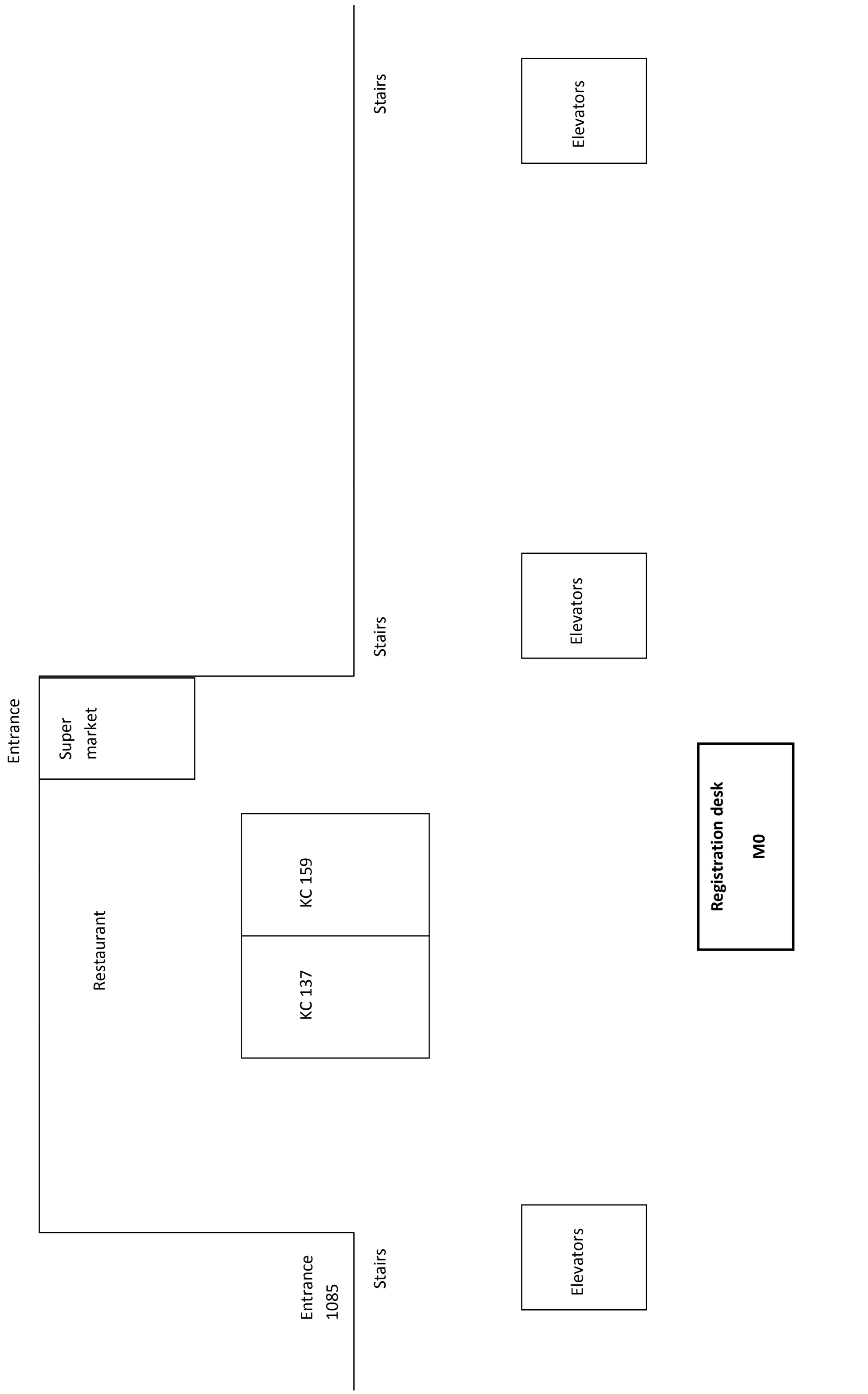
Rooms for the IWOTA 2014 conference are as follows:

- The opening session, the plenary lectures and the closing session will be held in KC 137.
- Semi-plenary lectures will be held in parallel in KC 137 and KC 159.
- Invited sessions and contributed parallel sessions are held on the sixth floor, in rooms in the M, P and S corridors. These can be reached by elevators or stairs. There are two elevators near the stairs, and a larger one in the main corridor.
- Coffee and tea breaks will be held in M 0, the main lobby of the Sciences building. The registration desk will also be there, as well as the booth of Birkhäuser.

For a layout of the ground floor see the next page.

There is wireless internet available via eduroam.

For discussions there are small lecture rooms available close to the rooms where the sessions will be held: in the M wing of the building on the sixth floor there are several rooms that are generally available. All lecture rooms are equipped with a blackboard, a computer and a beamer.


Stairs
Elevators
Stairs
Elevators
Entrance
Super market
Restaurant
KC 159
KC 137
Registration desk
M0
Entrance 1085
Stairs
Elevators

Program of IWOTA 2014

List of sessions

Title/Organizers	Acronym	Days
Linear operator theory, function theory, and linear systems Organizers: Joe Ball and Rien Kaashoek	OTFS	Mon,Tue,Thurs
Free noncommutative function theory and free real algebraic geometry Organizers: Bill Helton and Igor Klep	FN	Tue, Thurs
Spectral theory for Sturm-Liouville and differential operators Organizers: Jussi Behrndt and Carsten Trunk	SL	Mon, Fri
Clifford analysis and operator theory Organizers: Daniel Alpay, Fabrizio Colombo, Uwe Kähler, Frank Sommen	Cliff	Mon
Dynamics of linear operators Organizers: Sophie Grivaux, Catalin Badea, Frederic Bayart	DLO	Thur
Continuous and discrete hypercomplex analysis Organizers: Paula Cerejeiras and Irene Sabadini	CDHA	Tue
Infinite dimensional systems Organizers: Birgit Jacob and Hans Zwart	IDS	Thurs
Semigroups: theory and applications Organizers: Andras Batkai and Balint Farkas	STA	Tue, Thurs, Fri
Free probability and operator theory Organizers: Roland Speicher, Serban Belinschi and Moritz Weber	FPOT	Mon
Toeplitz operators and related topics Organizers: Sergei Grudsky and Nikolai Vasilevski	TOEP	Mon, Thurs
Partial differenatial operators and potential method Organizers: Roland Duduchava and Vladimir Rabinovich	PDO	Tue, Fri
Operator theory and harmonic analysis Organizers: Alfonso Montes-Rodriguez, Haakan Hedenhalm and Manuel Cepedello	OTHA	Thurs
Concrete operators Organizer: Alfonso Montes-Rodriguez	CO	Mon
Contributed talks	Contr	Mon, Tue, Thurs, Fri
Special day on the occasion of the 65th birthday of Leiba Rodman Operators, matrices and indefinite inner products Organizers: Christian Mehl and Michal Wojtylak	OMII	Thurs, Fri
Special day on the occasion of the 80th birthday of Damir Arov Dima Arov's world Organizers: Harry Dym and Olof Staffans	DA	Tue

Program at a glance

Monday	Time	Plenary	Semiplenary	Semiplenary	Session 1	Session 2	Session 3	Session 4	Session 5	Session 6	Session 7	Session 8	Session 9
Room		KC137	KC137	KC159	M607	M623	M639	M655	P647	P 663	S 623	S655	S 631
	08.50-09.00	Opening											
	09.00-10.00	Tisseur											
	10.30-11.15		Tretter	ter Elst									
	11.15-12.00		Wyss	Boettcher									
	13.30-15.15				Cliff	SL	FPOT	CO	OTFS	Toep	Contr	Contr	
	15.45-17.30				Cliff	SL	FPOT	CO	OTFS	Toep	Contr	Contr	
Tuesday													
	09.00-10.00	Arov											
	10.30-11.15		Dym	Batkai									
	11.15-12.00		Staffans	Haase									
	13.30-15.15				DA	STA	FN		PDO	OTFS	CDHA	Contr	
	15.45-17.30				DA	STA	FN		PDO	OTFS	CDHA	Contr	
Wednesday													
	09.00-10.00	Parrilo											
	10.30-11.15		Trunk	Opmeer									
	11.15-12.00		Behrndt	Frazho									
	13.30-17.00	Excursion											
Thursday													
	09.00-10.00	Rodman											
	10.30-11.15		Lancaster	Chill									
	11.15-12.00		Mehl	Shalit									
	13.30-15.15				OMII	IDS	FN	OTHA	DLO	OTFS	Toep	Contr	STA
	15.45-17.30				OMII	Contr	Contr	OTHA	DLO	OTFS	Toep	Contr	STA
Friday													
	09.00-10.00	Partington											
	10.30-11.15		Dritschel	Brown									
	11.15-12.00		Woerdeman	Malamud									
	13.30-15.15				PDO	SL	Contr	STA	OMII		Contr	Contr	
	15.45-17.30				Contr	Contr	Contr	STA			Contr	Contr	

Program of IWOTA 2014

Time	Monday July 14
08.00	Registration M0
08.50	Welcome by the rector of the VU KC 137 **Frank van der Duyn Schouten** Welcome by professors William Helton and Marinus Kaashoek on behalf of the IWOTA Steering Committee.
09.00	**Francoise Tisseur** *Structured matrix polynomials and their sign characteristic: classical results and recent developments*
10.00	Coffee break M0 **Semi-plenary lectures**

	KC 137	KC 159
10.30	**Christiane Tretter** *Quadratic and block numerical ranges (QNR and BNR) of operators and operator functions*	**Tom ter Elst** *On convergence of sectorial forms*
11.15	**Christian Wyss** *Dichotomy, spectral subspaces and unbounded projections*	**Albrecht Böttcher** *Markov inequalities, norms of Volterra operators and zeros of Bessel functions*

Time	Monday July 14
12.00	Lunch
13.30	**Sessions**

Cliff	**SL**	**FPOT**	**CO**	**OTFS**	**TOEP**	**Contr**
Invited session	Invited session	Invited session	Invited session	Invited session	Invited session	2 parallel sessions
M607	M623	M 639	M 655	P647	P 663	S623 and S655

Time	Monday July 14
15.15	Coffee break
15.45	**Sessions**

Cliff	**SL**	**FPOT**	**CO**	**OTFS**	**TOEP**	**Contr**
Invited session	Invited session	Invited session	Invited session	Invited session	Invited session	2 parallel sessions
M607	M623	M 639	M 655	P647	P663	S623 and S655

Time	Monday July 14
17.30	End of sessions

Time	Tuesday July 15
	KC 137
09.00	**Dima Arov**
	My way in mathematics
10.00	Coffee break M0
	Semi-plenary lectures

	KC 137	KC 159
10.30	**Harry Dym**	**Andras Batkai**
	Twenty years after	*Error bounds for exponential integrators*
11.15	**Olof Staffans**	**Markus Haase**
	The linear stationary state/signal systems story	*Enhancing Techniques in Operator Theory with Applications to Functional Calculi*

Time	
12.00	Lunch
13.30	**Sessions**

DA	**STA**	**FN**	**PDO**	**OTFS**	**CDHA**	**Contr**
Invited session	Invited session	Invited session	Invited session	Invited session	Invited session	
M607	M623	M 639	P647	P663	S623	S655

Time	
15.15	Coffee break M0
15.45	**Sessions**

DA	**STA**	**FN**	**PDO**	**OTFS**	**CDHA**	**Contr**
Invited session	Invited session	Invited session	Invited session	Invited session	Invited session	
M607	M623	M 639	P647	P663	S623	S655

Time	
17.30	End of sessions

Time	Wednesday July 16		
	KC 137		
09.00	**Pablo Parrilo**		
	Positive semidefinite rank		
10.00	Coffee break M0		
	Semi-plenary lectures		
		KC 137	KC 159
	10.30	**Carsten Trunk**	**Mark Opmeer**
		Perturbation and spectral theory for J-non-negative operators and applications	*The algebraic Riccati equation*
	11.15	**Jussi Behrndt**	**Jim Agler**
		Spectral and extension theory of elliptic partial differential operators	*The Nevanlinna-Pick and Cartan Extension Theorems in Non-Commuting Variables*
12.00	Lunch		
12.00-12.45	Steering Committee Lunch in P423		
13.30	**Excursion** Rijksmuseum		

Time	Thursday July 17
	KC 137
09.00	**Leiba Rodman**
	Quaternion linear algebra
10.00	**Steering committee announcements: Future Iwota's**
10.05	Coffee break M0
	Semi-plenary lectures

	KC 137	KC 159
10.30	**Peter Lancaster**	**Ralph Chill**
	Spectral analysis for matrix polynomials with symmetries	*Extrapolation of maximal regularity and interpolation*
11.15	**Christian Mehl**	**Orr Shalit**
	Generic rank-one perturbations: structure defeats sensitivity	*The isomorphism problem for complete Pick algebras*

Time	
12.00	Lunch
13.30	**Sessions**

OMII	**IDS**	**FN**	**OTHA**	**DLO**	**OTFS**	**TOEP**	**STA**	**Contr**
Invited session	Invited session	Invited session	Invited session	Invited session	Invited session	Invited session	Invited session	
M607	M623	M 639	M 655	P647	P663	S623	S631	S655

Time	
15.15	Coffee break M0
15.45	**Sessions**

OMII	**Contr**	**Contr**	**OTHA**	**DLO**	**OTFS**	**TOEP**	**STA**	**Contr**
Invited session				Invited session	Invited session	Invited session	Invited session	
M607	M623	M 639	M 655	P647	P663	S623	S631	S655

Time	
17.30	End of sessions
19.00	Conference Dinner Rosarium

Time	Friday July 18
	KC 137
09.00	**Jonathan Partington**
	Zero–one laws for functional calculus on operator semigroups
10.00	Coffee break M0
	Semi-plenary lectures
12.00	Lunch
13.30	**Sessions**
15.15	Coffee break M0
15.45	**Sessions**
17.05	End of sessions

Semi-plenary lectures

	KC 137	KC 159
10.30	**Michael Dritschel** *Rational and H^∞ dilation*	**Malcolm Brown** *Scattering and inverse scattering for a left-definite SturmLiouville problem*
11.15	**Hugo Woerdeman** *Stable and real-zero polynomials, and their determinantal representations*	**Mark Malamud** *Perturbation determinants and trace formulas for singular and additive perturbations*

Sessions (13.30)

PDO	**SL**	**STA**	**OMII**	**Contr**
Invited session	Invited session	Invited session	Invited session	3 parallel sessions
M607	M623	M 655	P647	M 639, S623 and S655

Sessions (15.45)

STA	**Contr**
	5 parallel sessions
M655	M607, M 623 M, 639, S623 and S655

Sessions on Monday afternoon

Time	M 607 Cliff	M 623 SL	M 639 FPOT	M 655 CO	P 647 OTFS	P 663 TOEP	S 623 Contr	S 655 Contr
13.30-13.55	M. Peloso	J. Brasche	D. Alpay	F. Botelho	R. G. Douglas	W. Bauer	M. Möller	T. Bhattacharyya
13.55-14.20	D. Kimsey	Y. Christodoulides	T. Mai	R. Marreiros	N. Young	Z. Cuckovic	S. Pyatkov	G. Wanjala
14.20-14.45	B. Schneider	P. A. Cojuhari	V. Vinnikov	A. C. Conceição	D. Estévez	M. Loaiza	T. Roth	S. Belyi
14.45-15.10	F. Colombo	M. Demuth	J. Williams	G. Kohr	E. Shamovich	R. Quiroga-Barranco	L. Rzepnicki	P. Junghanns
15.50-16.15	D. Eelbode	C. Kühn	U. Franz	Aneesh M	J. H. van Schuppen	G. Rozenblum	B. Zinsou	V. Prokhorov
16.15-16.40	M. Martin	M. Langer	F. Lemeux	M. Meyries	B. Hanzon	J. Taskinen	I. Sheipak	P. Stovicek
16.40-17.05	D. Alpay	V. Mogilevskii	S. Raum	H. Queffélec	A. J. Sasane	H. Upmeier	A. Boitsev	M. Sarih
17.05-17.30	R.S. Kraußhar	A. Posilicano	A. Thom	D. Seco	F. van Schagen	N. Vasilevski	T. Selig	

Sessions on Tuesday afternoon

Time	M 607 DA	M 623 STA	M 639 FN
13.30-13.55	V. Adamyan	C. Batty	V. Vinnikov
13.55-14.20	V. Adamyan	Y. Tomilov	J. Cimprič
14.20-14.45	D. Kaliuzhnyi-Verbovetskyi	D. Seifert	S. Burgdorf
14.45-15.10	N. Rozhenko	S. Król	M. Brešar
15.50-16.15	J. Rovnyak	S. Verduyn Lunel	J. Ball
16.15-16.40	D. Pik	J. Glück	S. Kuhlmann
16.40-17.05	M. Kurula	M. Gerlach	Š. Špenko
17.05-17.30	V. Katsnelson	M. Kunze	D. Kaliuzhnyi-Verbovetskyi

Time	P 647 PDO	P 663 OTFS	S 623 CDHA	S 655 Contr
13.30-13.55	R. Duduchava	N. Nikolski	V. V. Kravchenko	A. Makin
13.55-14.20	J. Cruz-Sampedro	N. Nikolski	F. Sommen	C. Bartels
14.20-14.45	A. Merzon	A. Olofsson	S.-L. Eriksson	A. Khrabustovskyi
14.45-15.10	S. Mikhailov	V. Vinnikov	P. Cerejeiras	V. Vlasov and N. Rautian
15.50-16.15	M. Lanza de Cristoforis	D. Alpay	M. Vajiac	S. Di Bella
16.15-16.40	P. Musolino	Z. Lykova	U. Kähler	G. Bellomonte
16.40-17.05	J. Toft	M. Ghasemi	S. Bernstein	F. Gómez-Cubillo
17.05-17.30	V. Kovtunenko	M. Infusino	I. Sabadini	Ji Eun Lee

Sessions on Thursday afternoon

Time	M 607 OMII	M 623 IDS/Contr	M 639 FN/Contr	M 655 OTHA	P 647 DLO
13.30-13.55	J. Ball	B.Jacob	D. Farenick	A. Aleman	E. Matheron
13.55-14.20	H. Bart	C. Le Merdy	T. Piovesan	N. Arcozzi	A. Peris
14.20-14.45	A.Ran	D. Yakubovich	C. Kleski	A. Baranov	A. Lishanskii
14.45-15.10	S.Hassi	H. Zwart	J.W. Helton and I. Klep	I. Chalendar	H. Klaja
15.50-16.15	D. Janse van Rensburg	P. de Jager	M. Amouch	K. Seip	Y. Tomilov
16.15-16.40	L. Leben	A. T. Gürkanlı	Z. Pan	N. Vasilevski	J.A. Conejero
16.40-17.05	M. Wojtylak	A. Sandikci	S. Plaksa	V.Vasyunin	Q. Menet
17.05-17.30	H. Winkler	A. Moura Santos	G. Gogus		G. Costakis

Time	P 663 OTFS	S 623 TOEP	S 631 STA	S 655 Contr
13.30-13.55	Y. Arlinskiĭ	M. C. Câmara	R. Denk	A. Kuznetsova
13.55-14.20	R.T.W. Martin	S. Grudsky	M. Veraar	G. Salomon
14.20-14.45	A. Sakhnovich	A. Karapetyants	S. Wegner	R. Yang
14.45-15.10	S. ter Horst	Yu. Karlovich	J. Rozendaal	H. Trivedi
15.50-16.15	J.Arazy	O. Karlovych	M. Meyries	M. Aukhadiev
16.15-16.40	H. T. Kaptanoğlu	A. Sánchez-Nungaray	L. Paunonen	E. Lipacheva
16.40-17.05	B. Solel	M. A. Bastos	H. Vogt	T. Grigorian
17.05-17.30	D. Markiewicz		M. Adler	L. Molnar

Sessions on Friday afternoon

Time	M 607 PDO/Contr	M 623 SL/Contr	M 639 Contr	M 655 STA	P 647 OMII	S 623 Contr	S 655 Contr
13.30-13.55	V. Rabinovich	J. Rohleder	R. del Rio	M. Kramar Fijavž	M. Kaashoek	K. Rost	R. Gohm
13.55-14.20	B.-W. Schulze	S. Torba	Wen-Chi Kuo	M. Waurick	P. Šemrl	S. Sontz	I. Valusescu
14.20-14.45	A. Selitskii	M. Winklmeier	J. Peláez	F.Schwenninger	C. Camara	M. Seidel	V. Khatskevich
14.45-15.10	M. Kohr	V. Lotoreichik	L. Tovar	L. Abadias	A. Klein	H. Rabe	S. Furuichi
15.50-16.15	A.K. Motovilov	M. Schelling	K. Min	P. J. Miana		P. Santos	K. Paul
16.15-16.40	D. Barseghyan	I. Popov	B. Güntürk	J. A. Conejero		H. Mascarenhas	D. Sain

Abstracts

Plenary Invited Lectures

Francoise Tisseur	*Structured matrix polynomials and their sign characteristic: classical results and recent developments*	Mon.	9.00	KC 137
Damir Arov	*My way in mathematics*	Tue.	9.00	KC 137
Pablo Parrilo	*Positive semidefinite rank*	Wed.	9.00	KC 137
Leiba Rodman	*Quaternion linear algebra*	Thu.	9.00	KC 137
Jonathan Partington	*Zero–one laws for functional calculus on operator semigroups*	Fri.	9.00	KC 137

Semi-plenary Invited Lectures

Christiane Tretter	*Quadratic and block numerical ranges (QNR and BNR) of operators and operator functions*	Mon.	10.30	KC 137
Christian Wyss	*Dichotomy, spectral subspaces and unbounded projections*	Mon.	11.15	KC 137
Tom ter Elst	*On convergence of sectorial forms*	Mon.	10.30	KC 159
Albrecht Böttcher	*Markov inequalities, norms of Volterra operators, and zeros of Bessel functions*	Mon.	11.15	KC 159
Harry Dym	*Twenty years after*	Tue.	10.30	KC 137
Olof Staffans	*The linear stationary state/signal systems story*	Tue.	11.15	KC 137
András Bátkai	*Error bounds for exponential integrators*	Tue.	10.30	KC 159
Markus Haase	*Enhancing Techniques in Operator Theory with Applications to Functional Calculi*	Tue.	11.15	KC 159
Carsten Trunk	*Perturbation and spectral theory for J-non-negative operators and applications*	Wed.	10.30	KC 137
Jussi Behrndt	*Spectral and extension theory of elliptic partial differential operators*	Wed.	11.15	KC 137
Mark Opmeer	*The algebraic Riccati equation*	Wed.	10.30	KC 159
Jim Agler	*The Nevanlinna-Pick and Cartan Extension Theorems in Non-Commuting Variables*	Wed.	11.15	KC 159
Peter Lancaster	*Spectral analysis for matrix polynomials with symmetries*	Thu.	10.30	KC 137
Christian Mehl	*Generic rank-one perturbations: structure defeats sensitivity*	Thu.	11.15	KC 137
Ralph Chill	*Extrapolation of maximal regularity and interpolation*	Thu.	10.30	KC 159
Orr Shalit	*The isomorphism problem for complete Pick algebras*	Thu.	11.15	KC 159
Michael Dritschel	*Rational and H^∞ dilation*	Fri.	10.30	KC 137
Hugo Woerdeman	*Stable and real-zero polynomials, and their determinantal representations*	Fri.	11.15	KC 137
Malcolm Brown	*Scattering and inverse scattering for a left-definite SturmLiouville problem*	Fri.	10.30	KC 159
Mark Malamud	*Perturbation determinants and trace formulas for singular and additive perturbations*	Fri.	11.15	KC 159

Abstracts of Plenary talks

My way in mathematics

Damir Z. Arov
In this talk I will briefly review some of the main mathematical themes that I have worked on, and how one theme led to another. But first, I wish to express my thanks to A. Bobrov (the advisor of my Master's thesis) and V. Potapov and M. Krein who influenced my mathematical taste.
Over the years I moved from the subject of my Master's thesis on entropy in ergodic theory; to scattering theory; to passive linear stationary systems (including the Darlington method); to J-inner matrix functions and their application to interpolation and extension problems, direct and inverse spectral problems for integral and differential systems and prediction problems for weakly stationary vector valued stochastic processes.
In this journey, I had the good fortune to work with V. Adamyan and M. Krein (on scattering theory and the Nehari problem); H. Dym (on J-inner matrix functions and their applications); M. Kaashoek and our student D. Pik (on passive systems); J. Rovnyak and my former student S. Saprikin (also on passive systems); O. Staffans and his student M. Kurula (on state/signal passive systems) and B. Fritsche and B. Kirstein (on the Darlington method and related problems).
I also had the good luck to be the advisor of a number of talented PhD students: L. Simakova, M. Nudelman, L. Grossman, D. Kalyujnii-Verbovetskii, M. Bekker, O. Nitz and N. Rozhenko, who contributed to the development of a number of the topics mentioned above.

Zero–one laws for functional calculus on operator semigroups

Jonathan R. Partington

For a strongly continuous semigroup $(T(t))_{t>0}$ of operators defined on a Banach space, it has long been of interest to obtain qualitative information from estimates of operator-valued functions defined in terms of the semigroup.
For example, the classical (and elementary) 0–1 law asserts that if

$$\limsup_{t\to 0^+} \|T(t)-I\| < 1,$$

then in fact $\|T(t)-I\| \to 0$ and hence the semigroup is uniformly continuous and of the form e^{tA} for some bounded operator A.
Deeper is Hille's result [3] that if $(T(t))_{t>0}$ is differentiable and

$$\limsup_{t\to 0^+} \|tT^{'}(t)\| < 1/e,$$

then the generator of the semigroup is bounded.
We review these and more recent results for semigroups defined on the half-line or a sector: these involve norm estimates of functions defined in terms of Laplace transforms of measures. As a well-known special case, for the discrete measure $\delta_1-\delta_2$ the functional calculus gives the quantity $T(t)-T(2t)$; here there is the sharp result that if

$$\limsup_{t\to 0+} \|T(t)-T(2t)\| < 1/4,$$

then the semigroup has a bounded generator, and so the lim sup is 0.
The talk is based on joint work [1, 2] with Isabelle Chalendar (Lyon) and Jean Esterle (Bordeaux).

[1] Z. Bendaoud, I. Chalendar, J. Esterle, and J. R. Partington: Distances between elements of a semigroup and estimates for derivatives. *Acta Math. Sin. (Engl. Ser.)*, 26 (2010), 2239–2254.

[2] I. Chalendar, J. Esterle, and J. R. Partington: Lower estimates near the origin for functional calculus on operator semigroups. Submitted, 2014.

[3] E. Hille: On the differentiability of semi-group operators. *Acta Sci. Math. Szeged*, 12 (1950), 19–24.

Positive semidefinite rank

Pablo A. Parrilo

The *positive semidefinite rank* (psd-rank) of a nonnegative matrix M is the smallest integer k for which there exist positive semidefinite matrices A_i, B_j of size $k \times k$ such that $M_{ij} = \langle A_i, B_j \rangle$. The psd-rank is a "matrix analogue" of nonnegative rank, and has many appealing geometric interpretations. In particular, this rank is intimately related with the existence of semidefinite representations of polyhedra, and has several applications in quantum information theory. In this talk we describe the main mathematical properties of psd-rank, including its geometry, relationships with other rank notions, and computational and algorithmic aspects. The talk is based on joint work with Hamza Fawzi (MIT), Joao Gouveia (U. Coimbra), Richard Robinson and Rekha Thomas (U. Washington).

[1] J. Gouveia, P. A. Parrilo, R. Thomas, Lifts of convex sets and cone factorizations, *Mathematics of Operations Research*, Vol. 38, No. 2, pp. 248-264, 2013.

Quaternion linear algebra

Leiba Rodman

In recent years, problems in linear algebra over the skew field of real quaternions and related topics have attracted many researchers. In the lecture, a review of some recent results in the area will be given, with emphasis on quaternion numerical ranges and applications of canonical forms of pairs of quaternion matrices with symmetries. Open problems will be formulated.

Structured matrix polynomials and their sign characteristic: classical results and recent developments

Françoise Tisseur

In the first part of the talk we describe the sign characteristic of analytic selfadjoint matrix functions and in particular of monic selfadjoint matrix polynomials, and we recall some basic theorems. This introductory part is based on decades of results from I. Gohberg, P. Lancaster, and L. Rodman [1], [2].
Regular matrix polynomials with singular leading coefficients have eigenvalues at infinity. In the second part of the talk we outline the difficulties in associating a sign characteristic to eigenvalues at infinity and propose a definition that allows the extension of some theorems to the case of a regular selfadjoint matrix polynomial with singular leading coefficient.
Matrix polynomials with coefficients which alternate between Hermitian and skew-Hermitian matrices, or even with coefficient matrices appearing in a palindromic way commonly arise in applications. In the third part of the talk we also introduce a definition of sign characteristics for other important classes of structured matrix polynomials. Finally we discuss applications of the sign characteristic in particular in control systems, in the solution of structured inverse polynomial eigenvalue problems and in the characterization of special structured matrix polynomials such as overdamped quadratics, hyperbolic and quasidefinite matrix polynomials.
The talk is based on joint work with *Maha Al-Ammari*, *Yuji Nakatsukasa*, and *Vanni Noferini*.

[1] I. Gohberg, P. Lancaster, and L. Rodman. Spectral analysis of selfadjoint matrix polynomials. *Ann. of Math. (2)*, 112(1): 33–71, 1980.

[2] I. Gohberg, P. Lancaster, and L. Rodman. *Indefinite Linear Algebra and Applications.* Birkhäuser, Basel, Switzerland, 2005.

Abstracts of Semi-Plenary talks

The Nevanlinna-Pick and Cartan Extension Theorems in Non-Commuting Variables

Jim Agler

Let $\mathcal{M}_n^d$ denote the set of d-tuples of $n \times n$ matrices with complex entries and define $\mathcal{M}^d$, the *d-dimensional nc universe*, by

$$\mathcal{M}^d = \bigcup_{n=1}^{\infty} \mathcal{M}_n^d.$$

There are many ways to put interesting and distinctive topologies on $\mathcal{M}^d$. A particularly fruitful topology is the *free topology*, defined to be the topology that has as a basis the sets of form

$$G_\delta = \{x \in \mathcal{M}^d \mid \|\delta(x)\| < 1\}$$

where $\delta = [\delta_{ij}]$ is an $I \times J$ rectangular matrix with entries in $\mathbb{P}_d$, the algebra of polynomials in d non-commuting variables with coefficients in $\mathbb{C}$. If U is open in the free topology and $f : U \to \mathcal{M}^1$ is a graded function (i.e., $f(x) \in \mathcal{M}_n^1$ whenever $x \in \mathcal{M}_n^d \cap U$), then we say that f is a *free holomorphic function* if f is an nc-function (i.e., f preserves similarities and direct sums) and f is locally bounded in the free topology. In this talk I will describe a solution to the following problem.

Nevanlinna-Pick Interpolation Problem. Fix a basic set $G_\delta \subseteq \mathcal{M}^d$, nodes $\lambda_1, \ldots, \lambda_m \in G_\delta$ and targets $z_1, \ldots, z_m \in \mathcal{M}^1$. Find necessary and sufficient conditions for there exist a function ϕ satisfying

1. ϕ is free holomorphic on G_δ,
2. $\sup_{x \in G_\delta} \|\phi(x)\| \leq 1$, and
3. $\phi(\lambda_i) = z_i$ for each $i = 1, \ldots, m$.

I will then derive as a corollary the following result.

Free Cartan Extension Theorem. Let G_δ be a basic free domain and let V be free variety (i.e., V is the common 0-set of some subset of $\mathbb{P}_d$). If f is a bounded free holomorphic function on $V \cap G_\delta$, then there exists a free holomorphic function F on G_δ such that $f = F|V$. Furthermore, F may be chosen so that

$$\sup_{x \in G_\delta} \|F(x)\| = \sup_{x \in V \cap G_\delta} \|f(x)\|.$$

The talk is based on joint work with John McCarthy.

Error bounds for exponential integrators

András Bátkai

We consider semilinear evolution equations of the form

$$u'(t) = Au(t) + f(t, u(t))$$

and show how exponential integrators are used to solve them numerically. The name "exponential integrator" refers to a time-discretization method based on the variation-of-constants formula

$$u(t) = \mathrm{e}^{tA}u_0 + \int_0^t \mathrm{e}^{(t-s)A} f(s, u(s))ds$$

and can be used when we have a very good knowledge about the semigroup generated by A. First we introduce well-known convergence results form the parabolic case, then we present the idea how to derive error bounds for hyperbolic equations. Known and new examples will be presented, like the cubic nonlinear Schrödinger equation, the KdV equation or the shallow water equation. The talk is based on joint work with Petra Csomós.

Spectral and extension theory of elliptic partial differential operators

Jussi Behrndt

In this talk we illustrate how abstract methods from extension theory of symmetric operators can be applied in the spectral analysis of elliptic partial differential operators. In this context we discuss results on Schrödinger operators with δ-interactions on hypersurfaces, spectral estimates and asymptotics of the difference of selfadjoint realizations of elliptic PDEs, Dirichlet-to-Neumann maps and their relation to the spectrum, as well as selfadjoint realizations and maximal trace maps for the Laplacian on bounded Lipschitz domains.

Markov inequalities, norms of Volterra operators, and zeros of Bessel functions

Albrecht Böttcher

When equipped with an appropriate norm, the space $\mathcal{P}_n$ of all algebraic polynomials of a given degree n is a finite-dimensional Hilbert space. The operator D^ν of taking the νth derivative is bounded on this space, and determining the norm of this operator is equivalent to asking for the best constant C in the inequality $\|D^\nu f\| \leq C\|f\|$ $(f \in \mathcal{P}_n)$. Such inequalities are called Markov-type inequalities. The talk is devoted to the asymptotics of the constant C as n goes to infinity. The leading coefficient in the asymptotics turns out to be the norm of a Volterra integral operator, and hence the talk also surveys some results on the norms of Volterra operators, which is a fascinating story by itself. For example, in the case of first order derivatives ($\nu = 1$), the norm of the Volterra operator may be expressed in terms of the zeros of Bessel functions. In the simplest situation the coefficient is $2/\pi$, a number which occurs in Erhard Schmidt's 1932 investigation of the best constant in a Markov inequality, in Paul Halmos' 1967 result on the norm of the Cesàro-Volterra operator, and which is also the inverse of the first positive zero of the cosine function. The talk is based on joint work with Peter Dörfler from the Montanuniversität in Leoben, Austria.

Scattering and inverse scattering for a left-definite Sturm-Liouville problem

B. Malcolm Brown

This work develops a scattering and an inverse scattering theory for the Sturm-Liouville equation $-u'' + qu = \lambda wu$ where w may change sign but $q \neq 0$. Thus the left-hand side of the equation gives rise to a positive quadratic form and one is led to a leftdefinite spectral problem. The crucial ingredient of the approach is a generalized transform built on the Jost solutions of the problem and hence termed the Jost transform and the associated Paley- Wiener theorem linking growth properties of transforms with support properties of functions. One motivation for this investigation comes from the Camassa- Holm equation for which the solution of the Cauchy problem can be achieved by the inverse scattering transform for $-u'' + 1/4u = \lambda wu$.

Extrapolation of maximal regularity and interpolation

Ralph Chill

We show that if a first order Cauchy problem on a Banach space has L^p-maximal regularity for some $p \in (1, \infty)$, then it has $\mathbb{E}_w$-maximal regularity for every rearrangement invariant Banach function space $\mathbb{E}$ with Boyd indices $p_{\mathbb{E}}$, $q_{\mathbb{E}} \in (1, \infty)$ $(p_{\mathbb{E}} \leq q_{\mathbb{E}})$ and every Muckenhoupt weight $w \in A_{p_{\mathbb{E}}}$. We show how this follows from general extrapolation results due to Rubio de Francia, Curbera, Garcia-Cuerva, Martell, Perez and others, and consider also nonautonomous Cauchy problems.
The talk is based on joint work with Alberto Fiorenza and Sebastian Król.

Rational and H^∞ dilation

Michael Dritschel

The von Neumann inequalities over the disk and bidisk are essentially statements that certain representations of the disk and bidisk algebras are contractive. The Sz.-Nagy and Andô dilation theorems then imply that these representations are completely contractive. Based on this, Halmos posed the rational dilation problem: if a bounded domain is a spectral set for an operator or commuting tuple of operators (that is, a version of the von Neumann inequality holds), do we have a dilation to a normal operator or tuple of commuting normal operators with spectrum supported on the boundary of the domain? Arveson showed that this was equivalent to asking if contractive representations of analogues of the disk algebra are completely contractive. We begin by discussing recent work in this area on spectral sets which are "distinguished" varieties of the bidisk. We then discuss progress on an apparently even more intractable problem: Is every contractive representation of $H^\infty(\mathbb{D})$ completely contractive? The difficulties arise because of the complex nature of the maximal ideal space in this situation. While trying to construct a counterexample, we instead discovered that this statement is in fact true, at least for finite dimensional representations.
This is joint work with Michael Jury and Scott McCullough.

Twenty years after

Harry Dym

This talk will present some highlights of a twenty two year collaboration with Dima Arov. The focus will be on inverse problems for canonical integral and differential systems and/or Dirac-Krein systems. Applications to prediction theory for vector valued stationary processes will also be discussed, if time permits. The talk will be expository.

(1) A. Dumas: *Vingt ans après.* Paris, 1845.

(2) D.Z. Arov and H, Dym:*J-Contractive Matrix Valued Functions and Related Topics.* Cambridge University Press, 2008.

(3) D.Z. Arov and H, Dym:*Bitangential Direct and Inverse Problems for systems of Integral and Differential Equations.* Cambridge University Press, 2012.

Enhancing Techniques in Operator Theory with Applications to Functional Calculi

Markus Haase

An enhancing technique is a construction or a result that allows to pass from a generic to a more structured ("enhanced") situation. Classical examples are the Sz.-Nagy-Foiaş model theory of Hilbert space contractions and the ergodic decomposition of a generic measure-preserving dynamical system into ergodic systems.
In operator theory, enhancing techniques are often based on representations of a generic operator from a certain class in terms of operators with additional properties. In my talk I will discuss some instances of such representations — more precisely dilations, transference principles and direct integrals — and sketch their role in the proof of certain recent boundedness results for functional calculi.

Spectral analysis for matrix polynomials with symmetries

Peter Lancaster

Two lines of attack in the spectral theory of $n \times n$ matrix polynomials will be outlined. The first is an algebraic approach based on the notion of isospectral linear systems in $\mathbb{C}^{ln}$ (the linearizations) and the second on analysis of associated matrix-valued functions acting on $\mathbb{C}^n$.

The first approach concerns real symmetric systems and leads to canonical forms consisting of real matrix triples, and thence to *canonical triples*. Furthermore, for real selfadjoint systems we describe *sefadjoint canonical triples* of real matrices and illustrate their properties.
It turns out that, in this context, there is a fundamental orthogonality property associated with the spectrum. It will be shown how this can play a role in inverse (spectral) problems, i.e. constructing systems with prescribed spectral properties.

The talk is based on joint work with U. Prells and I. Zaballa:

[1] P. Lancaster, U. Prells, I. Zaballa, *An orthogonality property for real symmetric matrix polynomials with application to the inverse problem* Operators and Matrices, **7**, 2013, 357-379.

[2] P. Lancaster, I. Zaballa, *A review of canonical forms for selfadjoint matrix polynomials* Operator Theory: Advances and Applications, **218**, 2012, 425-443.

[3] P. Lancaster, I. Zaballa, *On the inverse symmetric quadratic eigenvalue problem* SIAM J. Matrix Anal. Appl., to appear.

Perturbation determinants and trace formulas for singular and additive perturbations

Mark Malamud

Recall that a pair $\{H', H\}$ of closed linear operators in a Hilbert space $\mathfrak{H}$ with resolvent sets $\rho(H')$ and $\rho(H)$ are called resolvent comparable if $\rho(H') \cap \rho(H) \neq \emptyset$ and their resolvent difference is of trace class.
We will discuss trace formulas for pairs of self-adjoint, maximal dissipative and other types of resolvent comparable operators. In particular, *the existence of a complex-valued spectral shift function for a pair $\{H', H\}$ of maximal dissipative* resolvent comparable operators is proved. We also investigate the existence of a real-valued spectral shift function. Moreover, we treat in details the case of additive trace class perturbations. If H and $H' = H + V$ are m-dissipative and V is of trace class, it is shown that a complex-valued spectral shift function can be chosen to be summable. We also obtain trace formulas for a pair $\{A, A^*\}$ *assuming only that A and A^* are resolvent comparable.* In this case the determinant of a characteristic function of A is involved in trace formulas.
Our results improve and generalize certain classical results of M.G. Krein for pairs of self-adjoint and dissipative operators, the results of A. Rybkin for such pairs, as well as the results of V. Adamyan, B. Pavlov, and M.Krein for pairs $\{A, A^*\}$ with a maximal dissipative operator A.
If $\widetilde{A}'$, $\widetilde{A}$ are proper extensions of a symmetric operator A, we employ the technique of boundary triplets to express a perturbation determinant $\Delta_{\widetilde{A}'/\widetilde{A}}(\cdot)$ of a pair $\{\widetilde{A}',\ \widetilde{A}\}$ *as a ratio of two ordinary determinants involving only boundary operators and the corresponding Weyl function.* For instance, if A has finite deficiency indices $n_\pm(A) = n < \infty$, $\Pi = \{\mathcal{H}, \Gamma_0, \Gamma_1\}$ is a boundary triplet for the adjoint operator A^*, and $M(\cdot)$ the corresponding Weyl function, then one of the perturbation determinants of the pair $\{\widetilde{A}',\ \widetilde{A}\}$ is given by

$$\Delta^{\Pi}_{\widetilde{A}'/\widetilde{A}}(z) := \frac{\det(B' - M(z))}{\det(B - M(z))}, \qquad z \in \rho(\widetilde{A}') \cap \rho(\widetilde{A}). \tag{1}$$

Here B, B' are the boundary operators corresponding to extensions $\widetilde{A}'$, $\widetilde{A}$ in the triplet Π. In turn, this formula allows one to express the spectral shift function of the pair $\{\widetilde{A}', \widetilde{A}\}$ by means of $M(\cdot)$ and B, B'.
The results are applied to boundary value problems for differential equations. Formula (1) is involved in trace formulas for ordinary differential operators. For second order elliptic operators on domains with compact boundary the perturbation determinants, hence the corresponding spectral shift functions, are expressed by means of boundary operators and Dirichlet-to-Neumann map.
The talk is based on works [1] and [2] joint with H. Neidhardt.

[1] M. Malamud, H. Neidhardt, Perturbation determinants for singular perturbations, *Russian J. of Math. Phys.*, v. 21, No 1 (2014), p. 55-98.

[2] M. Malamud, H. Neidhardt, Perturbation determinants and trace formulas for singular perturbations. *Preprint, arXiv:1212.6887[math-ph]*, 2012.

Generic rank-one perturbations: structure defeats sensitivity

Christian Mehl

The behaviour of eigenvalues of matrices under perturbations is a frequently study topic in Numerical Linear Algebra. In particular, the study of low rank perturbations is well established and well understood by now. For example, in [1] it was shown that if $A \in \mathbb{C}^{n\times n}$ is a defective matrix having the eigenvalue λ with partial multiplicities $n_1 \geq \cdots \geq n_k$, then applying a generic rank one perturbation results in a matrix having the eigenvalue λ with partial multiplicities $n_2 \geq \cdots \geq n_k$. As simple arguments show that the geometric multiplicity can only decrease by one if a rank-one perturbation is applied, we find that it is the largest Jordan block that is the most sensitive one to generic rank-one perturbations.
The picture changes drastically if structured matrices are considered and if the class of perturbation matrices is restricted to perturbations that preserve the original structure of the matrix. Here, "structure" means a symmetry structure with respect to a given indefinite inner product. In contrast to the unstructured case, there are instances when a structured rank-one perturbation leads to an increase of the size of the largest Jordan block. From this point of view structure defeats sensitivity because the behaviour under perturbations of the largest (and most sensitive) Jordan block changes if structure-preservation is enforced.
In the talk, we will investigate and explain this and other peculiar behaviours that occur when structure-preserving rank-one perturbations are applied. Special emphasis is put on the case of symplectic and orthogonal matrices with respect to an indefinite inner product.
The talk is based on joint work with V. Mehrmann, A.C.M. Ran, and L. Rodman.

[1] J. Moro and F. Dopico. Low rank perturbation of Jordan structure. *SIAM J. Matrix Anal. Appl.*, 25 (2003), 495–506.

The algebraic Riccati equation

Mark R. Opmeer

We consider the algebraic Riccati equation where the coefficients are unbounded operators on Hilbert spaces. This equation arises in optimal control of partial differential equations, but also for example in realization theory. It is well-known that the "standard form" of the algebraic Riccati equation is not the correct equation in some situations. In the first part of the talk we will discuss an alternative "operator node algebraic Riccati equation". In the second part of the talk we will discuss how operator theory can be used to prove convergence of an algorithm for numerically approximating the solution of the algebraic Riccati equation.

The first part of the talk is based on joint work with Olof Staffans and the second part of the talk is based on joint work with Timo Reis.

The isomorphism problem for complete Pick algebras

Orr Shalit

The classical Nevanlinna-Pick interpolation theorem motivated the study of a class of reproducing kernel Hilbert spaces, those which have the so called *complete Pick property.* The multiplier algebras of these spaces — the *complete Pick algebras* — lie at the crossroad connecting operator theory, function theory and complex geometry. My aim in this talk is to describe the structure theory of these algebras. In a nutshell: every such an algebra is associated with a complex analytic variety in the unit ball of a Hilbert space, and the geometry of this variety is a complete invariant of the operator algebraic structure of the multiplier algebra.

The linear stationary state/signal systems story

Olof Staffans

I first met Dima Arov in the MTNS conference 1998 in Padova where he gave a plenay talk on "Passive Linear Systems and Scattering Theory". Five years later, in the fall of 2003, Dima came to work with me in Åbo for one month, and that was the beginning of our linear stationary state/signal systems story. We decided to join forces to study the relationship between the (external) reciprocal symmetry of a conservative linear system and the (internal) symmetry structure of the system in three different settings, namely the scattering, the impedance, and the transmission setting. Instead of writing three separate papers with three separate sets of results and proofs we wanted to rationalize and to find some "general setting" that would cover the "common part" of the theory. The basic plan was to first develop the theory in such a "general setting" as far as far as possible, before discussing the three related symmetry problems mentioned above in detail.
After a couple of days we realized that the "behavioral approach" of [1] seemed to provide a suitable "general setting". To make the work more tractable from a technical point of view we decided to begin by stuying the discrete time case. As time went by the borderline between the "general theory" and the application to the original symmetry problem keept moving forward. Our first paper had to be split in two because it became too long. Then the second part had to be split in two because it became too long, then the third part had to be split in to, and so on. Every time the paper was split into two the original symmetry problem was postponed to the second unfinished half, and our "general solution" to the symmetry problem was not submitted until 2011. By that time we had finished a total of 6 papers on discrete time systems, 4 papers on continuous time systems, and 3 "general" papers that apply to both discrete time and continuous time systems (in addition to numerous conference papers). Some of these papers were written together with Mikael Kurula. The total number of pages was 576, which makes an average length of about 50 pages per paper. The specific applications of our symmetry paper to the scattering, impedance, and transmission settings is still "work in progress".
Since 2009 Dima and I have spent most of our research time on writing a book on linear stationary systems in continuous time. A partial preliminary draft of the first volume of this book is available in [2].
In this talk I shall give an overview of the theory of linear stationary state/signal systems. Some aspects of this theory will also be discussed in the talk [3] by Mikael Kurula.

[1] J. A. Ball and O. J. Staffans: Conservative state-space realizations of dissipative system behaviors, *Integral Equations Operator Theory*, 54:151–213, 2006.

[2] D. Z. Arov and O. J. Staffans: Linear Stationary Systems in Continuous Time, book manuscript, available at
http://users.abo.fi/staffans/publ.html.

[3] M. Kurula: State/signal realizations of passive continuous-time behaviours, Lecture at MTNS 2014.

On convergence of sectorial forms

Tom ter Elst

If $\mathfrak{h}$ and $\mathfrak{k}$ are two positive symmetric sesquilinear forms in a Hilbert space H, then one defines $\mathfrak{h} \le \mathfrak{k}$ if $D(\mathfrak{k}) \subset D(\mathfrak{h})$ and $\mathfrak{h}(u,u) \le \mathfrak{k}(u,u)$ for all $u \in D(\mathfrak{k})$. There are two monotone convergence theorems due to Kato.

Theorem 1 *Let* $0 \leq \mathfrak{h}_1 \leq \mathfrak{h}_2 \leq \ldots$ *be positive symmetric closed sesquilinear forms in a Hilbert space* H*. Define the form* $\mathfrak{h}_\infty$ *by*

$$D(\mathfrak{h}_\infty) = \{u \in \bigcap_{n=1}^{\infty} D(\mathfrak{h}_n) : \sup_{n \in \mathbb{N}} \mathfrak{h}_n(u) < \infty\}$$

and $\mathfrak{h}_\infty(u, v) = \lim_{n \to \infty} \mathfrak{h}_n(u, v)$*. Then* $\mathfrak{h}_\infty$ *is a closed positive symmetric form. Suppose* $D(\mathfrak{h}_\infty)$ *is dense in* H*. Let* A_n *and* A_∞ *be the self-adjoint operators associated with* $\mathfrak{h}_n$ *and* $\mathfrak{h}_\infty$ *for all* $n \in \mathbb{N}$*. Then* $\lim_{n \to \infty} A_n = A_\infty$ *in the strong resolvent sense.*

Theorem 2 *Let* $\mathfrak{h}_1 \geq \mathfrak{h}_2 \geq \ldots$ *be positive symmetric closed densely defined sesquilinear forms in a Hilbert space* H*. For all* $n \in \mathbb{N}$ *let* A_n *be the self-adjoint operator associated with* $\mathfrak{h}_n$*. Then there exists a self-adjoint operator* A_∞ *in* H *such that* $\lim_{n \to \infty} A_n = A_\infty$ *in the strong resolvent sense.*

Simon was able to relate the operator A_∞ in Theorem 2 to the limit of the forms.

Theorem 3 *Adopt the assumptions and notation as in Theorem 2. Define the form* $\mathfrak{h}_\infty$ *by* $D(\mathfrak{h}_\infty) = \bigcup_{n=1}^{\infty} D(\mathfrak{h}_n)$ *and* $\mathfrak{h}_\infty(u, v) = \lim_{n \to \infty} \mathfrak{h}_n(u, v)$*. Then* A_∞ *is the operator associated to the closure of the regular part of* $\mathfrak{h}_\infty$*.*

The aim of this talk is to extend the first two theorems to (possibly nonsymmetric) sectorial forms and simplify the last theorem.

This talk is based on joint work with W. Arendt and C.J.K. Batty.

[1] Arendt, W. and Elst, A. F. M. ter, Sectorial forms and degenerate differential operators. *J. Operator Theory* **67** (2012), 33–72.

[2] Batty, C. J. K. and Elst, A. F. M. ter, On series of sectorial forms. *J. Evol. Equ.* **14** (2014), 29–47.

Quadratic and block numerical ranges (QNR and BNR) of operators and operator functions

Christiane Tretter

In this talk a panorama of recent generalizations of the numerical range of (linear) operators and operator functions is presented. These new concepts allow one to take into account more particular structures of the operators or operator functions involved. The main results include the spectral inclusion property, resolvent estimates, and some Perron-Frobenius type results.

Perturbation and spectral theory for J-non-negative operators and applications

Carsten Trunk

We present recent developments for J-non-negative operators in spaces with an indefinite metric. More precisely, we consider a Hilbert space $\mathcal{H}$ with positive definite inner product $(\cdot, \cdot)$ and a self-adjoint, bounded operator J with $J^2 = I$ which serves as the Gramian of

$$[x, y] := (Jx, y) \quad \text{for } x, y \in \mathcal{H}.$$

It is usual, to call the tuple $(\mathcal{H}, [\cdot, \cdot])$ a *Krein space*. A densely defined operator in $\mathcal{H}$ is called *J-non-negative*, if it has a non-empty resolvent set and satisfies for all x in its domain

$$[Ax, x] \geq 0.$$

It is well-known that the spectrum of such an operator is real and that there exists a spectral function with (possible) singularities at 0 and ∞, see [1, 2, 5]. In the talk we will discuss various

spectral properties like Jordan chains or the numerical range. Main focus of the talk is the description of the spectrum after a one-dimensional perturbation: We will give bounds on the lengths of Jordan chains, bounds for the non-real spectrum and the number of eigenvalues in gaps of the essential spectrum.
J-non-negative operators appear in the study of operator polynomials (e.g. [3, 4]) and indefinite Sturm-Liouville equations, see, e.g. [6]. Currently much effort is devoted to inverse spectral problems related to left-definite Sturm Liouville problems and the Camassa-Holm equation.

[1] T. Ando, *Linear Operators in Krein Spaces*, Sapporo, Japan 1979.

[2] T.Ya. Azizov and I.S. Iokhvidov, *Linear Operators in Spaces with an Indefinite Metric*, John Wiley and Sons, Chichester, New York, 1989.

[3] J. Bognar, *Indefinite Inner Product Spaces*, Springer, 1974.

[4] M.G. Krein, *Introduction to the theory of indefinite J-spaces and to the theory of operators in those spaces*, Amer. Math. Soc. Transl. (2) 93 (1970), 103–176

[5] H. Langer, *Spectral functions of definitizable operators in Krein spaces*, Lecture Notes in Mathematics 948, Springer (1982), 1–46.

[6] A. Zettl, *Sturm-Liouville Theory*, AMS, Providence, RI, 2005.

Stable and real-zero polynomials, and their determinantal representations

Hugo J. Woerdeman

It is of interest to represent multivariable polynomials in a way so that it is easy to read off certain properties of its zeroes. For instance, if K is a contractive matrix and $Z_n = \oplus_{k=1}^{d} z_k I_{n_k}$, $n = (n_1, \ldots, n_d) \in \mathbb{N}^d$, is a diagonal matrix with variables $z_1, \ldots, z_d$ on the diagonal, then the polynomial

$$p(z_1, \ldots, z_d) = \alpha \det(I - KZ_n), \tag{2}$$

with $\alpha \neq 0$, has no roots when $|z_1| < 1, \ldots, |z_d| < 1$ (i.e., p is *stable*).
As another example, if $A_1 = A_1^*$, ..., $A_d = A_d^*$ are Hermitian matrices and we put

$$p(x_1, \ldots, x_d) = \alpha \det(I + x_1 A_1 + \cdots + x_d A_d), \tag{3}$$

then p has the property that for any real $x_1, \ldots, x_d$ the one variable polynomial $t \mapsto p(tx_1, \ldots, tx_d)$ has only real roots (i.e., p is *real-zero*).
The interesting question is about the converses. Does every stable polynomial allow a representation (2) with $\|K\| \leq 1$? Does every real-zero polynomial allow a representation (3) with $A_1, \ldots, A_d$ Hermitian?
The exploration of these questions brings us to Féjer-Riesz factorizations, the von Neumann inequality, rational inner functions in the Schur-Agler class, and other topics of interest.
The talk is based on the following papers.

[1] A. Grinshpan, D. S. Kaliuzhnyi-Verbovetskyi, V. Vinnikov, and H. J. Woerdeman, Stable and real-zero polynomials in two variables, *Multidimensional Systems and Signal Processing*, to appear.

[2] A. Grinshpan, D. S. Kaliuzhnyi-Verbovetskyi, and H. J. Woerdeman, Norm-constrained determinantal representations of multivariable polynomials. *Complex Anal. Oper. Theory* 7 (2013), no.3, pp. 635–654.

[3] H. J. Woerdeman, Determinantal representations of stable polynomials, *Operator Theory: Adv. Appl.* 237 (2013), 241-246.

Dichotomy, spectral subspaces and unbounded projections

Christian Wyss

We consider a densely defined operator S on a Banach space X such that a strip around the imaginary axis is contained in the resolvent set of S and the resolvent is uniformly bounded on this strip. A fundamental question is whether there exist invariant subspaces X_+ and X_- corresponding to the spectrum in the right and left half-plane, respectively. S is called dichotomous if these subspaces exist and yield a decomposition $X = X_+ \oplus X_-$ with associated bounded spectral projections onto $X_\pm$.
A sufficient condition for dichotomy and, in particular, the existence of $X_\pm$ was established in [1]. Our main result is that the spectral subspaces $X_\pm$ exist even if S is not dichotomous. In this case $X_+ \oplus X_-$ is only a dense subspace of X and the spectral projections will be unbounded. Using these projections, we are then able to derive perturbation theorems for dichotomous operators, which significantly improve a similar theorem from [1].
Our results apply to bisectorial operators, for which a bisector around the imaginary axis belongs to the resolvent set, and to a Hamiltonian block operator matrix arising in systems theory.
The talk is based on joint work with Monika Winklmeier.

[1] H. Bart, I. Gohberg, M. A. Kaashoek: Wiener-Hopf factorization, inverse Fourier transforms and exponentially dichotomous operators, *Journal of Functional Analysis*, 68 (1986), 1–42.

Abstracts of Talks in Invited Sessions

Clifford analysis and operator theory (Cliff)

Organizers: Daniel Alpay, Fabrizio Colombo, Uwe Kähler, Franciscus Sommen

Speaker	title	time	place
		Monday	M 607
Marco Peloso	Function spaces of exponential growth on a half-plane, zero-sets and applications	13.30-13.55	
David Kimsey	Herglotz's theorem for the quaternions	13.55-14.20	
Baruch Schneider	On quaternionic analysis for the singular Cauchy integral operator and its relation with the Cimmino system of partial differential equations	14.20-14.45	
Fabrizio Colombo	A new resolvent equations of the slice hyperholomorphic functional calculi	14.45-15.10	
David Eelbode	Quaternionic hermitean (Clifford) analysis	15.50-16.15	
Mircea Martin	Clifford Analysis and Spin Geometry Techniques in Operator Theory	16.15-16.40	
Daniel Alpay	de Branges Rovnyak spaces of slice-hyperholomorphic functions	16.40-17.05	
Rolf Sören Kraußhar	A Selberg trace formula for hypercomplex analytic modular forms	17.05-17.30	

Clifford analysis and operator Theory

Function spaces of exponential growth on a half-plane, zero-sets and applications

Marco Peloso

We introduce a new class of spaces of mixed Hardy-Bergman type on a half-plane. We study some functional properties of these spaces, and their zero-sets. We apply these results to the Muntz-Szasz problem for the Bergman space. (This is joint work, in progress, with M. Salvatori.)

Herglotz's theorem for the quaternions

David P. Kimsey

We call a function $r : \mathbb{Z} \to \mathbb{H}^{s\times s}$, where $\mathbb{H}$ denotes the quaternions, *positive definite* if the Toeplitz matrix $(r(a-b))_{a,b=0}^{N}$ is positive semidefinite for all $N \geq 0$. We will show that a function $r : \mathbb{Z} \to \mathbb{H}^{s\times s}$ is positive definite if and only if there exists a $\mathbb{H}^{s\times s}$-valued measure σ on $[0, 2\pi)$ (which obeys a certain positivity condition) so that

$$r(n) = \int_0^{2\pi} e^{int} d\sigma(t), \qquad n \in \mathbb{Z}.$$

In this case, σ is unique. If $r : \mathbb{Z} \to \mathbb{C}^{s\times s}$ we recover a classical theorem of Herglotz. We will also show that a function $r : \{-N, \dots, N\} \to \mathbb{H}^{s\times s}$ so that $(r(a-b))_{a,b=0}^{N}$ is positive semidefinite can be extended to a positive definite function $\tilde{r} : \mathbb{Z} \to \mathbb{H}^{s\times s}$. Finally, we will establish an integral characterization for bounded functions $r : \mathbb{Z} \to \mathbb{H}^{s\times s}$ with a finite number of negative squares.
The talk is based on joint work with Daniel Alpay, Fabrizio Colombo and Irene Sabadini.

On quaternionic analysis for the singular Cauchy integral operator and its relation with the Cimmino system of partial differential equations

Baruch Schneider

The main gool of this talk is to make a one-to-one correspondence between quaternionic hyperholomorphic functions and solutions for Cimmino system of partial differential equations. We study properties of the singular Cauchy-Cimmino integrals operators.
The talk is based on joint work with R. Abreu Blaya and J. Bory Reyes.

A new resolvent equations of the slice hyperholomorphic functional calculi

Fabrizio Colombo

The S-functional calculus is a functional calculus for $(n+1)$-tuples of non necessarily commuting operators that can be considered a higher dimensional version of the classical Riesz-Dunford functional calculus for a single operator. In this last calculus, the resolvent equation plays an important role in the proof of several results. Associated with the S-functional calculus there are two resolvent operators: the left $S_L^{-1}(s,T)$ and the right one $S_R^{-1}(s,T)$, where $s = (s_0, s_1, \dots, s_n) \in R^{n+1}$ and $T = (T_0, T_1, \dots, T_n)$ is an $(n+1)$-tuple of noncommuting operators. In this talk we show a new resolvent equation which is the analog of the classical resolvent equation. It is interesting to note that the equation involves both the left and the right S-resolvent operators simultaneously.
The talk is based on joint work with D. Alpay, J. Gantner, I. Sabadini

Quaternionic hermitean (Clifford) analysis

David Eelbode

Whereas classical Clifford analysis is centered around the notion of a single (elliptic) conformally invariant Dirac operator which factorises the Laplace operator Δ_m in m real variables, there exist

refinements which are essentially based on a reduction of the underlying symmetry group. In even dimensions, this gives rise to Hermitean Clifford analysis (from orthogonal to unitary). One can go one step further, and consider the function theory associated to the symplectic refinement of this unitary group, which gives then rise to quaternionic Hermitean Clifford analysis. In this talk we will explain how this function theory arises, hereby focusing on the scalar-valued case (using harmonic polynomials as a starting point).
The talk is based on joint work with F. Brackx, H. De Schepper, R. Lávička and V. Souček

Clifford analysis and spin geometry techniques in operator theory

Mircea Martin

The talk will illustrate several possible uses of some concepts and techniques from Clifford analysis and spin geometry in operator theory. We will start by introducing two types of *self-commutator* — or *curvature* — *identities* for systems of Hilbert space operators. Under appropriate assumptions, the identities will be set up as specific forms of the *Weitzenböck* and *Kodaira identities* for some general *Dirac* and *Laplace operators*.
Motivated by *Bochner's method* in spin geometry, we will next analyze *seminormal systems of operators*, which are defined by assuming that the remainders in their associated self-commutator identities are semidefinite. A singular integral model of seminormal systems of operators that involves Riesz transforms and a Putnam type commutator inequality in higher dimension will be also briefly discussed.

de Branges Rovnyak spaces of slice-hyperholomorphic functions

Daniel Alpay

A study of Schur analysis and of the associated reproducing kernel spaces (of the kind introduced by de Branges and Rovnyak) in the slice-hyperholomorphic setting has been recently initiated. See for instance [1],[2],[3]. In the classical theory the reproducing kernel Hilbert spaces of functions with a reproducing kernel of the form

$$\frac{A(z)A(w)^* - B(z)B(w)^*}{z + \overline{w}}$$

play an important role. In the talk we study the counterpart of these spaces in the setting of slice-hyperholomorphic functions.
The talk is based on joint work with Fabrizio Colombo, David Kimsey and Irene Sabadini.

[1] D. Alpay, V. Bolotnikov, F. Colombo and I. Sabadini. Self-mappings of the quaternionic unit ball: multiplier properties, Schwarz-Pick inequality, and Nevanlinna-Pick interpolation problem. *Indiana University Mathematics Journal.* To appear.

[2] D. Alpay, F. Colombo and I. Sabadini. Schur functions and their realizations in the slice hyperholomorphic setting. *Integral Equations and Operator Theory*, vol. 72 (2012), pp. 253-289.

[3] D. Alpay, F. Colombo and I. Sabadini. Pontryagin de Branges-Rovnyak spaces of slice hyperholomorphic functions. *Journal d'Analyse Mathématique*, vol. 121 (2013), no. 1, 87-125

A Selberg trace formula for hypercomplex analytic modular forms

Rolf Sören Kraußhar

An important result in the development of a theory of hypercomplex analytic modular forms over Clifford algebras has been the proof of the existence of non-trivial cusp forms for important discrete arithmetic subgroups of the Ahlfors-Vahlen group. Examples of such cusp forms can be constructed in terms of k-holomorphic Cliffordian Poincaré series. Eisenstein series form a complementary space in the set of hypercomplex analytic modular forms. They are not cusp forms. An important question is to understand whether all hypercomplex modular forms can be expressed in terms of finetely many basic Eisenstein- and Poincaré series.

In this talk we present a Selberg trace formula for this class of modular forms. This tool in hand allows us to show that the dimension of the space of hypercomplex-analytic cusp forms is finite. Finally, we also manage to describe the space of Eisenstein series and give a dimension formula for the complete space of k-holomorphic Cliffordian modular forms. The dimension of the space of Eisenstein series turns out to be equal to the number of cusp classes. This provides another nice analogy to the classical theory of modular forms.
This talk is based on joined work with Dennis Grob from RWTH Aachen University.

Spectral theory for Sturm-Liouville and differential operators (SL)
Organizers: Jussi Behrndt and Carsten Trunk

Speaker	title	time	place
		Monday	M 623
Johannes Brasche	δ'-interactions on compact sets	13.30-13.55	
Yiannis Christodoulides	Averaging of spectral measures associated with the Weyl-Titchmarsh m-function	13.55-14.20	
Petru A. Cojuhari	On spectral analysis of elliptic differential operators	14.20-14.45	
Michael Demuth	Eigenvalue asymptotics of linear operators in Banach spaces	14.45-15.10	
Christian Kühn	Strong coupling asymptotics for Schrödinger operators with delta-potential	15.50-16.15	
Matthias Langer	Spectral properties of unbounded J-self-adjoint block operator matrices	16.15-16.40	
Vadim Mogilevskii	On characteristic matrices and spectral functions of first-order symmetric systems	16.40-17.05	
Andrea Posilicano	Self-adjoint realizations of the Laplace-Beltrami operator on conic and anti-conic surfaces	17.05-17.30	
		Friday	M623
Jonathan Rohleder	Eigenvalue inequalities for elliptic differential operators	13.30-13.55	
Sergii Torba	Transmutation operators and efficient solution of Sturm-Liouville spectral problems	13.55-14.20	
Monika Winklmeier	Sharp bounds for the eigenvalues of the angular Kerr-Newman-Dirac operator	14.20-14.45	
Vladimir Lotoreichik	Schrödinger operators with δ-interactions supported on conical surfaces	14.45-15.10	
Michael Schelling	A new proof of Gershgorin's theorem	15.50-16.15	

Spectral theory for Sturm-Liouville and differential operators

δ'-interactions on compact sets

Johannes Brasche

One has mainly concentrated on two kinds of one-dimensional point interactions on finite sets, namely δ-interactions and δ'-interactions. More generally one may investigate interactions which take place on a compact set of Lebesgue measure zero. It is clear what the analogue of a δ-interaction is in this case. However, for a long time it has been an open problem what the analogue of a δ'-interaction is in this case. We shall present the solution to this problem, show how to construct a large class of Schrödinger operators describing a δ'-interaction on a compact null set, and discuss their spectral properties. It is noteworthy that in many cases one gets Schrödinger operators with infinitely many negative eigenvalues, despite the fact that the interaction only takes place on a compact set.
The talk is based on joint work with Leonid Nizhnik.

Averaging of spectral measures associated with the Weyl-Titchmarsh m-function

Yiannis Christodoulides

We consider averages of spectral measures of the form

$$\kappa(A) = \int_a^b \rho_\theta(A) d\nu(\theta),$$

where $\{\rho_\theta\}$ is a family of spectral measures associated with the Weyl-Titchmarsh m-function for the Schrödinger equation on the half-line, and ν is an arbitrary Herglotz measure. We show that the measure κ corresponds to a composition of Herglotz functions, and we examine the properties of κ by considering the boundary values of the functions undergoing composition. We give precise conditions for absolute continuity and the discrete part of κ.

[1] Y. T. Christodoulides and D. B. Pearson: Spectral theory of Herglotz functions and their compositions,*Mathematical Physics, Analysis and Geometry*, 7(4) (2004), 333–345.

[2] E. A. Coddington and N. Levinson:*Theory of Ordinary Differential Equations.* McGraw-Hill, 1955.

[3] G. Herglotz G: Über Potenzreihen mit positivem, reellem Teil in Einheitskreis,*Sächsischen Academie der Wissenschaften zu Leipzig*, 63 (1911), 501–511.

[4] C. A. Marx: Continuity of spectral averaging, *Proceedings of the American Mathematical Society*, 139(1) (2011), 283–291.

[5] R. del Rio and O. Tchebotareva: Sturm-Liouville operators in the half axis with local perturbations, *Journal of Mathematical Analysis and Applications*, 329 (2007), 557–566.

On spectral analysis of elliptic differential operators

Petru A. Cojuhari

We propose to discuss spectral properties of higher order elliptic differential operators. Emphasis is placed on those which have mainly applications to scattering theory. Applications to Dirac and Pauli operators will be considered.

Eigenvalue asymptotics of linear operators in Banach spaces

Michael Demuth

Let Z_0 be a bounded operator in a Banach space X and let K be a nuclear perturbation of Z_0 in X. We estimate the asymptotics of the eigenvalues of $Z_0 + K$ if they approach to the essential spectrum of Z_0. Moreover we give bounds for the number of eigenvalues in certain regions of the complex plane. The general operator theoretical result is applied to the discrete Laplacian.
The talk is based on a joint work with F. Hanauska (Clausthal).

Strong coupling asymptotics for Schrödinger operators with delta-potential

Christian Kühn

Let Σ be a bounded plain surface in $\mathbb{R}^3$. Denote by A_β the Schrödinger operator in $L^2(\mathbb{R}^3)$ with a δ-potential of strength β supported on Σ, i.e. the operator associated to the formal differential expression $-\Delta - \beta\delta_\Sigma$. Denote by $\Lambda_j(\beta)$ the jth negative eigenvalue of A_β. We will investigate the asymptotic behaviour of $\Lambda_j(\beta)$ for $\beta \to \infty$.
The talk is based on a joint work with Jaroslav Dittrich and Pavel Exner.

Spectral properties of unbounded J-self-adjoint block operator matrices

Matthias Langer

We consider unbounded block operator matrices of the form

$$\begin{pmatrix} A & B \\ -B^* & D \end{pmatrix}$$

in the direct sum of two Hilbert spaces where A and D are self-adjoint operators, bounded from below and from above, respectively, and B is closed. Under two different relative boundedness assumptions (upper dominant and diagonally dominant cases) the spectrum of the block operator matrix is discussed. The Schur complement and the quadratic numerical range are used to find conditions for the spectrum to be real and to establish variational principles for eigenvalues and eigenvalues estimates. The results are applied to block operator matrices that are connected with differential operators that depend rationally on the eigenvalue parameter.
The talk is based on joint work with Michael Strauss.

On characteristic matrices and spectral functions of first-order symmetric systems

Vadim Mogilevskii

Let $\mathbb{H}$ be a finite-dimensional Hilbert space, let $[\mathbb{H}]$ be the set of all operators in $\mathbb{H}$ and let $J \in [\mathbb{H}]$ satisfies $J^* = J^{-1} = -J$. We will discuss first-order symmetric system

$$Jy' - B(t)y = \lambda\Delta(t)y + \Delta(t)f(t) \tag{4}$$

with the $[\mathbb{H}]$-valued coefficients $B(t) = B^*(t)$ and $\Delta(t) \geq 0$ defined on an interval $\mathcal{I} = [a, b)$ with the regular endpoint a. Let $T_{\min}$ be the minimal relation and let $Y(\cdot, \lambda)$ be the $[\mathbb{H}]$-valued solution of the system (1) with $f = 0$ satisfying $Y(0, \lambda) = I$. A spectral (pseudospectral) function of such a system is defined as an $[\mathbb{H}]$-valued distribution function $\Sigma(s)$, $s \in \mathbb{R}$, such that the Fourier transform

$$\widehat{f}(s) = \int_{\mathcal{I}} Y^*(t, s)\Delta(t)f(t)\,dt$$

is an isometry V (resp. partial isometry V with $\ker V = \operatorname{mul} T_{\min}$) from $L^2_\Delta(\mathcal{I})$ into $L^2(\Sigma)$. We describe all generalized resolvents $y = R(\lambda)f$, $f \in L^2_\Delta(\mathcal{I})$, of $T_{\min}$ in terms of λ-depending boundary conditions imposed on regular and singular boundary values of a function y at the endpoints a and b respectively. This enables us to parametrize all characteristic matrices $\Omega(\lambda)$

of the system (1) immediately in terms of boundary conditions. Such a parametrization is given both by the block-matrix representation of $\Omega(\lambda)$ and by the formula similar to the Krein formula for resolvents.
System (1) is called absolutely definite if the set $\{t \in \mathcal{I} : \Delta(t) \text{is invertible}\}$ has a nonzero Lebesgue measure. For an absolutely definite system the above parametrization of $\Omega(\lambda)$ gives rise to parametrization of all pseudospectral and spectral functions of this system by means of a Nevanlinna boundary parameter. Our results implies the parametrisations of pseudospectral and spectral functions obtained by Langer and Textorius [1] and Sakhnovich [3] for the particular case of the system (1) on a compact interval $\mathcal{I} = [a, b]$.
The results of the talk are partially specified in [2].

[1] H. Langer and B. Textorius: Spectral functions of a symmetric linear relation with a directing mapping, I, *Proc. Roy. Soc. Edinburgh Sect. A*, 97 (1984), 165-176.

[2] V.I. Mogilevskii: On generalized resolvents and characteristic matrices of first-order symmetric systems, arXiv:1403.3995v1 [math.FA] 16 Mar 2014.

[3] A.L. Sakhnovich: Spectral functions of a canonical system of order $2n$, *Math. USSR-Sb.*, 71 (1992), 355–369.

Self-adjoint realizations of the Laplace-Beltrami operator on conic and anti-conic surfaces

Andrea Posilicano

Let Δ_α^{min}, $\alpha \in \mathbb{R}$, be the minimal realization of the Laplace-Beltrami operator on $(\mathbb{R}\backslash\{0\}) \times \mathbb{T}$ equipped with the singular/degenerate Riemannian metric $g_\alpha(x, \theta) = \begin{pmatrix} 1 & 0 \\ 0 & x^{-2\alpha} \end{pmatrix}$. The symmetric operator Δ_α^{min} is essentially self-adjoint whenever $\alpha \notin (-3, 1)$, has deficiency indices $(2, 2)$ whenever $\alpha \in (-3, -1]$ and has infinite deficiency indices whenever $\alpha \in (-1, 1)$. We study the case $\alpha \in (-1, 1)$, show that the defect space of Δ_α^{min} is (isomorphic to) the fractional Sobolev space $H^s(\mathbb{T}; \mathbb{C}^2)$ with $s = \frac{1}{2} - \frac{\alpha}{1+\alpha}$ and provide all self-adjoint extensions of Δ_α^{min}. Here one profits of the decomposability Δ_α^{min} as an infinite direct sum of symmetric Sturm-Liouville operators on the real line. For some values of the parameter α, such operators turn out to be unitarily equivalent to Schrödinger operators with solvable potentials, thus allowing some spectral analysis of the corresponding self-adjoint extensions.
Joint work with Dario Prandi.

Eigenvalue inequalities for elliptic differential operators

Jonathan Rohleder

In this talk second order elliptic differential operators on bounded Lipschitz domains subject to various boundary conditions (Dirichlet, Neumann, Robin) are considered. The main focus is on inequalities between eigenvalues for different boundary conditions.

Transmutation operators and efficient solution of Sturm-Liouville spectral problems

Sergii Torba

An operator T is called a transmutation operator [1] for a pair of operators A and B if it is continuous and continuously invertible on a suitable topological space and satisfy the operator equality $AT = TB$.
Transmutation operators, introduced by Delsarte, were used mostly as a theoretical tool in the solution of inverse spectral problems. We propose an efficient method for the solution of direct Sturm-Liouville spectral problems based on an approximation of transmutation operators.

In the case when $A = -\partial^2 + q(x)$ and $B = -\partial^2$, a transmutation operator T can be realized in the form of the Volterra integral operator [2]

$$Tu(x) = u(x) + \int_{-x}^{x} K(x,t)u(t)dt$$

with the integral kernel K satisfying the particular Goursat problem, whose exact solution is known only for some potentials.
We show that it is possible to approximate the integral kernel K in a form

$$K(x,t) \approx \sum_{n=0}^{N} a_n(x)t^n,$$

where the coefficients $a_n(x)$ can be easily obtained from a known (at least numerically) particular solution of the equation $Au = 0$.
Since the two linearly independent solutions of the equation $Au = \omega^2 u$ are the images of the functions $\cos\omega t$ and $\frac{\sin\omega t}{\omega}$ under the action of transmutation operator T, their approximations can be easily obtained using the proposed approximation of the integral kernel. For example,

$$T[\cos\omega t](x) \approx \cos\omega x + \sum_{n=0}^{N} a_n(x) \int_{-x}^{x} t^n \cos\omega t\, dt,$$

and the approximation is convenient because all the integrals can be evaluated exactly.
On the base of proposed approximations a new method of solution of spectral problems is developed having the following remarkable property: it allows to compute thousands of eigenvalues and eigenfunctions with uniform error bounds and with easy error control.
The talk is based on joint works with V. V. Kravchenko [3,4].

[1] B. M. Levitan: *Inverse Sturm-Liouville problems.* VSP, Zeist, 1987.

[2] V. A. Marchenko: *Sturm-Liouville operators and applications.* Birkháuser, Basel, 1986.

[3] V. V. Kravchenko and S. M. Torba: Construction of transmutation operators and hyperbolic pseudoanalytic functions, *Complex Anal. Oper. Theory* (2014), 51pp, doi:10.1007/s11785-014-0373-3.

[4] V. V. Kravchenko and S. M. Torba. Analytic approximation of transmutation operators and applications to highly accurate solution of spectral problems, submitted, available at arXiv:1306.2914, 32pp.

Sharp bounds for the eigenvalues of the angular Kerr-Newman-Dirac operator

Monika Winklmeier

The angular part of the Dirac equation in the Kerr-Newman metric is the block operator matrix

$$\mathcal{A} = \begin{pmatrix} -am\cos\theta & \frac{\mathrm{d}}{\mathrm{d}\theta} + \frac{\kappa}{\sin\theta} + am\cos\theta \\ \frac{\mathrm{d}}{\mathrm{d}\theta} + \frac{\kappa}{\sin\theta} + am\cos\theta & am\cos\theta \end{pmatrix}$$

which acts on functions in $L_2(0,\pi)^2$. Here $\kappa \in \mathbb{Z} + \frac{1}{2}$ and a, m and ω are real parameters. It can be shown that this operator has only point spectrum. To the best of our knowledge, analytic formulae for the eigenvalues are available only in special cases. I will show how the so-called second order spectrum allows us to find numerical approximations of the eigenvalues with guaranteed error bounds and I will compare the numerical values found my this method with those available in the literature.

The talk is based on joint work with Lyonell Boulton.

Schrödinger operators with δ-interactions supported on conical surfaces

Vladimir Lotoreichik

We prove that the self-adjoint three-dimensional Schrödinger operator $-\Delta_{\alpha,\mathcal{C}_\theta}$ with attractive δ-interaction of constant strength $\alpha > 0$ supported on the conical surface

$$\mathcal{C}_\theta := \big\{(x,y,z) \in \mathbb{R}^3 \colon z = \cot(\theta)\sqrt{x^2+y^2}\big\}, \qquad \theta \in (0,\pi/2),$$

has the essential spectrum $[-\alpha^2/4, +\infty)$ and its discrete spectrum below the point $-\alpha^2/4$ is infinite. Asymptotic estimates for the eigenvalues of $-\Delta_{\alpha,\mathcal{C}_\theta}$ are obtained. Our results remain valid if the hypersurface $\mathcal{C}_\theta$ is locally deformed (with Lipschitz regularity preserved).
This talk is based on the joint work with Jussi Behrndt and Pavel Exner.

A new proof of Gershgorin's theorem

Michael Schelling

We derive a new proof for Gershgorin's theorem, based on the Schur complement, which gives an estimate for the eigenvalues of an operator matrix. This version also holds for unbounded diagonal entries and relatively-bounded off-diagonal entries. Furthermore we also improved the estimates given by the so called Cassini-ovals.
The talk is based on joint work with Anna Dall'Acqua and Delio Mugnolo.

Free probability and operator theory (FPOT)
Organizers: Serban Belinschi, Roland Speicher and Moritz Weber

Speaker	title	time	place
		Monday	M639
Daniel Alpay	On free stochastic processes and their derivatives	13.30-13.55	
Tobias Mai	On the calculation of distributions and Brown measures	13.55-14.20	
Victor Vinnikov	Noncommutative Hardy classes	14.20-14.45	
John Williams	Computational results in operator-values free probability	14.45-15.10	
Uwe Franz	Lévy processes on free compact quantum groups	15.50-16.15	
François Lemeux	Combinatorial and operator algebraic properties for compact quantum groups	16.15-16.40	
Sven Raum	The classification of easy quantum groups	16.40-17.05	
Andreas Thom	Random walks and invariant random forests	17.05-17.30	

Free probability and operator theory

On free stochastic processes and their derivatives

Daniel Alpay

In the commutative setting, Hida's white noise space gives a framework to construct stochastic processes with stationary increments, and their derivatives exist in the Kondratiev's space of stochastic distributions (see [1]). This space is in fact a topological algebra, and has a special structure, in terms of the Våge inequalities, which relate the product with the underlying norms.

In the talk we discuss the non-commutative version of the above analysis. We first discuss non-commutative spaces of stochastic test functions and distributions. The latter space contains the non-commutative white noise space, and is a topological algebra. See [3].

Then we present a family of free stochastic processes with stationary increments. The values of the derivatives are now continuous operators from the space of stochastic test functions into the space of stochastic distributions. Finally we define a stochastic integral for the given family of free processes. See [2].

The talk is based on joint works with Palle Jorgensen (University of Iowa) and Guy Salomon (Ben-Gurion University).

[1] D. Alpay, H. Attia and D. Levanony. On the characteristics of a class of Gaussian processes within the white noise space setting. *Stochastic Processes and their Applications*, vol. 120 (issue 7), pp. 1074-1104 (2010).

[2] D. Alpay, P. Jorgensen and G. Salomon, On free stochastic processes and their derivatives. ArXiv e-prints 1311.3239 (2013).

[3] D. Alpay and G. Salomon. Non-commutative stochastic distributions and applications to linear systems theory. *Stochastic Processes and Applications*, vol. 123 (2013), pp. 2303-2322.

On the calculation of distributions and Brown measures

Tobias Mai

The linearization trick in its self-adjoint version due to G. Anderson allows, in combination with results about the operator-valued free additive convolution, an algorithmic solution to quite general questions in free probability theory, which also have applications in random matrix theory. For instance:

(1) How can we compute the distribution of self-adjoint non-commutative polynomials in free random variables? (answered in joint work with S. Belinschi and R. Speicher)

(2) How can we compute the Brown measure of non-commutative polynomials in free random variables? (answered by S. Belinschi, P. Sniady, and R. Speicher)

In my talk, I will present these solutions and I will discuss, how they can be extended from non-commutative polynomials to the case of non-commutative rational functions.

Noncommutative Hardy classes

Victor Vinnikov

I will discuss the emerging theory of Hardy classes on the noncommutative unit ball over a finite dimensional operator space with a variety of operator space norms. The methods combine the general theory of nc functions with asymptotic freeness results of D.-V. Voiculescu and formulae for integration over unitary groups of B. Collins and P. Sniady. It is the first step in developing a general theory of nc bounded symmetric domains.
This is a joint work with Mihai Popa.

Computational results in operator-valued free probability.

John D. Williams

In the 90's and 2000's, the theory of free probability was extended to operator-valued distributions in order to study amalgamated free product phenomena in operator theory. While much theoretical work has been done in studying this field there are few concrete calculations of the convolution operations and transforms. In this talk, we present several new techniques in calculating the convolution operations (arising from both analytic and combinatorial methods) and several concrete examples of the convolution operation with explicit identification of the moments of the convolved distributions.

Lévy processes on free compact quantum groups

Uwe Franz

We recall the basic theory of Lévy processes on involutive bialgebras and CQG algebras and discuss recent classification results for Lévy processes on the free orthogonal quantum groups O_N^+ and the free permutation quantum groups S_N^+. We also study in more detail some examples of Lévy processes on these quantum groups and on the free unitary quantum group U_N^+.
Based on joint work with Fabio Cipriani, Anna Kula, and Adam Skalski.

[1] F. Cipriani, U. Franz, A. Kula: Symmetries of Lévy processes on compact quantum groups, their Markov semigroups and potential theory. J. Funct. Anal. 266 (2014), no. 5, 2789-2844.

[2] U. Franz, A. Kula, A. Skalski: Lévy processes on quantum symmetry groups, in preparation.

Combinatorial and operator algebraic properties for compact quantum groups

François Lemeux

After recalling a few definitions about Wang's free quantum groups such as free orthogonal and unitary quantum groups and quantum permutation groups, I will give the definition of Bichon for free wreath product quantum groups. I will explain how to compute the fusion rules of certain free wreath products by the quantum permutation groups. To do this I will focus on giving a combinatorial description of the intertwiner spaces in such free wreath products. I will give some applications of the knowledge of such fusion rules such as approximation properties and factoriality.

The classification of easy quantum groups

Sven Raum

Motivated by results in free probability theory and combinatorics, Banica and Speicher introduced in 2009 easy quantum groups. In free probability theory they are used to describe non-classical symmetries of specific distributions.
The class of easy quantum groups contains the important free orthogonal quantum group and the quantum permutation group. They were introduced by Wang and constitute original examples of quantum groups, which cannot be obtained by deforming classical groups. However, there are many more easy quantum groups. Contributions to their classification were made by Banica-Speicher, Banica-Curran-Speicher and Weber. Recently it was completed by Raum-Weber.
In this talk we introduce the concept of easy quantum groups, demonstrate their connection with free probability and describe their classification. Based on this classification, we point out some further directions of research in this field.
The talk is based on joint work with Moritz Weber.

Random walks and invariant random forrests

Andreas Thom

We show that every finitely generated non-amenable group has finite generating sets with arbitrarily small spectral radius. Together with a new estimate for the expected degree for the free minimal spanning forest, this gives some new insight on percolation of non-amenable Cayley graphs – and confirms a weak form of a conjecture due to Itai Benjamini and Oded Schramm.
We also give some applications to a characterization of amenability of group von Neumann algebras in terms of operator spaces and apply the methods to particular cases of Dixmiers problem about amenability of unitarisable groups.

[1] A. Thom: The expected degree of minimal spanning forests, to appear in IMRN, arXiv:1306.0303.

[2] A. Thom: A remark about the spectral radius, submitted, arXiv:1306.1767.

Concrete operators (CO)
Organizers: Alfonso Montes

Speaker	title	time	place
		Monday	M655
Fernanda Botelho	Classes of operators on some spaces of analytic functions	13.30-13.55	
Rui Marreiros	On a singular integral operator with non-Carleman shift and conjugation	13.55-14.20	
Ana C. Conceição	On the kernel of some classes of singular integral operators	14.20-14.45	
Gabriela Kohr	The generalized Loewner differential equation associated to univalent subordination chains in $\mathbb{C}^n$ and complex Banach spaces. Applications	14.45-15.10	
Aneesh M	Supercyclicity and frequent hypercyclicity in the space of self-adjoint operators	15.50-16.15	
Martin Meyries	Pointwise multiplication on vector-valued function spaces with power weights	16.15-16.40	
Hervé Queffélec	Approximation numbers of composition operators on the Dirichlet space	16.40-17.05	
Daniel Seco	Cyclicity in some function spaces in two variables	17.05-17.30	

Concrete Operators

Classes of operators on some spaces of analytic functions

Fernanda Botelho

In this talk we describe the form for the surjective linear isometries and for the bounded hermitian operators on the vector valued little Bloch space and scalar valued Zygmund space.

[1] F. Botelho: Surjective Isometries and Hermitian Operators on Zygmund spaces, 2014.

[2] F. Botelho and J. Jamison: Isometries on the vector valued little Bloch space, 2013.

On a singular integral operator with non-Carleman shift and conjugation

Rui Marreiros

On the Hilbert space $L_2(\mathbb{T})$ we consider the singular integral operator with non-Carleman shift and conjugation $K = P_+ + (aI + AC)P_-$, where $P_\pm$ are the Cauchy projectors, $A = \sum_{j=0}^{m} a_j U^j$, a, a_j, $j = \overline{1, m}$, are continuous functions on the unit circle $\mathbb{T}$, U is the shift operator and C is the operator of complex conjugation. Some estimates for the dimension of the kernel of the operator K are obtained.
This talk is based on a joint work with Ana Conceição.

On the kernel of some classes of singular integral operators

Ana C. Conceição

The main goal of this talk is to show how symbolic computation can be used to compute the kernel of some classes of singular integral operators. The design of our algorithms was focused on the possibility of implementing on a computer, all, or a significant part, of the extensive symbolic and numeric calculations present in the analytical algorithms. The methods developed rely on innovative techniques of Operator Theory and have a great potential of extension to more complex and general problems. In addition, we present some results on the dimension of the kernel of some classes of singular integral operators whose kernel, in general, can not be determined in an explicit form. Some nontrivial examples computed with the computer algebra system *Mathematica* are presented.
This talk is based on joint work with Rui C. Marreiros and José C. Pereira.

[1] Ana C. Conceição, José C. Pereira: An Overview of Symbolic Computation on Operator Theory, *Proceedings of the 1st International Conference on Algebraic and Symbolic Computation - SYMCOMP 2013 (ECCOMAS Thematic Conference) (pen drive), Instituto Superior Técnico, Lisbon, Portugal*, (2013), IP02, 39–71.

[2] Ana C. Conceição, José C. Pereira: Exploring the spectra of singular integral operators with rational coefficients, *Proceedings of the 1st International Conference on Algebraic and Symbolic Computation - SYMCOMP 2013 (ECCOMAS Thematic Conference) (pen drive), Instituto Superior Técnico, Lisbon, Portugal*, (2013), ID10, 175–194.

[3] Ana C. Conceição, Viktor G. Kravchenko, José C. Pereira: Computing some classes of Cauchy type singular integrals with *Mathematica* software, *Advances in Computational Mathematics*, 39 (2) (2013), 273–288.

[4] Viktor G. Kravchenko, Rui C. Marreiros, Juan S. Rodriguez: An estimate for the number of solutions of an homogeneous generalized Riemann boundary value problem with shift, *Operator Theory: Advances and Applications*, 220 (2012), 163–177.

[5] Ana C. Conceição, Viktor G. Kravchenko, José C. Pereira: Rational Functions Factorization Algorithm: a symbolic computation for the scalar and matrix cases, *Proceedings of the 1st National Conference on Symbolic Computation in Education and Research - CSEI 2012 (CD-ROM), Instituto Superior Técnico, Lisbon, Portugal*, (2012), P02, 1–13.

[6] Ana C. Conceição, Viktor G. Kravchenko: About explicit factorization of some classes of non-rational matrix functions, *Mathematische Nachrichten*, 280 (9-10) (2007), 1022–1034.

The generalized Loewner differential equation associated to univalent subordination chains in $\mathbb{C}^n$ and complex Banach spaces. Applications

Gabriela Kohr

In this talk we survey recent results related to extreme points, support points and reachable families of biholomorphic mappings generated by the generalized Loewner differential equation on the unit ball B^n in $\mathbb{C}^n$. Certain applications and conjectures are also considered.
Let $\mathbb{C}^n$ denote the space of n complex variables $z = (z_1, \dots, z_n)$ with the Euclidean inner product $\langle z, w\rangle = \sum_{j=1}^n z_j \overline{w}_j$ and the Euclidean norm $\|z\| = \langle z, z\rangle^{1/2}$. Let B^n be the unit ball in $\mathbb{C}^n$. Also, let $A \in L(\mathbb{C}^n)$ be a linear operator, $k_+(A)$ be the upper exponential index of A, and let $m(A) = \min\{\Re\langle A(z), z\rangle : \|z\| = 1\}$. Under the assumption $k_+(A) < 2m(A)$, we are concerned with the family $S^0_A(B^n)$ of mappings which have A-parametric representation, that is $f \in S^0_A(B^n)$ if there exists an A-normalized univalent subordination chain $f(z,t)$ such that $f = f(\cdot, 0)$ and $\{e^{-tA} f(\cdot, t)\}_{t\geq 0}$ is a locally uniformly bounded family on B^n. We shall present various properties of extreme points and support points associated with the compact family $S^0_A(B^n)$. These results generalize to higher dimensions related results due to R. Pell, W.E. Kirwan and G. Schober.
In the second part of the talk, we use ideas from control theory to consider extremal problems related to bounded mappings in $S^0_A(B^n)$. For this aim, we investigate the time-log M-reachable family $\tilde{\mathcal{R}}_{\log M}(\mathrm{id}_{B^n}, \mathcal{N}_A)$ generated by the Carathéodory mappings, where $M \geq 1$. These results are generalizations to $\mathbb{C}^n$ of well known results due to K. Loewner, Ch. Pommerenke and O. Roth.
In the last part of the talk we present recent results related to univalent subordination chains and the Loewner differential equation in reflexive complex Banach spaces.

[1] P. Duren, I. Graham, H. Hamada, and G. Kohr: Solutions for the generalized Loewner differential equation in several complex variables, *Math. Ann.*, 347(2010), 411–435.

[2] I. Graham, H. Hamada, G. Kohr, and M. Kohr: Asymptotically spirallike mappings in several complex variables, *J. Anal. Math.*, 105 (2008), 267–302.

[3] I. Graham, H. Hamada, G. Kohr, and M. Kohr: Parametric representation and asymptotic starlikeness in $\mathbb{C}^n$, *Proc. Amer. Math. Soc.*, 136 (2008), 3963–3973.

[4] I. Graham, H. Hamada, G. Kohr, and M. Kohr: Univalent subordination chains in reflexive complex Banach spaces, *Contemporary Math.* (AMS), 591 (2013), 83–111.

[5] I. Graham, H. Hamada, G. Kohr, and M. Kohr: Extremal properties associated with univalent subordination chains in $\mathbb{C}^n$, *Math. Ann.*, to appear.

Supercyclicity and frequent hypercyclicity in the space of self-adjoint operators

Aneesh M

Let $\mathcal{L}(H)$ be the class of all bounded operators on a separable infinite- dimensional Hilbert space H and $S(H)$ be the real subspace of $\mathcal{L}(H)$ consisting of all self-adjoint operators. Let C_T be the conjugate operator $S \to TST^*$ on $S(H)$, where $T \in \mathcal{L}(H)$. We then prove the following results.

a. If T satisfies the frequent hypercyclicity criterion, then C_T is frequently hypercyclic on $S(H)$.

b. If T satisfies the supercyclicity criterion, then C_T is supercyclic on $S(H)$.

Here $S(H)$ is equipped with the compact-open topology. Examples of unilateral and bilateral shifts are given to illustrate our results.

Pointwise multiplication on vector-valued function spaces with power weights

Martin Meyries

We investigate pointwise multipliers on vector-valued function spaces over $\mathbb{R}^d$, equipped with Muckenhoupt weights. The main result is that in the natural parameter range, the characteristic function of the half-space is a pointwise multiplier on Bessel-potential spaces H with values in a UMD Banach space. This is proved for a class of power weights, including the unweighted case, and extends the classical result of Shamir and Strichartz. The multiplication estimate is based on the paraproduct technique and a randomized Littlewood-Paley decomposition. An analogous result is obtained for Besov spaces B and Triebel-Lizorkin spaces F. Here the underlying Banach space may be arbitrary.
The talk is based on joint work with Mark Veraar.

Approximation numbers of composition operators on the Dirichlet space

Hervé Queffélec

Let $\varphi : \mathbb{D} \to \mathbb{D}$ be analytic (non-constant) and $C_\varphi(f) = f \circ \varphi$. It is known that the operator C_φ is always bounded on the Hardy space H^2, and NSC for its compactness or membership in Schatten classes were given in the nineties. More recently and more precisely (Li-Q-Rodriguez-Piazza, 2012), its approximation numbers $a_n(C_\varphi)$ were studied, and the following parameter emerged:

$$\beta(C_\varphi) = \liminf_{n\to\infty} [a_n(C_\varphi)]^{1/n}, \quad 0 \le \beta(C_\varphi) \le 1.$$

Note that

$$\beta(C_\varphi) > 0 \iff a_n(C_\varphi) \ge \ c\,r^n \text{ for some } r > 0.$$
$$\beta(C_\varphi) < 1 \iff a_n(C_\varphi) \le \ Cr^n \text{ for some } r < 1.$$

It was proved that $\beta(C_\varphi) > 0$ and that $\beta(C_\varphi) = 1 \iff \|\varphi\|_\infty = 1$.

As concerns the Dirichlet space $\mathcal{D}$, the situation is more intricate: first, C_φ is nots always bounded on $\mathcal{D}$. Second, it can be "very" compact on H^2 and bounded, but not compact, on $\mathcal{D}$. More specifically, the approximation numbers of C_φ on H^2 and $\mathcal{D}$ can behave very differently.
In that talk, we shall present some parallel results on those approximation numbers of C_φ on $\mathcal{D}$, like:

1. $\beta(C_\varphi) > 0$ and moreover $a_n(C_\varphi)$ can tend to 0 arbitrarily slowly.

2. $\beta(C_\varphi) = 1 \iff \|\varphi\|_\infty = 1$.

3. For somes cusp maps, we have $a_n(C_\varphi) \approx e^{-n/\log n}$ on H^2 while $a_n(C_\varphi) \approx e^{-\sqrt{n}}$ on $\mathcal{D}$.

If one compares with the H^2-case, the proofs often necessitate several additional ingredients.

This is joint work with P.Lefevre, D.Li, L.Rodriguez-Piazza.

Cyclicity in some function spaces in two variables.

Daniel Seco

A function f in a function space X is said to be cyclic if the polynomial multiples of f form a dense subspace of X. We consider the question of characterizing the cyclicity of a polynomial f in the family of Dirichlet-type spaces, D_α, in two variables, that is, the space of holomorphic functions over the unit bidisk with Taylor coefficients $\{a_{k,l}\}$ such that $\sum_{k,l=0}^{\infty} |a_{k,l}|^2 (k+1)^\alpha (l+1)^\alpha < \infty$.

Linear operator theory, function theory, and linear systems (OTFS)
Organizers: Joseph A. Ball and Marinus A. Kaashoek

Speaker	title	time	place
		Monday	P 647
Ronald G. Douglas	The Arveson conjecture and the odd GRR theorem	13.30-13.55	
Nicholas Young	A new domain related to μ-synthesis	13.55-14.20	
Daniel Estévez	Separating structures and operator vessels	14.20-14.45	
Eli Shamovich	Taylor joint spectrum and linear systems	14.45-15.10	
Jan H. van Schuppen	Rational observers of rational systems	15.50-16.15	
Bernard Hanzon	Non-negativity of impulse response functions	16.15-16.40	
Amol J. Sasane	A new ν-metric in control theory	16.40-17.05	
Frederik van Schagen	The inverse problem for Ellis-Gohberg orthogonal matrix functions	17.05-17.30	
		Tuesday	P 663
Nikolai Nikolski	The spectrum of $L^2(w)$ Fourier multipliers for weights, Part I	13.30-13.55	
Nikolai Nikolski	The spectrum of $L^2(w)$ Fourier multipliers for weights, Part II	13.55-14.20	
Anders Olofsson	Operator identities for standard weighted Bergman shift and Toeplitz operators	14.20-14.45	
Victor Vinnikov	Transfer-function realization and zero/pole structure for multivariable rational matrix functions	14.45-15.10	
Daniel Alpay	On algebras which are inductive limits of Banach spaces	15.50-16.15	
Zinaida Lykova	A rich structure related to the construction of analytic matrix functions	16.15-16.40	
Mehdi Ghasemi	Integral representation of linear functionals on commutative algebras	16.40-17.05	
Maria Infusino	The infinite dimensional moment problem on semi-algebraic sets	17.05-17.30	
		Thursday	P 663
Yury Arlinskiĭ	Around the Van Daele–Schmüdgen theorem	13.30-13.55	
R.T.W. Martin	Symmetric extensions of symmetric operators	13.55-14.20	
Alexander Sakhnovich	Inverse problem for Dirac systems with locally square-summable potentials	14.20-14.45	
Sanne ter Horst	A pre-order and equivalence relation for Schur class functions and invariance under linear fractional Redheffer maps	14.45-15.10	
Jonathan Arazy	On certain convolution operators on symmetric cones	15.50-16.15	
H. Turgay Kaptanoğlu	Integral operators with harmonic Besov kernels acting on L^p with applications to harmonic Besov spaces on the unit ball of $\mathbb{R}^n$	16.15-16.40 1	
Baruch Solel	Matricial families and weighted shifts	6.40-17.05	
Daniel Markiewicz	Tensor algebras and subproduct systems arising from stochastic matrices	17.05-17.30	

Linear operator theory, function theory, and linear systems

The Arveson conjecture and the odd GRR theorem

Ronald G. Douglas

Extending techniques from Several Complex Variables and Partial Differential Equations, Tang, Yu and I establish the Arveson conjecture in many cases when the zero variety is ”nice”. Related results have also been obtained by Fnglis and Eschmeier. The framework for this approach reveals the relationship of the Arveson problem to the Grothendieck-Riemann-Roch Theorem.

A new domain related to μ-synthesis

Nicholas Young

I will describe the basic complex geometry and function theory of the *pentablock* $\mathcal{P}$, which is the bounded domain in $\mathbb{C}$ given by

$$\mathcal{P} = \{(a_{21}, \operatorname{tr} A, \det A) : A = \left[a_{ij}\right]_{i,j=1}^{2} \in \mathbb{B}\}$$

where $\mathbb{B}$ denotes the open unit ball in the space of 2×2 complex matrices. We prove several characterizations of the domain. We describe its distinguished boundary and exhibit a 4-parameter group of automorphisms of $\mathcal{P}$. We show that $\mathcal{P}$ is intimately connected with the problem of μ-synthesis for a certain cost function μ on the space of 2×2 matrices defined in connection with robust stabilization by control engineers. We demonstrate connections between the function theories of $\mathcal{P}$ and $\mathbb{B}$. We show that $\mathcal{P}$ is polynomially convex and starlike.
The talk is based on joint work with Jim Agler and Zinaida Lykova.

[1] Jim Agler, Zinaida A. Lykova and N. J. Young: The complex geometry of a domain related to μ-synthesis, *arXiv: 1403.1960v1 [math.CV]*, 8 March 2014, 1–36.

Separating structures and operator vessels

Daniel Estévez

In this talk, we will study the so called *separating structures*. These are formed by a pair of commuting selfadjoint operators A_1, A_2 on a Hilbert space K with a decomposition $K = H_- \oplus H_+$ which almost reduces A_j, $j = 1, 2$, in the sense that the operators $P_{H_-} A_j P_{H_+}$ and $P_{H_+} A_j P_{H_-}$ have finite rank. An algebraic curve in $\mathbb{C}^2$ called the discriminant curve can be associated to a separating structure. This curve is given by a determinantal representation which is very similar to the one that appears in the theory of operator vessels of Livšic and Vinnikov.
We show that under certain conditions, the discriminant curve is separated, which means that its real points divide it in two halves. This can be seen as a first step in constructing a model of the separating structure using the operators of multiplication by the coordinate variables on the Hardy spaces of the two halves of the curve.
We also give a notion of a generalized compression of a linear operator. This allows us to define the compression of a separating structure. The result of this compression is an operator vessel whose discriminant curve coincides with that of the separating structure.
This is joint work with Dmitry Yakubovich (Autónoma Univ. of Madrid) and Victor Vinnikov (Ben-Gurion Univ. of the Negev).

Taylor joint spectrum and linear systems

Eli Shamovich

In this talk we'll recall first the notion of the Taylor joint spectrum for an n-tuple of commuting operators defined by J. L. Taylor in [3]. Then we will show using Taylor's methods that for a linear

time invariant overdetermined system, as defined in [1] and [2], the existence of a joint transfer function is implied by vanishing of certain cohomology groups.
Next we'll generalize to the non-commutative setting. We will consider linear translation invariant overdetermined systems on a connected and simply connected Lie group associated to a representation of the associated Lie algebra on a Hilbert space. We will define the joint transfer function and show that the sufficient conditions from the commutative case generalize to this non-commutative setting. The Taylor spectrum in the general setting was described by J. L. Taylor in [4].
The talk is based on joint work with Victor Vinnikov.

[1] J. A. Ball and V. Vinnikov: Overdetermined multidimensional systems: state space and frequency domain methods, *IMA Vol. Math. Appl.*, 134 (2003), 63–119.

[2] M. S. Livšic, N. Kravitsky, A. S. Markus and V. Vinnikov: *Theory of commuting nonselfadjoint operators.*, Mathematics and its Applications 332, Kluwer Academic Publishers Group, 1995.

[3] J. L. Taylor: A joint spectrum for several commuting operators, *Journal of Functional Analysis*, 6 (1970), 172–191.

[4] J. L. Taylor: A general framework for a multi-operator functional calculus, *Advances in Mathematics*, 9 (1972), 183–252.

Rational observers of rational systems

Jan H. van Schuppen

An observer of a control system is a control system itself which takes as inputs the input and the output of another system and provides a state of the observer and predictions of the output of the system. Observers are used in system identification, in prediction, and in control with partial observations. The main reference of an observer of a polynomial system for a chemical reaction network is that of M. Chaves and E.D. Sontag (2002). In this lecture a realization approach to observer synthesis is explored. The problem is to determine a rational realization of the original rational system of which the state is a function of the observations. Note the restriction that the observer is itself also a rational system. Nash observers of Nash systems, a generalization of rational systems, are also investigated. This problem is different from the observer synthesis problems of the literature. The viewpoint is based on the realization interpretation of the Kalman filter. Necessary and sufficient conditions for the existence of an observer are derived. Properties of observers will be described. The approach is based on the algebraic approach to realization of rational and of Nash systems which was developed by the authors over the past years. Examples of rational observers of rational systems will be provided.
The talk is based on joint work with Jana Němcovä (Inst. Chemical Technology, Prague, Czech Republic).

Non-negativity of impulse response functions

Bernard Hanzon

In [1] a sufficient condition (SC) is provided for non-negativity of the impulse response function of a stable continuous-time linear dynamical time-invariant system (also called EPT function, Euler-d'Alembert function etc). In the present contribution it will be shown how the SC can be verified algebraically (exploiting the fact that the class of real exponential-polynomial functions forms a ring over the real numbers and that such functions have at most finitely many real sign-changing zeros, all of which can be determined by the generalized Budan-Fourier (GBF) algorithm). The class of impulse response functions satisfying the SC is closed under addition and multiplication. It will be shown that it is also closed under convolution. This is important for applications in non-negative systems theory, and also in financial mathematics and probability theory, in which

case non-negative, non-identically-zero, impulse response functions can be considered as probability density functions after normalization through division by the L_1 norm. The result obtained for convolution of two non-negative impulse response functions uses the well-known theorem from almost periodic function theory that a *non-negative* almost periodic function which is not identically zero, has positive mean value (see for instance p. 12, section 3 of [2]).
An analogous sufficient condition can be formulated for the class of non-negative impulse functions of discrete-time stable linear time-invariant dynamical systems. We will present result for this SC analogous to the continuous time case. Furthermore it will be shown that systems that allow for a non-negative state-space realization also satisfy the SC as well as impulse response functions that can be written as a sum of squares of impulse response functions.
As no constructive method is known to check non-negativity of arbitrary impulse response functions, the new results on the SC could potentially be very useful in various applications to linear systems theory, probability theory, financial mathematics, financial and economic time series modelling (e.g. GARCH modeling), etc.
The talk is based on joint work with Finbarr Holland.
Research of B. Hanzon is supported by the Science Foundation Ireland under grant numbers RFP2007-MATF802 and 07/MI/2008.

[1] B. Hanzon, F. Holland: "Non-negativity Analysis for Exponential-Polynomial-Trigonometric Functions on $[0, \infty)$", pp. 399-412 in: W. Arendt, J. Ball, J. Behrndt, K.H. Forster, V. Mehrmann, C. Trunk(eds): Spectral Theory, Mathematical System Theory, Evolution Equations, Differential and Difference Equations, *Operator Theory: Advances and Applications*, Springer, Basel, 2012.

[2] A.S Besicovitch: "Almost Periodic Functions", Cambridge University Press/Dover 1954.

A new ν-metric in control theory

Amol J. Sasane

The need for measuring distances between control systems is basic in control theory. For example, in robust control theory, one knows that the unstable control system P to be stabilized is just an approximation of reality, and so one would really like the stabilizing controller to not only stabilize the nominal control system P, but also all sufficiently close systems P' to P. The question of what one means by "closeness" of control systems thus arises naturally. So one needs a metric on the set of unstable systems which is amenable to computation and which has good properties in the robust stabilization problem. Such a desirable metric was introduced by Glenn Vinnicombe in 1993, and is called the "ν-metric". There essentially only linear control systems described by constant coefficient ODEs were considered, and the problem of what happens in the case of linear systems described by PDEs or delay-differential equations was left open. In this talk, we address this issue, and give an extension of the Vinnicombe ν-metric.
The talk is based on joint work with Joseph Ball.

The inverse problem for Ellis-Gohberg orthogonal matrix functions

Frederik van Schagen

The talk deals with the inverse problem for the class of orthogonal functions that was introduced by Ellis and Gohberg in 1992 for the scalar case. The problem is reduced to a linear equation with a special right hand side. This reduction allows one to solve the inverse problem for square matrix functions under conditions that are natural generalizations of those appearing in the scalar case. These conditions lead to a unique solution. Special attention is paid to the polynomial case. A number of partial results are obtained for the non-square case. Examples will be presented to illustrate the main results.
The talk is based on joint work with M.A. Kaashoek.

The spectrum of $L^2(w)$ Fourier multipliers for weights, Part I

Nikolai Nikolski

The spectrum of Fourier multipliers (convolutors) on a weighted L^2 space is shown to be the closure of eigenvalues ("no hidden spectrum"), if the weight function w has only a finite number of singularities of Lévy-Khinchin type. The latter are described as weights for which $\mathcal{F}L^2(w)$ are discrete Besov-Dirichlet spaces. For the same class of weights, the multiplier algebra $Mult(L^2(w))$ permits a complete description (in terms of capacitary inequalities related to Besov-Dirichlet spaces). If time permits, the nature of the hidden spectrum in the case of infinitely many Lévy-Khinchin singularities will be discussed.
This is a report on joint research with Igor Verbitsky.

The spectrum of $L^2(w)$ Fourier multipliers for weights, Part II

Nikolai Nikolski

This part II of the talk is a continuation of Part I.

Operator identities for standard weighted Bergman shift and Toeplitz operators

Anders Olofsson

It is well-known that Toeplitz operators on the Hardy space of the unit disc are characterized by the equality

$$S^*TS = T,$$

where S is the Hardy shift operator. In this talk we discuss a generalized equality of this type which characterizes Toeplitz operators with harmonic symbols in the class of standard weighted Bergman spaces of the unit disc containing the Hardy space and the unweighted Bergman space. The operators satisfying this equality are also naturally described using a slightly extended form of the Sz.-Nagy-Foias functional calculus for contractions. This leads us to consider Toeplitz operators as integrals of naturally associated positive operator measures in order to take properties of balayage into account.
The talk is based on joint works with Issam Louhichi and Aron Wennman.

Transfer-function realization and zero/pole structure for multivariable rational matrix functions

Victor Vinnikov

Reconstruction of a nondegenerate matrix-valued rational function of a single complex variable from its zero and pole data, including the directional information, has been studied at length by Israel Gohberg and collaborators in the 1980s. The solution appears as the transfer function of an input/state/output linear system. This talk will give an update from an ongoing journey towards extending these ideas to the multivariable case, namely, a nondegenerate matrix-valued rational function of d complex variables. Zeroes and poles are now algebraic hypersurfaces in the d-dimensional affine space $\mathbb{C}^d$ (or in its projective compactification $\mathbb{P}^d$), their directions are vector bundles, or more general torsion free sheaves, on these hypersurfaces, and the solution appears as the transfer function of a Fornasini–Marchesini multidimensional linear systems. We will pay a special attention to the case $d = 2$ where one can profitably use a well developed theory of determinantal representations of plane algebraic curves. Even the case of scalar functions is interesting, giving new insights into minimality properties of multivariable realizations.
The talk is based on joint work with Joe Ball and Quanlei Fang.

On algebras which are inductive limits of Banach spaces

Daniel Alpay

Motivated by the theory of non commutative stochastic distributions, see [1], we introduce algebras which are inductive limits of Banach spaces and carry inequalities which are counterparts of the inequality for the norm in a Banach algebra. We then define an associated Wiener algebra, and prove the corresponding version of the well-known Wiener theorem. Finally, we consider factorization theory in these algebra, and in particular, in the associated Wiener algebra. The results are reported in [2].
The talk is based on joint work with Guy Salomon.

[1] D. Alpay and G. Salomon. Non-commutative stochastic distributions and applications to linear systems theory. *Stochastic Processes and Applications*, vol. 123 (2013), pp. 2303-2322.

[2] D. Alpay and G. Salomon. On algebras which are inductive limits of Banach spaces. ArXiv e-prints 1302.3372 (2013).

A rich structure related to the construction of analytic matrix functions

Zinaida Lykova

We present a duality between the space $\mathrm{Hol}(\mathbb{D},\Gamma)$ of analytic functions from the disc $\mathbb{D}$ to the symmetrized bidisc $\Gamma = \{(z_1+z_2, z_1z_2) : z_1, z_2 \in \overline{\mathbb{D}}\}$ and a subset of the Schur class $\mathcal{S}_2$ of the polydisc. We use Hilbert space models for functions in $\mathcal{S}_2$ to obtain necessary and sufficient conditions for solvability of the interpolation problem in the space $\mathrm{Hol}(\mathbb{D},\Gamma)$. The rich structure related to the construction of analytic matrix functions can be summarised diagrammatically as

$$\begin{array}{ccc} \mathcal{S}^{2\times 2} & \longleftrightarrow & \mathcal{R} \\ \updownarrow & & \updownarrow \\ \mathrm{Hol}(\mathbb{D},\Gamma_2) & \longleftrightarrow & \mathcal{S}_2, \end{array} \tag{5}$$

where $\mathcal{S}^{2\times 2}$ is the matricial Schur class and $\mathcal{R}$ is the set of pairs of positive kernels on the disc subject to a certain boundedness condition. In the diagram (5), $\mathcal{S}^{2\times 2}$ and $\mathcal{S}_2$ are much-studied classical objects, whereas $\mathrm{Hol}(\mathbb{D},\Gamma_2)$ and $\mathcal{R}$ were introduced over the past two decades in connection with the *robust stabilization problem* that arises in control engineering.
The talk is based on joint work with Jim Agler, David Brown and Nicholas Young.

[1] Jim Agler, Zinaida A. Lykova and N. J. Young: A case of μ-synthesis as a quadratic semidefinite program, *SIAM Journal on Control and Optimization*, 2013, 51(3), 2472–2508.

Integral representation of linear functionals on commutative algebras

Mehdi Ghasemi

Let A be a vector space of real valued functions on a non-empty set X and $L : A \to \mathbb{R}$ a linear functional. Given a σ-algebra $\mathcal{A}$, of subsets of X, we present a necessary and sufficient condition for L to be representable as an integral with respect to a measure μ on X such that elements of $\mathcal{A}$ are μ-measurable. We apply this result to give an integral representation for all continuous positive semidefinite linear functionals on a locally multiplicatively convex commutative $\mathbb{R}$-algebra.
The talk is based on joint work with M. Marshall and S. Kuhlmann.

[1] M. Ghasemi, M. Marshall, S. Kuhlmann: *Application of Jacobi's representation theorem to locally multiplicatively convex topological* $\mathbb{R}$*algebras*, JFA, 266 (2014) 1041–1049.

[2] M. Ghasemi: *Integral representation of linear functionals on function spaces*, submitted.

The infinite dimensional moment problem on semi-algebraic sets

Maria Infusino

The infinite dimensional moment problem naturally arises from applied fields dealing with complex systems like many-body systems in statistical mechanics, spatial ecology, stochastic geometry, etc. The essence of the analysis of such a system is to evaluate selected characteristics (usually correlation functions), which encode the most relevant properties of the system. These characteristics are indeed the only ones that give a reasonable picture of the qualitative behaviour of the system. It is therefore fundamental to investigate whether a given candidate correlation function represents the actual correlation function of some random distribution. This problem is well-known as *realizability problem.*

In this talk, I present a recent result about the realizability problem on a generic closed basic semi-algebraic subset $\mathcal{S}$ of the space of generalized functions on $\mathbb{R}^d$. Our approach is based on the analogy between the classical multivariate moment problem and the moment problem on nuclear spaces, which goes beyond the operator techniques. Combining the classical results in [1] with the methods developed in [3] to treat the moment problem on basic semi-algebraic sets of $\mathbb{R}^d$, we derive necessary and sufficient conditions for an infinite sequence of generalized functions to be the moment functions of a finite measure concentrated on $\mathcal{S}$.

Our realizability conditions can be more easily verified than the well-known Haviland type conditions introduced by Lenard in [4]. As concrete examples, I show how to apply our theorem to the set of all Radon measures, the set of all sub-probabilities, the set of all simple point configurations. The talk is based on joint work with Tobias Kuna and Aldo Rota, [2].

[1] Y. M. Berezansky and Y. G. Kondratiev: *Spectral Methods in Infinite-Dimensional Analysis*, Naukova Dumka, Kiev, 1988 (In Russian).

[2] M. Infusino, T. Kuna, and A. Rota: The full infinite dimensional moment problem on semi-algebraic sets of generalized functions, *ArXiv e-prints, arXiv:1311.2175*, submitted.

[3] J. B. Lasserre: The K-moment problem for continuous linear functionals, *Trans. Amer. Math. Soc.*, 365, no.5 (2013), 2489–2504.

[4] A. Lenard: States of classical statistical mechanical systems of infinitely many particles I, *Arch. Rational Mech. Anal.*, 59 (1975), 219–239.

Around the Van Daele–Schmüdgen theorem

Yury Arlinskiĭ

For a bounded non-negative self-adjoint operator acting in a complex, infinite-dimensional, separable Hilbert space $\mathcal{H}$ and possessing a dense range $\mathcal{R}$ we propose a new approach to characterisation of phenomenon concerning the existence of subspaces (closed linear manifolds) $\mathfrak{M} \subset \mathcal{H}$ such that $\mathfrak{M} \cap \mathcal{R} = \mathfrak{M}^\perp \cap \mathcal{R} = \{0\}$. The existence of such subspaces leads to various pathological properties of unbounded self-adjoint operators related to von Neumann theorem [1], which states that for any unbounded self-adjoint operator A in $\mathcal{H}$ there exists a self-adjoint operator B such that intersection of their domains is trivial: $\operatorname{dom} A \cap \operatorname{dom} B = \{0\}$. We revise the Van Daele [2] and Schmüdgen [3] assertions to refine them. We also develop a new systematic approach, which allows to construct for any unbounded densely defined symmetric/self-adjoint operator T infinitely many pairs $\langle T_1, T_2 \rangle$ of its closed densely defined restrictions $T_k \subset T$ such that $\operatorname{dom}(T^* T_k) = \{0\}$ ($\Rightarrow \operatorname{dom} T_k^2 = \{0\}$) $k = 1, 2$ and $\operatorname{dom} T_1 \cap \operatorname{dom} T_2 = \{0\}$, $\operatorname{dom} T_1 \dot{+} \operatorname{dom} T_2 = \operatorname{dom} T$.

The talk is based on joint work with Valentin A. Zagrebnov (Université d'Aix-Marseille).

[1] J. von Neumann: Zur Theorie der Unbeschränkten Matrizen, *J. Reine Angew. Math.* 161 (1929), 208–236.

[2] A. Van Daele: On pairs of closed operators, *Bull. Soc. Math. Belg., Ser. B* 34 (1982), 25–40.

[3] K. Schmüdgen: On restrictions of unbounded symmetric operators, *J. Oper. Theory*, 11 (1984), 379–393.

Symmetric extensions of symmetric operators

R.T.W. Martin

We provide a complete function-theoretic characterization of the family of all symmetric extensions of a closed simple symmetric operator B with equal deficiency indices defined in a separable Hilbert space $\mathcal{H}$ in the case where the Livsic characteristic function Θ_B of B is an inner function. This problem is equivalent to the characterization of the set of all partial isometric extensions of a partial isometry with equal defect indices. Our results show, in particular, that if A is any self-adjoint extension of B, then one can construct a natural unitary invariant $\Phi[A;B]$ which is a contractive analytic (matrix) function on $\mathbb{C}_+$ and obeys $\Phi[A;B] \geq \Theta_B$, *i.e.* $\Theta_B^{-1}\Phi[A;B]$ is contractive and analytic on $\mathbb{C}_+$. Moreover $\Phi[A_1;B] = \Phi[A_2,B]$ if and only if A_1 is unitarily equivalent to A_2 via a unitary that fixes $\mathcal{H}$ and the map $A \mapsto \Phi[A;B]$ is a bijection onto the set of all contractive analytic functions greater than Θ_B. Given another simple symmetric operator B_2 we further provide necessary and sufficient conditions on Θ_{B_2} so that B_1 is unitarily equivalent to a restriction of B_2.

Inverse problem for Dirac systems with locally square-summable potentials and rectangular Weyl functions; some applications to operators with distributional matrix-valued potentials

Alexander Sakhnovich

Inverse problem for Dirac systems with locally square summable potentials and rectangular Weyl functions is solved. For that purpose we use a new result on the linear similarity between operators from a subclass of triangular integral operators and the operator of integration. Borg-Marchenko type uniqueness theorem is derived also. In this way we generalize some of the statements from [1]. The main result has applications to the case of Schrödinger-type operators with distributional matrix-valued potentials.
The talk and research were supported by the Austrian Science Fund (FWF) under Grant No. P24301.
The talk is based partly on joint work with J. Eckhardt, F. Gesztesy, R. Nichols and G. Teschl.

[1] A.L. Sakhnovich, L.A. Sakhnovich, and I.Ya. Roitberg: *Inverse Problems and Nonlinear Evolution Equations. Solutions, Darboux Matrices and Weyl–Titchmarsh Functions*, Studies in Mathematics, Vol. 47, de Gruyter, 2013.

A pre-order and equivalence relation for Schur class functions and invariance under linear fractional Redheffer maps

Sanne ter Horst

Motivated by work of Yu.L. Shmul'yan in [2] a pre-order and an equivalence relation on the set of operator-valued Schur class functions are introduced and the behavior of Redheffer linear fractional transformations (LFTs) with respect to these relations is studied. In particular, it is shown that Redheffer LFTs preserve the equivalence relation, but not necessarily the pre-order. The latter does occur under some additional assumptions on the coefficients in the Redheffer LFT. The talk is based on [1].

[1] S. ter Horst A pre-order and an equivalence relation on Schur class functions and their invariance under linear fractional transformations, to appear in *J. Operator Theory.*

[2] Yu.L. Shmul'yan, Generalized fractional-linear transformations of operator spheres (Russian), *Sibirsk. Mat. Zh.* **21** (1980), no. 5, 114–131; translation in *Siberian Math. J.* **21** (1991), no. 3, 496–506.

On certain convolution operators on symmetric cones

Jonathan Arazy

I will report on a current project of stuying convolution operators on finite dimensional symmetric cones Ω which are invariant with respect to the action of the maximal solvable group of automorphisms of Ω. Of particular interest is the study of the spectral properties and the L^p-boundedness of these operators. The study of the general case is motivated by the special case of the one-dimensional cone $(0, \infty)$, where simple methods (essentially - Schur's lemma on integral operators) yield surprisingly sharp results.

Integral operators with harmonic Besov kernels acting on L^p with applications to harmonic Besov spaces on the unit ball of $\mathbb{R}^n$

H. Turgay Kaptanoğlu

The weighted Lebesgue spaces we work on are the L^p_q with respect to the measure $d\nu_q(x) = (1-|x|^2)^q\, d\nu(x)$ on the unit ball $\mathbb{B}$ of $\mathbb{R}^n$ with $q \in \mathbb{R}$, ν indicating volume, and $p \geq 1$. The kernels of the integral operators we study have one of two forms. Either $K_{s,t}(x,y) = (1-|x|^2)^t\, R_{s+t}(x,y)\,(1-|y|^2)^s$ or $K_{s,t}(x,y) = \dfrac{(1-|x|^2)^t\,(1-|y|^2)^s}{\{x,y\}^{(n+s+t)/2}}$, where $\{x,y\} = 1 - 2x \cdot y + |x|^2|y|^2$ and $s, t \in \mathbb{R}$. Here $R_q(x,y)$ is the reproducing kernel of the Hilbert harmonic Besov space b^2_q. The Besov spaces b^p_q with $q \in \mathbb{R}$ consist of harmonic functions on $\mathbb{B}$ whose radial derivatives lie in the harmonic Bergman spaces for which $q > -1$. The R_q are weighted infinite sums of zonal harmonics, and they are obtained in [1]. The second form of the kernel of the integral operators actually is an upper bound on the first form.
We characterize in terms of s, t those integral operators that are bounded on the L^p_q using a Schur test and detailed pointwise and integral growth rate estimates on the R_q and their derivatives near the boundary of $\mathbb{B}$. These estimates form the most difficult and lengthy part of this work. The integral operators are then used to show that the order of the radial derivative used in the integral definition of the b^p_q can be chosen freely as long as it is above a certain threshold. Another application is to the characterization of bounded Bergman projections from the L^p_q onto the b^p_q.
The talk is based on joint work with S. Gergün and A. E. Üreyen.

[1] S. Gergün, H. T. Kaptanoğlu, A. E. Üreyen: Reproducing Kernels for Harmonic Besov Spaces on the Ball, *C. R. Math. Acad. Sci. Paris*, **347** (2009), 735–738.

Matricial families and weighted shifts

Baruch Solel

Let $H^\infty(E)$ be the Hardy algebra of a W^*-correspondence E over a W^*-algebra M. These algebras are generated by a copy of M and shifts (defined by the elements of E). Each element $F \in H^\infty(E)$ gives rise to a family $\{\widehat{F_\sigma}\}$ of analytic operator valued functions where σ runs over the normal representations of M and $\widehat{F_\sigma}$ is defined on the (open) unit ball of the operator space $E^{\sigma *}$ (associated with E and σ). Such a family exhibits "matricial structure" that we studied in previous works (inspired by works of Joseph Taylor, Kaliuzhnyi-Verbovetskyi and Vinnikov, D. Voiculescu and others).
In this talk I will show that one can study matricial families of operator-valued functions defined on more general matricial sets (not necessarily unit balls) by studying Hardy algebras generated by a copy of M and *weighted* shifts. This work generalizes some results of G. Popescu.
The talk is based on joint work with Paul S. Muhly.

Tensor algebras and subproduct systems arising from stochastic matrices

Daniel Markiewicz

In analogy with the “classical” tensor algebra over Fock space, given a subproduct system X in the sense of Shalit and Solel, one can define a *tensor algebra* associated to X. It is the non-selfadjoint operator algebra generated by shift operators over the Fock W*-correspondence of X.
In this talk we will be interested in the subproduct systems and the tensor algebras associated to stochastic matrices over countable state spaces (possibly infinite). We will discuss the classification of the tensor algebras in this case, and how much they remember about the matrices. More precisely, let P and Q be two stochastic matrices over the same state space, with tensor algebras $\mathcal{T}_+(P)$ and $\mathcal{T}_+(Q)$. We show for example that if P and Q are recurrent, then $\mathcal{T}_+(P)$ and $\mathcal{T}_+(Q)$ are isometrically isomorphic if and only if P and Q are the same up to permutation of indices. We also show that an algebraic isomorphism between $\mathcal{T}_+(P)$ and $\mathcal{T}_+(Q)$ is automatically bounded, leading to strong results on classification up to algebraic isomorphism when the state space is finite.
This talk is based on joint work with Adam Dor-On.

[1] A. Dor-On, D. Markiewicz: Operator algebras and subproduct systems arising from stochastic matrices, *preprint* arxiv:1401.7032.

Toeplitz operators and related topics (Toep)
Organizers: Sergei Grudsky and Nikolai Vasilevski

Speaker	title	time	place
		Monday	P 663
Wolfram Bauer	Commuting Toeplitz operators with harmonic symbols	13.30-13.55	
Zeljko Cuckovic	From the $\overline{\partial}$-Neumann operator to Axler-Zheng theorem	13.55-14.20	
Maribel Loaiza	On Toeplitz operators on the pluriharmonic Bergman space	14.20-14.45	
Raul Quiroga-Barranco	Commuting Toeplitz operators defined by isometric group actions	14.45-15.10	
Grigori Rozenblum	On the d-bar equation in a class of distrbutions and a finite rank theorem for Toeplitz operators in the Fock space	15.50-16.15	
Jari Taskinen	Recent results on boundedness of Toeplitz operators	16.15-16.40	
Harald Upmeier	Homogeneous vector bundles and intertwining operators	16.40-17.05	
Nikolai Vasilevski	On compactness of commutators and semi-commutators of Toeplitz operators on the Bergman space	17.05-17.30	
		Thursday	S 623
M. Cristina Câmara	Asymmetrically truncated Toeplitz operators in H_p	13.30-13.55	
Sergei Grudsky	Eigenvalues of Hermitian Toeplitz matrices with smooth simple-loop symbols	13.55-14.20	
Alexey Karapetyants	Mixed norm weighted Bergman spaces and Topelitz operators with unbounded symbol	14.20-14.45	
Yuri Karlovich	C^*-algebras of Bergman type operators in polygonal domains	14.45-15.10	
Oleksiy Karlovych	On a weighted singular integral operator with shifts and slowly oscillating data	15.50-16.15	
Armando Sánchez-Nungaray	Commutative C^*-algebra of Toeplitz operators on the superball	16.15-16.40	
M. Amélia Bastos	An invertibility theory for classes of convolution type operators	16.40-17.05	

Toeplitz operators and related topics

Commuting Toeplitz operators with harmonic symbols

Wolfram Bauer

In this talk we characterize commuting Toeplitz operators with harmonic symbols acting on the Segal-Bargmann space over the complex plane. We assume a certain growth condition of the symbols at infinity. The vanishing of the commutant turns out to be equivalent to a fix point equation of the Berezin transform, which we then study in the case of arbitrary dimensions. The results are compared to what is known in case of Toeplitz operators acting on the Bergman space over the unit disc or the Hardy space over the circle. New effects arise due to the unboundedness of the symbols.
The talk is based on joint work with Boo Rim Choe and Hyungwoon Koo, Korea University.

From the $\overline{\partial}$-Neumann operator to Axler-Zheng theorem

Zeljko Cuckovic

When studying compactness of Toeplitz and Hankel operators on domains in $\mathbb{C}^n$ we show that the $\overline{\partial}$-Neumann operator plays an important role. We use this approach to prove a version of Axler-Zheng Theorem on a class of smooth bounded pseudoconvex domains.
This is joint work with Sonmez Sahutoglu.

On Toeplitz operators on the pluriharmonic Bergman space

Maribel Loaiza

Toeplitz operators on the harmonic Bergman space of the unit disk and of the upper half plane behave quite different from those acting on the corresponding Bergman space.The harmonic Bergman space of the unit disk (and of the upper half-plane) is closed related to the Bergman and the anti-Bergman spaces and every Toeplitz operator acting on it can be represented in terms of Toeplitz operators acting on the Bergman or on the anti-Bergman space and operators between these two spaces. In [1] we studied different algebras of Toeplitz operators acting on the harmonic Bergman space of the unit disk (and of the upper half-plane) and we found very interesting differences between these operators and the corresponding operators acting on the Bergman space. Inspired by the results in [1] and the work of Bauer, Quiroga-Barranco and Vasilevski concerning to commutative algebras of Toeplitz operators on the unit ball, we study Toeplitz operators acting on the pluriharmonic Bergman space of the Siegel domain in $\mathbb{C}^n$ and the C^*-algebras generated by these operators. In this talk we are mainly interested in the C^*-algebra generated by Toeplitz operators whose symbols are invariant under the action of the quasi-parabolic group of biholomorphisms of the Siegel domain.
The talk is based on joint work with C. Lozano.

[1] M. Loaiza, C. Lozano: On C^*-algebras of Toeplitz Operators on the harmonic Bergman space, *Integral Equations and Operator Theory*, 76 (2013), 105–130.

Commuting Toeplitz operators defined by isometric group actions

Raul Quiroga-Barranco

Let D be a bounded symmetric domain whose group of isometries has a connected component given by the simple Lie group G. It is well know that there is a unitary representation of G on $\mathcal{A}^2_\lambda(D)$, where the latter denotes a weighted Bergman space of holomorphic square-integrable functions on D for a suitably weighted Lebesgue measure.
On the other hand, for $a \in L^\infty(D)$ the Toeplitz operator on $\mathcal{A}^2_\lambda(D)$ is defined by

$$T_a^{(\lambda)}(f) = B^{(\lambda)}(af)$$

where $B^{(\lambda)}$ is the orthogonal projection $L^2_\lambda(D) \to \mathcal{A}^2_\lambda(D)$.
A natural way to make use of both of these pieces of information is to consider symbols invariant under a given closed subgroup of G. More precisely, for H a closed subgroup of G we denote by $\mathcal{A}_H$ the subspace of $L^\infty(D)$ consisting of H-invariant symbols. In the work of Grudsky, Karapetyants, Quiroga-Barranco, Vasilevski and others it has been shown that the space $\mathcal{A}_H$ defines commuting Toeplitz operators for some choices of H. More specifically, this has been proved for H a maximal Abelian subgroup of G when $D = \mathbf{B}^n$ is the unit n-ball. However, except for very few exceptions, a similar phenomenon for other irreducible bounded symmetric domains has not been established. In this talk we will exhibit the existence of many other subgroups H of G for which the set of symbols $\mathcal{A}_H$ defines commuting Toeplitz operators. For example, we will see that for any irreducible bounded symmetric domain D, if K denotes the maximal compact subgroup of G, then the set of symbols $\mathcal{A}_K$ yields commuting Toeplitz operators on every weighted Bergman space $\mathcal{A}^2_\lambda(D)$. We will also present several other results for non-compact groups.
The talk is based on joint work with Matthew Dawson and Gestur Ólafsson.

On the d-bar equation in a class of distrbutions and a finite rank theorem for Toeplitz operators in the Fock space

Rozenblum Grigori

The problem on describing finite Toeplitz operators in Bergman type spaces has recently attracted much attention. We describe some most recent results on this topic. We consider symbols belonging to certain class of distributions in the plane. This class is dual to the space of functions which, together with a certain number of derivatives, have Gaussian decay at infinity. We define Toeplitz operators in the Fock space with such symbols. The main result is that if such Toeplitz operator has finite rank, the symbol must be a finite linear combination of the delta-distributions at some points and their derivatives. The main analytical tool is a theorem on the solvability of the d-bar equation in such classes of distributions and on the estimates of the solutions.

Recent results on boundedness of Toeplitz operators

Jari Takinen

We extend some of the results of the paper [1] for Dirichlet and Besov spaces on the open unit disc of $\mathbb{C}$ and also for the weighted harmonic Bergman space on the unit ball $\mathbb{C}^N$. The results include sufficient conditions for the boundedness of a Toeplitz operator in terms of a weak averaging condition for the symbol of the operator. Compactness and Fredholm theory are also treated in some cases. In the case of Besov spaces we show how the approach can be returned back to the Bergman space setting of [1]. In the case of spaces on $\mathbb{C}^N$, we give the detailed, combinatorially challenging generalisation of the original proof to this multivariable setting. (We also include a fix of a minor flaw in the argument of [1] concerning the use of the mean value property.)
The talk is based on joint work with Congwen Liu, Antti Perälä and Jani Virtanen.

[1] J.Taskinen, J.Virtanen: Toeplitz operators on Bergman spaces with locally integrable symbols, *Rev.Math.Iberoamericana* , 26,2 (2010), 693–706.

Homogeneous vector bundles and intertwining operators

Harald Upmeier

The well-known Bergman spaces of holomorphic functions can be generalized to holomorphic sections of suitable vector bundles. This is particularly important for symmetric domains $D = G/K$, where the holomorphic sections of homogeneous vector bundles give rise to irreducible representations of G, the so-called holomorphic discrete series. In the talk we present an explicit decomposition of the multiplicity-free hermitian vector bundles into G-irreducible components. Already for the unit disk and the unit ball, this leads to interesting formulas for the corresponding

intertwining operators. We also discuss the general case of symmetric domains of arbitrary rank, and construct intertwining operators using the combinatorial properties of integer partitions. From an operator theoretic point of view, these vector-valued Bergman type spaces are of interest, since the Toeplitz C^*-algebra is still irreducible although the unitary group action is not irreducible any more.
The result for the unit ball (rank 1) is based on joint work with Gadhadar Misra, Bangalore.

On compactness of commutators and semi-commutators of Toeplitz operators on the Bergman space

Nikolai Vasilevski

Given a C^*-subalgebra $\mathcal{A}(\mathbb{D})$ of $L_\infty(\mathbb{D})$, the Toeplitz operators with symbols from $\mathcal{A}(\mathbb{D})$ and acting on the Bergman space over the unit disk $\mathbb{D}$ may have (or have not) compact semi-commutator and compact commutator property. In the talk we discuss the next questions.

- How big (essential) is the gap between algebras $\mathcal{A}(\mathbb{D})$ possessing compact semi-commutator and compact commutator properties?
- How to measure the differences between such algebras?
- How do the properties of related operator algebras respond to that differences?

An invertibility theory for classes of convolution type operators

M. Amélia Bastos

Classes of convolution type operators on unions of bounded intervals whose kernels have Fourier transforms which are related to corona problems are studied. Under the assumption of existence of a nontrivial solution of an homogeneous Riemann-Hilbert problem on Hardy spaces with matrix coefficient G, criteria for the existence of a generalized factorization of symbols G of Toeplitz operators were found as well as explicit formulas for its factors in terms of solutions of two corona problems [1]. This theory allowed the invertibility study for classes of convolution type operators with symbols satisfying some conditions. In this talk a generalization of this invertibility theory is established, and formulas for the inverse operators are given, using a generalization of the portuguese transformation for $n \times n$ matrix functions.
The talk is based on joint work with P. Lopes.

[1] M.A. Bastos, Yu.I. Karlovich, A.F. dos Santos: Oscillatory Riemann-Hilbert problems and the corona theorem, *J. Func. Anal.*, 197 (2003), 347–397

Asymmetrically truncated Toeplitz operators in H_p

M. Cristina Câmara

Asymmetrically truncated Toeplitz operators in the Hardy spaces H_p are introduced and discussed, using their relation with Toeplitz operators with matrix symbols to which they are shown to be equivalent after extension.
The talk is based on joint work with Jonathan Partington.

Eigenvalues of Hermitian Toeplitz matrices with smooth simple-loop symbols

Sergei Grudsky

The report presents higher-order asymptotic formulas for the eigenvalues and eighenvectors of large Hermitian Toeplitz matrices with moderately smooth symbols which trace out a simple loop on the real line. The formulas are established not only for the extreme eigenvalues, but also for the inner eigenvalues. The results extend and make more precise existing results, which so far pertain to banded matrices or to matrices with infinitely differentiable symbols. A fixed-point equation for the eigenvalues is also given. Note that this equation can be solved numerically by an iteration method.

Mixed norm weighted Bergman spaces and Topelitz operators with unbounded symbols

Alexey Karapetyants

Starting with the papers of S. Bergman and M. Djrbashian, the spaces of p - integrable with respect to σ finite measure analytic functions on open connected sets in the complex plane have been intensively studied by many authors. The study of Toeplitz operators (and algebras of Toeplitz operators) acting in these spaces served as an important objective of the entire theory. However, the symbols of Toeplitz operators under consideration were bounded functions (or measures), in general. N.L. Vasilevski proposed an approach, which provides effective study of Toeplitz operators with unbounded (special) symbols in weighed Bergman spaces. These techniques and methods were developed later in the papers by N.L. Vasilevski, S.M. Grudsky and A.N. Karapetyants.
The purpose of the talk is to discuss some generalizations of the mentioned Bergman spaces. We consider mixed norm weighted Bergman spaces on the unit disc D and the upper half plane Π. For instance, in the case of the unit disc D the mixed norm Lebesgue space $L^{2,p}_\lambda(D)$, $\lambda > -1$ is defined to consist of measurable functions $f(z) = f(rt)$ on D such that

$$\|f\|_{L^{2,p}_\lambda(D)} = \left(\int_T \left| \int_0^1 |f(rt)|^p \nu_\lambda(r) r dr \right|^{\frac{2}{p}} \frac{1}{\pi} \;\; d\sigma(t) \right)^{\frac{1}{2}}.$$

Here $d\sigma(t) = \frac{dt}{it}$, and $\nu_\lambda(r) = (\lambda+1)(1-r^2)^\lambda$, $\lambda > -1$. Now the mixed norm weighted Bergman space $A^{2,p}_\lambda(D)$ is defined to be the subspace of $L^{2,p}_\lambda(D)$ consisting of analytic functions.
It goes without saying that when studying analytic p - integrable functions, the main attention should be paid to behavior near the boundary. That is why we introduce and study the mixed norm spaces with p norm taken along the lines that are orthogonal to boundary. Certainly, further study of Toeplitz operators and C^*-algebras of Toeplitz operators with unbounded special symbols in these spaces is an another problem to undertake.
The talk is based on joint work with Irina Smirnova (see e.g. [1]).

[1] A.N. Karapetyants, I Yu. Snirnova: On a connection of weighted mixed norm Bergman spaces on upper half plane and unit disc with Hardy spaces, *Izvestia Vuzov. Severo-Kavkazskii Region*, 4 (2012), 15–17.

C^*-algebras of Bergman type operators in polygonal domains

Yuri Karlovich

Fredholm symbol calculi for C^*-algebras of Bergman type operators with continuous coefficients are constructed for a class of convex polygonal domains. Fredholm criteria for operators in these algebras are established.

On a weighted singular integral operator with shifts and slowly oscillating data

Oleksiy Karlovych

Let α, β be orientation-preserving diffeomorphism (shifts) of $\mathbb{R}_+ = (0, \infty)$ onto itself with the only fixed points 0 and ∞ and U_α, U_β be the isometric shift operators on $L^p(\mathbb{R}_+)$ given by $U_\alpha f = (\alpha')^{1/p}(f \circ \alpha)$, $U_\beta f = (\beta')^{1/p}(f \circ \beta)$, and $P_2^\pm = (I \pm S_2)/2$ where

$$(S_2 f)(t) := \frac{1}{\pi i} \int\limits_0^\infty \left(\frac{t}{\tau}\right)^{1/2-1/p} \frac{f(\tau)}{\tau - t} \, d\tau, \quad t \in \mathbb{R}_+,$$

is the weighted Cauchy singular integral operator. We prove that if α', β' and c, d are continuous on $\mathbb{R}_+$ and slowly oscillating at 0 and ∞, and

$$\limsup_{t \to s} |c(t)| < 1, \quad \limsup_{t \to s} |d(t)| < 1, \quad s \in \{0, \infty\},$$

then the operator $(I - cU_\alpha)P_2^+ + (I - dU_\beta)P_2^-$ is Fredholm on $L^p(\mathbb{R}_+)$ and its index is equal to zero. Moreover, its regularizers are described.
The talk is based on a joint work with Yuri Karlovich and Amarino Lebre.

Commutative C^*-algebra of Toeplitz operators on the superball

Armando Sánchez-Nungaray

In this talk we study Toeplitz operators acting on the super Bergman space on the superball. We consider five different types of commutative super subgroups of the biholomorphisms of the superball or the super Siegel domain and we prove that the C^*-algebras generated by Toeplitz operators whose symbols are invariant under the action of these groups are commutative.
The talk is based on joint work with R. Quiroga-Barranco.

Dima Arov's world (DA)
Harry Dym and Olof Staffans

Speaker	title	time	place
		Tuesday	M607
Vadim Adamyan	Unitary couplings of semiunitary operators	13.30-13.55	
Vadim Adamyan	Equivalent couplings of semiunitary operators and the Nehari problem for operator functions	13.55-14.20	
Dmitry Kaliuzhnyi-Verbovetskyi	Learning from Arov: Scattering systems	14.20-14.45	
Natalia Rozhenko	Passive impedance systems theory and its applications	14.45-15.10	
James Rovnyak	Operator identities and generalized Carathéodory functions	15.50-16.15	
Derk Pik	Separation of sounds of muscial instruments	16.15-16.40	
Mikael Kurula	State/signal realizations of passive continuous-time behavious	16.40-17.05	
Victor Katsnelson	Approximation by scapar rational function with preassigned poles. An approach based on analytic matrix functions theory	17.05-17.30	

Dima Arov's world

Special day in honour of Dima Arov on the occasion of his 80th birthday

Unitary couplings of semiunitary operators

Vadim Adamyan

Let $V_\pm$ be two simple semiunitary operators in separable Hilbert spaces $\mathcal{D}_\pm$, respectively. A unitary operator U in a Hilbert space $\mathcal{H}$ is a unitary coupling of the operators $V_\pm$ if U is a unitary extension of V_+ and at the same time U^{-1} is a unitary extension of V_-. The theory of unitary couplings of arbitrary semiunitary operators, which was developed in its general form in early works of D. Arov and the author, has proved to be a rather effective tool in the study of classical and quantum scattering problems, in problems that can be reduced to interpolation problems in the theory of analytic matrix-valued functions, and in the theory of stationary stochastic processes. This talk is an attempt to highlight the origins of the general theory of unitary couplings, to describe its main concepts, and to draw attention to some of its striking applications.

Equivalent couplings of semiunitary operators and the Nehari problem for operator functions

Vadim Adamyan

Let $V_\pm$ be two simple semiunitary operators in separable Hilbert spaces $\mathcal{D}_\pm$, respectively. A minimal isometric extension V of V_+ such that V^* is a minimal isometric extension of V_- is called an isometric coupling of $V_\pm$. Unitary couplings U of $V_\pm$, which are unitary extensions of the same isometric coupling V of $V_\pm$ are called *equivalent*. In this talk we show that there is a one-to-one correspondence between a class of equivalent couplings and the set of all solutions of a certain Nehari problem for scalar or operator-valued functions. This correspondence was actually a universal key to the main results in the series of joint works of D.Z. Arov, M.G. Krein, and the author.

Learning from Arov: Scattering systems

Dmitry S. Kaliuzhnyi-Verbovetskyi

D.Z. Arov was one of the first to realize that the scattering systems formalism can be used as an alternative description and a useful tool for studying passive (in particular, conservative) linear 1D systems. In my talk, I will discuss multivariable scattering systems and their interplay with various types of conservative linear dD systems.

Passive impedance systems theory and its applications

Natalia Rozhenko

The talk is dedicated to the 80th birthday of Damir Arov and it will be based on our recent results on the development of the Darlington method for passive impedance systems with losses in the scattering channels [1]–[5].

A new model of a passive impedance system with a bilaterally stable evolutionary semigroup will be considered, and some special types of passive impedance realizations (minimal, optimal, *-optimal, minimal and optimal, and minimal and *-optimal) will be presented. It will be shown how the results of passive systems theory can be applied to present a p-dimensional regular weak stationary discrete time stochastic process $y(t)$ as the output data of a linear bi-stable discrete time dynamical system.

The talk is based on joint work with Damir Arov.

[1] Arov D.Z., Rozhenko N.A.: Realizations of stationary stochastic processes: applications of passive system theory, *Methods Funct. Anal. Topology*, vol. 18, no. 4 (2012), 305–331.

[2] Arov D.Z., Rozhenko N.A.: On the relation between the Darlington realizations of matrix functions from the Caratheodory class and their $J_{p,r}$-inner SI-dilations, *Mathematical Notes*, vol. 90, no. 6 (2011), 103–114.

[3] Arov D.Z., Rozhenko N.A.: To the theory of passive systems of resistance with losses of scattering channels, *Journal of Mathematical Sciences*, vol. 156, (2009), 742–760.

[4] Arov D.Z., Rozhenko N.A.: $J_{p,m}$-inner dilations of matrix-valued functions that belong to the Caratheodory class and admit pseudocontinuation, *St. Petersburg Mathematical Journal*, 19, (2008), 375–395.

[5] Arov D.Z., Rozhenko N.A.: Passive impedance systems with losses of scattering channels, *Ukr. Math. Journal*, 159, (2007), 678–707.

Operator identities and generalized Carathéodory functions

James Rovnyak

This talk will discuss indefinite cases of operator identities related to interpolation problems in the unit disk. The Krein–Langer integral representation is adapted to the disk in a generalization of the classical Herglotz representation of analytic functions having nonnegative real part in the unit disk. The representation is used to associate an operator identity with a given matrix-valued generalized Carathéodory function. The abstract interpolation problem is to reverse this assignment and characterize all generalized Carathéodory functions which are associated with a given operator identity.
This is joint work with Lev Sakhnovich.

Separation of sounds of musical instruments

Derek Pik

When listening to music, we are able to concentrate on one particular instrument. Somewhere between the perceiving of a sound by the ear and our experience of the sound a selection is made: which component of the sound is interesting and which is not.
The human ear makes extensive use of the stereo image of the sound. We will not do this: we have studied a mono version of sound selection. If we are given a mono sample of a piece of music, can we filter out one particular instrument? Such filter, were it to exist, has many applications. It can be used for analyzing natural sounds, for studio purposes, and as an intelligent means of reducing natural noise from a sound sample.
This research reports on work by my doctoral student Bert Greevenbosch, performed under my guidance. The thesis is available on the internet at
http://www.bertgreevenbosch.nl/msc/scriptie.pdf

State/signal realizations of passive continuous-time behaviours

Mikael Kurula

In the talk I present parts of [1], an extension of the isometric, co-isometric, and unitary functional models of a Schur function on the complex unit disk by de Branges and Rovnyak [2,3]. The differences from the classical setting are essentially the following:

1. Instead of working on the unit disk $\mathbb{D}$, which corresponds to discrete-time systems, the paper is about realizations in terms of continuous-time systems.

2. A Schur function ϕ describes the frequency-domain input/output behaviour of its realization. We wish to dispose of the distinction between input and output in a system; therefore we take as data to be realized an arbitrary maximal dissipative right-shift invariant subspace $\mathfrak{W}_+$ of $L^2([0,\infty);\mathcal{W})$, where $\mathcal{W}$ is some Kreĭn space.

3. The realizations that we develop are *state/signal systems* rather than contractions between Hilbert spaces.

With respect to an arbitrary fundamental decomposition $\mathcal{W} = -\mathcal{Y} \boxplus \mathcal{U}$ of $\mathcal{W}$ into an input space $\mathcal{U}$ and output space $\mathcal{Y}$, $\mathfrak{W}_+$ determines a certain Schur function $\varphi : \mathbb{C}^+ \to \mathcal{L}(\mathcal{U}; \mathcal{Y})$. This function φ can moreover be realized in a fashion similar to [2,3]; see [4]. The results in [1], however, are not formulated in terms of any arbitrary choice of fundamental decomposition.
The talk is based on joint work with Damir Arov and Olof Staffans.

[1] D. Z. Arov, M. Kurula, and O. J. Staffans: Canonical state/signal shift realizations of passive continuous time behaviors, *Complex Anal. Oper. Theory*, 5(2):331–402, 2011.

[2] L. de Branges and J. Rovnyak: Canonical models in quantum scattering theory, In *Perturbation Theory and its Applications in Quantum Mechanics (Proc. Adv. Sem. Math. Res. Center, U.S. Army, Theoret. Chem. Inst., Univ. of Wisconsin, Madison, Wis., 1965)*, pages 295–392. Wiley, New York, 1966.

[3] L. de Branges and J. Rovnyak: *Square summable power series.* Holt, Rinehart and Winston, New York, 1966.

[4] J. A. Ball, M. Kurula, O. J. Staffans, and H. Zwart: De Branges-Rovnyak realizations of operator-valued Schur functions on the complex right half-plane, *Complex Anal. Oper. Theory Online First*, `http://dx.doi.org/10.1007/s11785-014-0358-2`, 2014.

Approximation by scalar rational function with preassigned poles. An approach based on analytic matrix functions theory

Victor Katsnelson

In classical complex function theory there are problems for scalar functions which can be better understood and studied in the framework of J-contractive matrix-valued functions. This point of view was initiated by V.P. Potapov who created the theory of J-contractive matrix-valued functions (originally with other motivations). The validity of this point of view was confirmed by V.P. Potapov's own work and by investigations of his followers on extrapolation and interpolation problems for functions belonging to special classes and on approximation problems for scalar functions. We present the solution of an approximation problem for scalar functions which was posed by G.Ts. Tumarkin in the late 60s and which was solved by the speaker in 1993. It is essentially based on a result of D.Z. Arov on rational approximation of pseudocontinuable functions.
The talk is based on the article [1].

[1] V. Katsnelson: Description of a class of functions which admit an approximation by rational functions with preassigned poles. I., *Operator Theory: Advances and Applications*, Vol. 72 (1994), 87–132.

Semigroups: theory and applications (STA)

Organizers: András Bátkai and Bálint Farkas

Speaker	title	time	place
		Tuesday	M623
Charles Batty	L^p-Tauberian theorems	13.30-13.55	
Yuri Tomilov	A new approach to approximation of operator semigroups	13.55-14.20	
David Seifert	The non-analytic growth bound and a Katznelson-Tzafriri theorem for measures	14.20-14.45	
Sebastian Król	Extrapolation of L^p-maximal regularity for second order Cauchy problems	14.45-15.10	
S. Verduyn Lunel	Completeness theorems, characteristic matrices and small solutions	15.50-16.15	
J. Glück	Eventual positivity of operator semigroups	16.15-16.40	
Moritz Gerlach	Stability of strong Feller semigroups	16.40-17.05	
Markus Kunze	Diffusion with nonlocal boundary conditions	17.05-17.30	
		Thursday	S 631
Robert Denk	A strongly damped plate equation with Dirichlet-Neumann boundary conditions	13.30-13.55	
Mark Veraar	A new approach to maximal regularity with time dependent generator	13.55-14.20	
Sven-Ake Wegner	Growth bound and spectral bound for semigroups on Fréchet spaces	14.20-14.45	
Jan Rozendaal	Functional calculus on interpolation spaces for groups	14.45-15.10	
Martin Meyries	Traces and embeddings of anisotropic function spaces	15.50-16.15	
Lassi Paunonen	Robustness of polynomial stability of semigroups	16.15-16.40	
Hendrik Vogt	Sharp heat kernel estimates for Schrödinger semigroups	16.40-17.05	
Martin Adler	Point delays for flows in networks	17.05-17.30	
		Friday	M655
Marjeta Kramar Fijavž	Semigroups of max-plus linear operators	13.30-13.55	
Marcus Waurick	Does the non-autonomous Schrödinger equation depend continuously on the potential?	13.55-14.20	
Felix Schwenninger	On measuring the unboundedness of the H^∞-calculus for analytic semigroups	14.20-14.45	
Luciano Abadias	Algebraic properties and sharp extensions for resolvent families	14.45-15.10	
Pedro J. Miana	Approximation of C_0-semigroups and resolvent operators by Laguerre expansions	15.50-16.15	
J. Alberto Conejero	Chaos for the hyperbolic bioheat equation	16.15-16.40	

Semigroups: theory and applications

L^p-Tauberian theorems

Charles Batty

There is an established Tauberian theory for bounded functions and bounded semigroups based on pointwise estimates. It originated with results of Ingham and Karamata in the 1930s and has recently been developed to provide estimates of rates of convergence. One may regard this as a particular case of Tauberian theory for L^p-functions. The talk will describe the L^p-theory for functions, and its interaction with semigroup theory and rates of decay of energy of wave equations.
The talk is based on joint work with Alexander Borichev and Yuri Tomilov.

A new approach to approximation of operator semigroups

Yuri Tomilov

The talk will concern operator dynamics which is regular in a sense of convergence to zero with a certain rate. We start with presenting L^p-analogues of the classical tauberian theorem of Ingham and Karamata, and its variations giving rates of decay. We show how these results can be applied to derive L^p-decay of operator families arising in the study of the decay of energy for damped wave equations and local energy for wave equations in exterior domains. Finally, we give several examples showing that the L^p-rates of decay obtained in this way are optimal in a sense.

This is joint work with Charles Batty (Oxford) and Alexander Borichev (Marseille).

The non-analytic growth bound and a Katznelson-Tzafriri theorem for measures

David Seifert

The Katznelson-Tzafriri theorem is a cornerstone of the asymptotic theory of operator semigroups. This talk will introduce the result in its original form and present a recent extension based on results obtained in [1], where the Hilbert space case is treated. The main ingredient required to extend the result to the setting of general Banach spaces is the so-called non-analytic growth bound of a C_0-semigroup, which leads to a simpler proof even in the Hilbert space case.

[1] C.J.K. Batty, R. Chill and Y. Tomilov: Fine scales of decay of operator semigroups, *Journal of the European Mathematical Society*, to appear.

Extrapolation of L^p-maximal regularity for second order Cauchy problems

Sebastian Król

We present a new extrapolation theorem for singular integral operators with operator-valued kernels which may be considered as a refined version of the result by Rubio de Francia, Ruiz & Torrea in the case of weighted Lebesque spaces [1]. It extends also the recent result by Curbera, García-Cuerva, Martell & Pérez on extrapolation of Calderón-Zygmund operators in the case of weighted rearrangement invariant Banach function spaces [2].
Our assumptions on kernels are asymmetric, and in each coordinate weaker than the standard conditions. Nevertheless, we get the same conclusion as in the case of the above-mentioned extrapolation result for Calderón-Zygmund operators for which the Coifman-Fefferman inequality applied. Our assumptions are in a sense best possible.
We derive from this theorem an extrapolation result which is particularly adapted to convolution operators on the positive half-line, and thus, for example, to the second order Cauchy problem. We show that if the second order problem

$$\ddot{u} + B\dot{u} + Au = f \text{ on } \mathbb{R}_+, \quad u(0) = \dot{u}(0) = 0, \tag{6}$$

has L^p-maximal regularity for some $p \in (1, \infty)$, i.e. for every $f \in L^p_{loc}(\mathbb{R}_+; X)$, (6) admits a unique solution $u \in W^{2,1}_{loc}(\mathbb{R}_+; X)$ satisfying

$$u, \dot{u}, \ddot{u}, B\dot{u}, Au \in L^p_{loc}(\mathbb{R}_+; X),$$

then, in particular, it has $\mathbb{E}_w$-maximal regularity for every rearrangement invariant Banach function space $\mathbb{E}$ with Boyd indices $p_{\mathbb{E}}, q_{\mathbb{E}} \in (1, \infty)$ and for every Muckenhoupt weight $w \in A_{p_{\mathbb{E}}}$.
The talk is based on joint work with R. Chill.

[1] J. Rubio de Francia, F. J. Ruiz and J. L. Torrea: Calderón-Zygmund theory for operator-valued kernels, *Adv. in Math.* 62 (1986), 7–48.

[2] G. P. Curbera, J. García-Cuerva, J. M. Martell and C. Pérez: Extrapolation with weights, rearrangement-invariant function spaces, modular inequalities and applications to singular integrals, *Adv. Math.* 203 (2006), 256–318.

[3] R. Chill, S. Król: Extrapolation of L^p-maximal regularity for second order Cauchy problems, *Banach Center Publications*, 2014, to appear.

Completeness theorems, characteristic matrices and small solutions

Sjoerd Verduyn Lunel

In this talk we present necessary and sufficient conditions for completeness of the span of eigenvectors and generalized eigenvectors for a large class of classes of nonselfadjoint operators including Hilbert-Schmidt operators of order one. The results are based on the notion of a characteristic matrix and precise resolvent estimates near infinity using the growth theory for subharmonic functions and, in particular, the Phragmén-Lindelöf indicator function. Our result extends a number of classical results for trace class operators to Hilbert-Schmidt operators of order one, and allows us to extend Keldysh type theorems to operators that are not close to selfadjoint. Finally, we discuss the close relation between superstable semigroups, non-completeness of the underlying infinitesimal generator and the associated characteristic matrix.
The talk is based on joint work with Rien Kaashoek.

Eventual positivity of operator semigroups

Jochen Glück

Positive one-parameter semigroups play a prominent role in operator theory both due to their importance in applications and due to their beautiful structure theory. A possible generalization are eventually positive semigroups, i.e. time discrete or time continuous semigroups which become positive for large times. By now, this phenomenon has mainly been studied in the finite dimensional setting.
In this talk, we consider the concept of eventual positivity in the setting of infinite dimensional Banach lattices. We give an overview over several characterizations and spectral results and we present some special phenomena that only occur in infinite dimensions. This talk is based on joint work with Wolfgang Arendt, Daniel Daners and James Kennedy.

Stability of strong Feller semigroups

Moritz Gerlach

A bounded operator T on the space of all signed Borel measures on a Polish space Ω is continuous with respect to the weak topology induced by the bounded measurable functions if and only if T is given by

$$T\mu = \int_\Omega k(x, \cdot\,) d\mu(x)$$

for some bounded transition kernel k. We show in [1] that these weakly continuous operators form a sublattice of all bounded operators where the modulus $|T|$ is given by the transition kernel $|k|$. Using a theorem of Greiner, this gives rise to a short and purely analytic proof of Doob's theorem on convergence of Markovian and irreducible strong Feller semigroups.
If the semigroup is not irreducible but weakly ergodic, then it is still strongly stable, i.e. it converges strongly to the ergodic projection onto its fixed space [2]. Taking the characterization of weak ergodicity from [3] into account, this shows that Markovian strong Feller semigroups are strongly stable if and only if its fixed space separates the points of the fixed space of the adjoint semigroup on the continuous functions.

[1] M. Gerlach and M. Kunze: On the lattice structure of weakly continuous operators on the space of measures, preprint 2014.

[2] M. Gerlach: A Tauberian theorem for strong Feller semigroups, to appear in *Arch. Math. (Basel)*, 2014.

[3] M. Gerlach and M. Kunze: Mean ergodic theorems on norming dual pairs, to appear in *Ergodic Theory Dynam. Systems*, 2014.

Diffusion with nonlocal boundary conditions

Markus Kunze

On a bounded, Dirichlet regular set $\Omega \subset \mathbb{R}$, we consider the Laplace Operator Δ_μ with domain

$$D(\Delta_\mu) = \Big\{ u \in C(\overline{\Omega}) : u(z) = \int_\Omega u(x)\, d\mu_z(x) \Big\}.$$

Here $\mu : \partial\Omega \to \mathcal{M}(\overline{\Omega})$ is a weak*-continuous measure-valued map such that $0 \le \mu_z(\Omega) \le 1$ and $\mu_z(\partial\Omega) = 0$ for all $z \in \partial\Omega$.
We show that the Operator Δ_μ generates a (in general not strongly continuous) positive contraction semigroup on $C(\overline{\Omega})$ which is strongly Feller. We also discuss the asymptotic behavior of this semigroup.
This is joint work with Wolfgang Arendt and Stefan Kunkel.

A strongly damped plate equation with Dirichlet-Neumann boundary conditions

Robert Denk

We investigate sectoriality and maximal regularity in L^p-L^q-Sobolev spaces for the strongly damped plate equation with Dirichlet-Neumann (clamped) boundary conditions. More precisely, we consider the strongly damped plate equation and the related first-order system in the whole space, the half-space, and in bounded and sufficiently smooth domains. It turns out that the first-order system related to the scalar equation generates a C_0-semigroup in the whole case only after a shift in the operator. Moreover, for the boundary value problem, the first-order system is not sectorial, even if we admit a shift. In fact, we have to include zero boundary conditions in the underlying function space in order to obtain sectoriality and maximal regularity.
The talk is based on joint work with Roland Schnaubelt (Karlsruhe).

A new approach to maximal regularity with time dependent generator

Mark Veraar

Maximal regularity can often be used to obtain a priori estimates which give global existence results. For example, using maximal regularity it is possible to solve quasi-linear and fully nonlinear PDEs by elegant linearization techniques combined with the contraction mapping principle.
In this talk I will explain a new approach to maximal L^p-regularity for parabolic PDEs with time dependent generator $A(t)$. Here we do not assume any continuity properties of $A(t)$ as a function

of time. We show that there is an abstract operator theoretic condition on A(t) which is sufficient to obtain maximal L^p-regularity. As an application I will obtain an optimal $L^p(L^q)$ regularity result in the case each $A(t)$ is a system of $2m$-th order elliptic differential operator on $\mathbb{R}^d$ in non-divergence form. The main novelty in is that the coefficients are merely measurable in time and we allow the full range $1 < p, q < \infty$.
This talk is based on joint work with Chiara Gallarati

Growth bound and spectral bound for semigroups on Fréchet spaces

Sven-Ake Wegner

Let X be a complete metrizable locally convex space. A family of operators $T = (T(t))_{t\geq 0} \subseteq L(X)$ which satisfies the evolution property is a C_0-semigroup if in addition all its orbits are continuous. The generator $(A, D(A))$ is defined exactly as in the case of a Banach space.
In the talk, we first generalize the concept of the growth bound $\omega_0(T)$ from Banach spaces to Fréchet spaces. Then we employ classical and recent approaches for non Banach spectral theories to define the spectral bound $\mathrm{s}(A)$.
We show on the one hand that the Banach space inequality $\mathrm{s}(A) \leq \omega_0(T)$ extends to the new setting. On the other hand we prove that the forward shift on the space of all complex sequences endowed with the topology of pointwise convergence generates a uniformly continuous semigroup for which the above bounds are distinct—a phenomenon which cannot occur on a Banach space.

Functional calculus on interpolation spaces for groups

Jan Rozendaal

In this talk some recent results on functional calculus for generators of strongly continuous groups on real interpolation spaces will be explained. Interpolation versions of known transference principles for groups are established that link harmonic analysis to functional calculus theory. These allow one to use results about Fourier multipliers on vector-valued Besov spaces to obtain statements on the boundedness of certain functional calculi for group generators on real interpolation spaces. From this results about principal value integrals, sectorial operators and generators of cosine functions can be deduced.
This talk is based on joint work with Markus Haase.

Traces and embeddings of anisotropic function spaces

Martin Meyries

Trace spaces of a general class of function spaces of intersection type with mixed regularity scales are characterized. We can overcome the difficulty of mixed scales by employing a microscopic improvement in weighted Sobolev and mixed derivative embeddings with fixed integrability parameter. As an application we prove maximal L^p-L^q-regularity for the linearized, fully inhomogeneous two-phase Stefan problem with Gibbs-Thomson correction.
The talk is based on joint work with Mark Veraar.

Robustness of polynomial stability of semigroups

Lassi Paunonen
In this presentation we study a strongly continuous semigroup $T(t)$ generated by $A : \mathcal{D}(A) \subset X \to X$ on a Hilbert space X. The semigroup is called *polynomially stable* if $T(t)$ is uniformly bounded, if $i\mathbb{R} \subset \rho(A)$, and if there exist constants $\alpha > 0$ and $M > 0$ such that [1]

$$\|T(t)A^{-1}\| \leq \frac{M}{t^{1/\alpha}} \qquad \forall t > 0.$$

We consider the preservation of the polynomial stability of $T(t)$ under finite-rank perturbations $A+BC$, where $B \in \mathcal{L}(\mathbb{C}^m, X)$ and $C \in \mathcal{L}(X, \mathbb{C}^m)$. We assume that for some $\beta, \gamma \geq 0$ the operators B and C satisfy

$$\mathcal{R}(B) \subset \mathcal{D}((-A)^\beta) \qquad \text{and} \qquad \mathcal{R}(C^*) \subset \mathcal{D}((-A^*)^\gamma).$$

As the main result of the presentation we show that if $\beta+\gamma \geq \alpha$, then there exists $\delta > 0$ such that the semigroup generated by $A+BC$ is polynomially stable whenever $\|(-A)^\beta B\| \cdot \|(-A^*)^\gamma C^*\| < \delta$.

[1] A. Borichev and Y. Tomilov. Optimal polynomial decay of functions and operator semigroups. *Math. Ann.*, 347(2):455–478, 2010.

[2] L. Paunonen. Robustness of polynomial stability with respect to unbounded perturbations. *Systems Control Lett.*, 62:331–337, 2013.

Sharp heat kernel estimates for Schrödinger semigroups

Hendrik Vogt

It is well known that C_0-semigroups on $L_2(\mathbb{R}^d)$ generated by Schrödinger operators $-H_V = \Delta - V$ with Kato class potentials $V\colon \mathbb{R}^d \to \mathbb{R}$ have integral kernels satisfying Gaussian bounds

$$p_t(x,y) \leq c\, e^{-\omega t} \exp\left(-\frac{|x-y|^2}{at}\right)$$

with any $\omega < -E_0(H_V)$ and $a > 4$, where $E_0(H_V) := \inf \sigma(H_V)$.
The estimate is also valid with $\omega = E_0(H_V)$ and $a = 4$ if polynomial correction terms are added: instead of $e^{-\omega t}$ one only needs $(1+t)^{d/2} e^{-E_0(H_V)t}$, and the term $\exp\bigl(-\frac{|x-y|^2}{at}\bigr)$ can be replaced with

$$\left(1 + \frac{|x-y|^2}{4t}\right)^{\frac{d-1}{2}} \exp\left(-\frac{|x-y|^2}{4t}\right),$$

due to a general result of Sikora. We will discuss conditions under which the power $\frac{d-1}{2}$ can be reduced. In some cases, the term $\bigl(1 + \frac{|x-y|^2}{4t}\bigr)$ can be avoided altogether.

Point delays for flows in networks

Martin Adler

In [1] the authors investigated a vertex delay problem on a finite network. But they could only prove their result for bounded delay operators.
In this talk I will present a network approach using [2] that covers the case of delay operators being unbounded both in the space and time component.
This approach further generalizes to network structures with boundary conditions depending on each other.
The talk is based on joint work with Miriam Bombieri and Klaus-Jochen Engel.

[1] F. Bayazit, B. Dorn and A. Rhandi: Flows in networks with delay in the vertices, *Math. Nachr.*, 285 (2012), 1603–1615.

[2] M. Adler, M. Bombieri and K. Engel: On perturbations of generators of C_0-semigroups, submitted to *Abstract and Applied Analysis* 2014.

Semigroups of max-plus linear operators

Marjeta Kramar Fijavž

We consider operators on max-plus vector space (or idempotent space) that are linear with respect to the max-plus operations. We call such operators *max-plus linear operators*. We define the *(one-parameter) max-plus semigroup* $(T(t))_{t\geq 0}$ as a family of max-plus linear operators satisfying the semigroup properties. We study properties of strongly continuous max-plus semigroups and present some examples of them.
The talk is based on joint work with Aljoša Peperko and Eszter Sikolya.

Does the non-autonomous Schrödinger equation depend continuously on the potential?

Marcus Waurick

We consider the non-autonomous Schrödinger equation with bounded potential V in a bounded domain Ω with Dirichlet boundary conditions, zero initial conditions and non-homogeneous right-hand side f. Given a sequence of bounded potentials $(V_n)_n$ such that $V_n \to V \in L^\infty(\mathbb{R}\times\Omega)$ in $\sigma(L^\infty, L^1)$, we show that under additional assumptions on $(V_n)_n$, the sequence $(u_n)_n$ of solutions of

$$(\partial_t + V_n(t,x) + i\Delta)\, u_n(t,x) = f(t,x) \quad ((t,x)\in\mathbb{R}\times\Omega)$$

converges weakly to the solution of

$$(\partial_t + V(t,x) + i\Delta)\, u(t,x) = f(t,x) \quad ((t,x)\in\mathbb{R}\times\Omega).$$

We outline possible generalizations of the results obtained, see also [1].

[1] M. Waurick: Continuous dependence on the coefficients for a class of non-autonomous evolutionary equations, arxiv.org:1308.5566.

On measuring the unboundedness of the H^∞-calculus for analytic semigroups

Felix Schwenninger

The boundedness of the H^∞-calculus is investigated by estimating the bound $\kappa(\varepsilon)$ of the mapping $H^\infty \to \mathcal{L}(X)$: $f \mapsto f(A)T(\varepsilon)$ for ε near zero. Here, $-A$ generates an analytic, exponentially stable C_0-semigroup T and H^∞ denotes the space of bounded analytic functions on a sector strictly containing the spectrum of A. We show that $\kappa(\varepsilon) = \mathcal{O}(|\log(\varepsilon)|)$ in general, whereas $\kappa(\varepsilon) = \mathcal{O}(1)$ if and only if the calculus is bounded.
Such estimates first appeared in [3] in a slightly different form. In [1] it was shown that one can even get the logarithmic behavior for non-analytic semigroups, if X is a Hilbert space.
It is well-known that square function estimates for A and A^* characterize bounded calculi in Hilbert spaces. We show that having such estimates for A or A^* only, gives $\kappa(\varepsilon) = \mathcal{O}(\sqrt{|\log\varepsilon|})$, thus interpolates the general behavior.
The talk is based on joint work with Hans Zwart.

[1] M. Haase, J. Rozendaal: Functional calculus for semigroup generators via transference, *J. Funct. Anal.*, 265 (2013), no. 1, 3345–3368.

[2] F.L. Schwenninger, H. Zwart: On measuring the unboundedness of the H^∞-caclulus, In preparation, (2014).

[3] P. Vitse: A Besov class functional calculus for bounded holomorphic semigroups, *J. Funct. Anal.*, 228 (2005), no. 2, 245–269.

Algebraic properties and sharp extensions for resolvent families

Luciano Abadias

In this talk we give extensions of local (a,k)-regularized resolvent families. In particular, we extend local solutions of fractional Riemann-Liouville Cauchy problems without loss of regularity. The main technique to obtain these extensions is the use of algebraic structure of these solutions, which are defined by a new version of the Duhamel formula. To obtain these results we apply methods of Laplace transform in several variables. Finally, we consider some examples in fractional equations. The talk is based on joint work with Carlos Lizama and Pedro J. Miana.

[1] L. Abadias, C. Lizama, P. J. Miana: *Algebraic properties and sharp extensions for solution families of bounded operators.* Prepint (2014).

[2] V. Keyantuo, P.J. Miana, L. Sánchez-Lajusticia: *Sharp extensions for convoluted solutions of abstract Cauchy problems.* Integr. Equ. Oper. Theory, 77 (2013), no. 2, 211-241.

Approximation of C_0-semigroups and resolvent operators by Laguerre expansions

Pedro J. Miana

In this talk we introduce Laguerre expansions to approximate C_0-semigroups and resolvent operators. We give the rate of convergence of Laguerre expansion to the C_0-semigroup and compare with other known approximations. To do that, we need to study Laguerre functions and the convergence of Laguerre series in Lebesgue spaces. To finish, we consider concrete C_0-semigroups: shift, convolution and holomorphic semigroups where some of these results are improved.
The talk is based on joint work with Luciano Abadias.

[1] L. Abadias, P. J. Miana: *C_0-semigroups and resolvent operators approximated by Laguerre expansions.* ArXiv:1311.7542 (2013).

[2] G. Szegö: *Orthogonal polynomials,* American Mathematical Society Colloquium Publications Volume XXIII. American Mathematical Society (1967).

Chaos for the hyperbolic bioheat equation

J. Alberto Conejero

The Hyperbolic Heat Transfer Equation describes heat processes in which extremely short periods of time or extreme temperature gradients are involved. It is already known that there are solutions of this equation which exhibit a chaotic behaviour, in the sense of Devaney, on certain spaces of analytic functions with certain growth control [1,3]. Similar results were previously stated for the solutions of the Parabolic Heat Transfer Equation by Herzog in [4].
We show that this chaotic behaviour still appears when we add a source term to this equation, i.e. in the Hyperbolic Bioheat Equation. These results can also be applied for the Wave Equation and for a higher order version of the Hyperbolic Bioheat Equation.
The talk is based on joint work with F. Rdenas and M. Trujillo [2]

[1] J.A. Conejero, A. Peris, and M. Trujillo: Chaotic asymptotic behavior of the hyperbolic heat transfer equation solutions, *Int. J. Bifurcation and Chaos*, 20, no.9 (2010), 2943–2947.

[2] J.A. Conejero, F. Ródenas, and M. Trujillo: Chaos for the hyperbolic bioheat equation, *Preprint*, (2014).

[3] K.G. Grosse-Erdmann and A. Peris Manguillot: *Linear Chaos.* Universitext. Springer, London, 2011.

[4] G. Herzog: On a universality of the heat equation, *Math. Nachr.*, 188 (1997), 169–171.

Continuous and discrete hypercomplex analysis (CDHA)
Organizers: Paula Cerejeiras and Irene Sabadini

Speaker	title	time	place
		Tuesday	S 623
Vladislav V. Kravchenko	Time-dependent Maxwell's system for inhomogeneous media and bicomplex pseudoanalytic functions	13.30-13.55	
Franciscus Sommen	Clifford superanalysis	13.55-14.20	
Sirkka-Liisa Eriksson	Hyperbolic function theory	14.20-14.45	
Paula Cerejeiras	Analysis for fractional Dirac operators	14.45-15.10	
Mihaela Vajiac	A new type of hypercomplex analysis in $\mathbb{R}^3$	15.50-16.15	
Uwe Kähler	Hardy decompositions and Riemann problems for discrete monogenic functions	16.15-16.40	
Swanhild Bernstein	Fractional monogenic signals in $\mathbb{R}^3$ and $\mathbb{R}^4$	16.40-17.05	
Irene Sabadini	Krein-Langer factorization in the slice hyperholomorphic setting	17.05-17.30	

Continuous and discrete hypercomplex analysis

Time-dependent Maxwell's system for inhomogeneous media and bicomplex pseudo-analytic functions

Vladislav V. Kravchenko

Using the relation between the time-dependent Maxwell system for inhomogeneous media with the bicomplex Vekua equation [1] we solve the problem of electromagnetic scattering by an inhomogeneous layer in terms of transmutation operators (see, e.g., [2]). With the aid of the recent results from [3] the representation of solution obtained is converted into an efficient numerical technique. Numerical examples are discussed.

[1] V. V. Kravchenko, M. P. Ramirez, On Bers generating functions for first order systems of mathematical physics. Advances in Applied Clifford Algebras, 2011, v. 21, 547-559.

[2] V. A. Marchenko *Sturm-Liouville operators and applications* (Basel: Birkhäuser, 1986).

[3] V. V. Kravchenko, S. M. Torba Analytic approximation of transmutation operators and applications to highly accurate solution of spectral problems. Submitted, available at arxiv.org.

Clifford superanalysis

Franciscus Sommen

We first introduce some basic concepts about superspace and superanalysis using the Vladimirov-Volovich approach. Then we introduce the basics for Clifford analysis in superspace, leading to the expected ortho-symplectic structure. Next we recall what we think should be the correct rules for differential forms on superspace and explain how one can integrate superforms. This leads to a way to introduce the Berezin integral and, more in general, to integration in "deepspace". All this can be applied to Clifford-superforms.

Hyperbolic function theory

Sirkka-Liisa Eriksson

Our aim is to build a function theory based on hyperbolic metric and Clifford numbers. We consider harmonic functions with respect to the Laplace-Beltrami operator of the Riemannian metric $ds^2 = x_2^{-\frac{2k}{n-1}} \left(\sum_{i=0}^{n} dx_i^2\right)$ and their function theory in $\mathbb{R}^{n+1}$. H. Leutwiler in 1992 discovered that the power function, calculated using the Clifford product, is a conjugate gradient of a hyperbolic harmonic function. He started to research these type of functions, called H-solutions, satisfying a modified Dirac equation, connected to the hyperbolic metric. All usual trigonometric and exponential functions have a natural extensions to H-solutions. S.-L. Eriksson and H. Leutwiler, extended H-solutions to total algebra valued functions, called hypermonogenic functions. Function theory of based on hyperbolic metric is providing a new style of analysis, since the hyperbolic metric is invariant under isometries of the upper half-space.
We study generalized hypermonogenic functions, called $k-$hypermonogenic functions. For example the function $|x|^{k-n+1}\, x^{-1}$ is k-hypermonogenic. Note that 0-hypermonogenic are monogenic and $n-1$-hypermonogenic functions are hypermonogenic
We prove in odd case the Cauchy type integral formulas for k-hypermonogenic functions where the kernels are calculted using the hyperbolic distance of the Poincare upper half space model. Earlier these results have been proved for hypermonogenic functions. The coauthor of the work is Heikki Orelma, Tampere University of Technology.

[1] Eriksson-Bique, S.-L., k -hypermonogenic functions, *In Progress in analysis: H. Begerh, R. Gilbert, and M.W.Wong, eds*, World Scientific, Singabore, 2003, 337-348.

[2] S.-L. Eriksson: Integral formulas for hypermonogenic functions,*Bull. of the Belgian Math. Soc.* 11 (2004), 705-717.

[3] S.-L., Eriksson and H. Leutwiler:An improved Cauchy formula for hypermonogenic functions,*Adv. appl. Clifford alg.* 19 (2009), 269–282.

[4] S.-L. Eriksson: A Hyperbolic Dirac Operator and its Kernels, Complex Variables and Elliptic Equations. *Complex Variables and Elliptic Equations: An International Journal* Volume 58, Issue 6 (2013), 767-781.

[5] S.-L. Eriksson and H. Orelma: Hyperbolic Laplace Operator and the Weinstein Equation in $\mathbb{R}^3$, *Adv. in Appl. Clifford Algebras* 24 (2014), 109-124.

Analysis for fractional Dirac operators

Paula Cerejeiras

In recent years one can observe a growing interest in fractional calculus. This theory applied to modelling of problems - be it optics or quantum mechanics - provides a wider degree of freedom which can be used for a more complete characterization of the object under study or as an additional relevant parameter.
In this talk we present a higher-dimensional function theory based on fractional Weyl relations. We introduce a fractional Dirac operator and discuss some of its properties as well as the relevant toolkit for the function theory.
The talk is based on joint work with N. Vieira and U. Kähler.

A new type of hypercomplex analysis in $\mathbb{R}^3$

Mihaela Vajiac

We introduce a new algebra structure in $\mathbb{R}^3$ coming from quotients of polynomial rings. In particular, such an algebra is generated by a unit which cubes to -1, and it turns out to be isomorphic to $\mathbb{C} \times \mathbb{R}$. Furthermore, we employ in full detail a hypercomplex analysis on these algebras, and in this context we study classical problems such as Severi's analytic continuation theorem. We conclude with a generalization of Clifford Analysis for such type of algebras.

Hardy decompositions and Riemann problems for discrete monogenic functions

Uwe Kähler

In recent years one can observe an increasing interest in obtaining discrete counterparts for various continuous structures, especially in a discrete equivalent to continuous function theory. This is not only driven by the idea of creating numerical algorithms for different continuous methods of studying partial differential equations, but also for true discrete purposes, as can be seen, among others, by recent results of S. Smirnov in connecting complex discrete function theory with problems in probability and statistical physics or the introduction of finite element exterior calculus in analyzing variational crimes While methods for a discrete function theory are very much developed in the complex case the higher-dimensional case is yet underdeveloped. This is mainly due to the fact that while discrete complex analysis is under (more or less) continuous development since the 1940's discrete Clifford analysis started effectively only in the eighties and nineties with the construction of discrete Dirac operators either for numerical methods of partial differential equations or for quantized problems in physics. The development of genuinely function theoretical methods only started quite recently. In this talk we would like to address the problem of discrete boundary values and introduce discrete Hardy spaces as function spaces of such boundary values. After presenting Hardy decompositions for discrete Hardy spaces we will consider the Riemann problem for discrete monogenic functions as an application.
The talk is based on joint work with P. Cerejeiras, M. Ku, and F. Sommen.

Fractional monogenic signals in $\mathbb{R}^3$ and $\mathbb{R}^4$

Swanhild Bernstein

Fractional analytic signals play an important role in optics because they can be obtained by experiment and they can be described by the fractional Fourier transform. The experimental way leads to another, surprisingly identical definition, the fractional analytic signal can be obtain by rotating the analytic signal.
One analogue of an analytic signal in higher dimensions is the monogenic signal. Because in higher dimension it is difficult to understand what a fractional Fourier transform is we follow the way to define the fractional monogenic signal by a rotated versions of the monogenic signal. Rotations in $\mathbb{R}^3$ and $\mathbb{R}^4$ can be described perfectly by quaterions. For a signal defined by a function of three real variables, that will be identified with the scalar-part of a quaternionic function, three Riesz components can be added to give a quaterion-valued signal which now will be rotated. Unfortunately, it is not really clear what the signal is. Therefore we restrict ourselfs to so-called isoclinic rotation and consider them.
The situation for a signal defined by a function of two variables is different. We have to identify the signal with the direction e_3 and than add two Riesz components to obtain a monogenic signal that can be rotated in $\mathbb{R}^3$ and the rotation is described by pure quaterions. If the rotation is restricted to a rotation in $\mathbb{R}^2$ perpendicular to e_3 we get a signal that is composed of the signal f and rotated Riesz component = generalized Riesz transform in the sense of Unser.
We will present some properties of these fractional monogenic signals.

Krein-Langer factorization in the slice hyperholomorphic setting

Irene Sabadini

A classical result of Krein and Langer states that S is a generalized Schur function with κ negative squares if and only if it is the restriction to Ω of a function of the form

$$B_0(z)^{-1}S_0(z),$$

where S_0 is a $\mathbb{C}^{r\times s}$-valued Schur function and B_0 is $\mathbb{C}^{r\times r}$-valued Blaschke product of degree κ.
In this talk we discuss the theorem of Krein and Langer in this setting of quaternionic valued slice hyperholomorphic functions. We treat both the unit ball and half-space cases. To this end, we need the notion of negative squares and of reproducing kernel Pontryagin spaces in the quaternionic setting as well as the notion of generalized Schur function and of Blaschke product.
The talk is based on joint work with D. Alpay and F. Colombo.

Free noncommutative function theory and free real algebraic geometry (FN)
Organizers: J. William Helton & Igor Klep

Speaker	title	time	place
		Tuesday	M 639
Victor Vinnikov	Noncommutative (nc) integrability	13.30-13.55	
Jakob Cimprič	A real Nullstellensatz for free modules	13.55-14.20	
Sabine Burgdorf	Tracial optimization and semidefinite relaxation	14.20-14.45	
Matej Brešar	Functional identities	14.45-15.10	
Joseph Ball	The noncommutative Fock space as a reproducing kernel Hilbert space	15.50-16.15	
Salma Kuhlmann	Test sets for positivity of invariant polynomials and applications	16.15-16.40	
	to sum of squares representations and moment poblems		
Špela Špenko	Free function theory through matrix invariants	16.40-17.05	
Dmitry Kaliuzhnyi-Verbobetskyi	Implicit/inverse function theorems for free noncommutative functions	17.05-17.30	
		Thursday	M 639
Douglas Farenick	The noncommutative Choquet boundary of some finite-dimensional operator systems	13.30-13.55	
Teresa Piovesan	Conic approach to quantum graph parameters using the completely positive semidefinite cone	13.55-14.20	
Craig Kleski	Hyperrigidity of operator systems	14.20-14.45	
J. William Helton and Igor Klep	Matrix variables, free real algebraic geometry and convexity	14.45-15.10	

Free noncommutative function theory and free real algebraic geometry

The noncommutative Fock space as a reproducing kernel Hilbert space

Joseph Ball

The noncommutative Fock space, which we denote here as $H^2_{\mathcal{Y}}(\mathcal{F}_d)$, consisting of formal power series $f(z) = \sum_{\alpha \in \mathcal{F}_d} f_\alpha z^\alpha$ with vector coefficients $f_\alpha \in \mathcal{Y}$ ($\mathcal{Y}$ equal to a coefficient Hilbert space) in freely noncommuting indeterminates (so $z^\alpha = z_{i_N} \cdots z_{i_1}$ if α is the word $i_N \cdots i_1$ in the free semigroup $\mathcal{F}_d$ (multiplication via concatenation) generated by the alphabet $\{1, \dots, d\}$ in d symbols) has figured prominently in the literature over the years, especially in Popescu's extension of the Sz.-Nagy dilation theorem and the Sz.-Nagy-Foias model theory to the case of a possibly noncommuting d-tuple $T = (T_1, \dots, T_d)$ of operators for which the row $\begin{bmatrix} T_1 & \cdots & T_d \end{bmatrix}$ is a contraction. This space can be viewed as the free noncommutative analogue of the Drury-Arveson space, the reproducing kernel Hilbert space on the unit ball in $\mathbb{C}^d$ with reproducing kernel $k_d(z, w) = \frac{1}{1 - z_1 \overline{w}_1 - \cdots - z_d \overline{w}_d}$. This kernel has a certain universal property with respect to the family of kernels (called Pick kernels) for which positivity of a single Pick matrix associated with the interpolation data is always both necessary and sufficient for solvability of the associated multiplier interpolation problem (see [1]). In work with Victor Vinnikov [2], the speaker introduced the notion of noncommutative formal positive kernel and associated a reproducing kernel Hilbert space consisting of formal power series with vector coefficients with such a kernel. By substituting square matrices for the indeterminates, one can view the elements of such spaces as well as the associated formal positive kernel as noncommutative functions on a certain noncommutative domain. This talk will identify the noncommutative Fock space as the reproducing kernel Hilbert space associated with a formal noncommutative analogue of the Drury-Arveson kernel, and discuss interpolation problems for multipliers on this space and the closely related notion of noncommutative Pick kernel.

[1] J. Agler and J.E. McCarthy: *Pick Interpolation and Hilbert Function Spaces*, Graduate Studies in Mathematics Volume 44, American Math. Soc., 2002.

[2] J.A. Ball and V. Vinnikov: Formal reproducing kernel Hilbert spaces: The commutative and noncommutative settings, in: *Reproducing Kernel Spaces and Applications*, pp. 1–60, Birkhäuser, 2005.

Functional identities

Matej Brešar

A functional identity on an algebra A is an identical relation that involves arbitrary elements from A along with functions which are considered as unknowns. The fundamental example of such an identity is

$$\sum_{k=1}^{m} F_k(\overline{x}_m^k) x_k = \sum_{k=1}^{m} x_k G_k(\overline{x}_m^k) \quad \text{for all } x_1 \dots, x_m \in A,$$

where $\overline{x}_m^k = (x_1, \dots, \hat{x}_k, \dots, x_m)$ and F_k, G_k are arbitrary functions from A^{m-1} to A. The description of these functions, which is the usual goal when facing such an identity, has turned to be applicable to various mathematical problems.

The theory of functional identities has been so far mainly developed in the context of infinite dimensional algebras; see [1]. In the talk we will focus on the recent works [2,3,4] with Claudio Procesi and Špela Špenko that consider the case where A is the matrix algebra $M_n(F)$.

[1] M. Brešar, M. A. Chebotar, W. S. Martindale 3rd, *Functional identities.* Birkhäuser Verlag, 2007.

[2] M. Brešar, C. Procesi, Š. Špenko, Quasi-identities and the Cayley-Hamilton polynomial, arXiv:1212.4597v3.

[3] M. Brešar, Š. Špenko, Functional identities in one variable, *J. Algebra*, 401 (2014), 234–244.

[4] M. Brešar, Š. Špenko, Functional identities on matrix algebras, arXiv:1403.5454.

Tracial optimization and semidefinite relaxation

Sabine Burgdorf

Polynomial optimization where one optimizes the value of a given polynomial over a semi-algebraic set in $\mathbb{R}^n$ is an active field of research with plenty of applications. It is naturally extended to the free non-commutative context where one evaluates polynomials in operators instead of real or complex numbers. One interesting class of problems is given by tracial optimization where one optimizes the trace of a non-commutative polynomial over bounded operators or matrices. This is used e.g. to get estimates for quantum graph parameters.

A classical method to solve polynomial optimization problems is to construct a hierarchy of semidefinite programs (based on real algebraic representation theorems) which relax the problem and additionally converge to the optimal value. Whereas this works well in the classical setup, the best relaxation procedure for tracial optimization is still unknown, e.g. following the Lasserre approach one cannot verify the convergence of the corresponding hierarchy. In this talk we will discuss several possibilities to relax the tracial optimization problem and will evaluate which one might be most promising for application.

A real Nullstellensatz for free modules

Jakob Cimprič

Let $\mathcal{A}$ be the algebra of all $n \times n$ matrices with entries from $\mathbb{R}[x_1, \ldots, x_d]$ and let $G_1, \ldots, G_m, F \in \mathcal{A}$. We will show that $F(a)v = 0$ for every $a \in \mathbb{R}^d$ and $v \in \mathbb{R}^n$ such that $G_i(a)v = 0$ for all i if and only if F belongs to the smallest real left ideal of $\mathcal{A}$ which contains $G_1, \ldots, G_m$. Here a left ideal J of $\mathcal{A}$ is real if for every $H_1, \ldots, H_k \in \mathcal{A}$ such that $H_1^T H_1 + \ldots + H_k^T H_k \in J + J^T$ we have that $H_1, \ldots, H_k \in J$. We call this result the one-sided Real Nullstellensatz for matrix polynomials. We first prove by induction on n that it holds when $G_1, \ldots, G_m, F$ have zeros everywhere except in the first row. This auxiliary result can be formulated as a Real Nullstellensatz for the free module $\mathbb{R}[x_1, \ldots, x_d]^n$.

[1] J. Cimprič, A Real Nullstellensatz for free modules, J. Algebra 396 (2013), 143-150.

The noncommutative Choquet boundary of some finite-dimensional operator systems

Douglas Farenick

One of the most basic function systems is the one spanned by the constant function 1, the coordinate function z, and the complex conjugate $\overline{z}$, all considered as continuous complex-valued functions defined on the unit circle. The Choquet boundary of this function system is the unit circle itself. In this talk I will describe Arveson's theory of the noncommutative Choquet boundary for noncommutative function systems (aka, operator systems), with particular attention paid to low-dimensional operator systems spanned by certain Hilbert space operators.

Matrix variables, free real algebraic geometry and convexity.

J. William Helton and *Igor Klep*

The talk concerns inequalities for functions having matrix variables. The functions are typically free (noncommutative) polynomials or rational functions. A focus of much attention are the

inequalities corresponding to convexity. Such mathematics is central to linear systems problems which are specified entirely by a signal flow diagram and L^2 performance specs on signals.
At this point we have:

(A) Versions of the classical real algebraic geometry description of when one polynomial p is nonnegative on the domain where another polynomial q is nonnegative.

(B) Classification of convex noncommutative polynomials, rational functions and varieties. Now we know that such matrix convexity typically forces the presence of some linear matrix inequality (LMI).

(C) Some theory of matrix convex hulls.

(D) Some theory of changes of variables to achieve free noncommutative convexity.

(E) Other.

The talk will select a recent topic from this. The work originates in trying to develop some theory for studying the matrix inequalities which are ubiquitous in linear engineering systems and control. Most of the work is done jointly by J. William Helton, Igor Klep and Scott A McCullough with collaborators.
The talks of Bill Helton and Igor Klep in this session are both on this topic and will be coordinated.

Hyperrigidity of operator systems

Craig Kleski

In 2011, Arveson conjectured that if every irreducible representation of a C*-algebra A generated by a separable operator system S has the unique extension property relative to S, then every representation must also have the unique extension property; this is not known to be true in general even for commutative C*-algebras. Nevertheless, when A is Type I, we discuss a theorem that reveals the structure of such A and we also discuss progress toward a solution when A is nuclear.

Test sets for positivity of invariant polynomials and applications to sum of squares representations and moment poblems

Salma Kuhlmann

In this paper, we began a systematic study of positivity and moment problems in an equivariant setting. Given a reductive group acting on an affine variety, we consider the dual action on the coordinate ring. In this setting, given an invariant closed semi-algebraic set K, we study the problem of representation of invariant positive polynomials on K by invariant sums of squares. In case the group is semi-algebraically compact, invariant positive polynomials are associated to positive polynomials on the corresponding orbit variety. This orbit variety is a semi algebraic set by a theorem of Procesi and Schwarz (see e.g. corollary 3.9 in our paper). Based on this work, the aim is to compute explicit semi-algebraic descriptions of orbit varieties for certain important groups (e.g. linear algebraic groups) and use this to deduce a general testing set for positivity. Of particular interest is the symmetric group, and the hope would be to obtain in this way a new proof of Timofte's half degree principle, as well as results on representation of positive invariant forms by sums of squares. This approach is interesting for applications to optimization, in view of symmetry reductions.

[1] J. Cimpric, S. Kuhlmann and C. Scheiderer, Sums of Squares and Moment Problems in Equivariant Situations. Transactions Amer. Math. Soc., 361, 735–765 (2009).

Conic approach to quantum graph parameters using the completely positive semidefinite cone

Teresa Piovesan

The completely positive semidefinite matrix cone $\mathcal{CS}_+^n$ is a new matrix cone, consisting of all symmetric $n \times n$ matrices that admit a Gram representation by positive semidefinite matrices (of any size). This new cone is used to model quantum analogues of the classical independence and chromatic graph parameters, which are roughly obtained by allowing variables to be positive semidefinite matrices instead of binary scalar variables in the programs defining the classical graph parameters.
We investigate relationships between the cone $\mathcal{CS}_+^n$ and the completely positive and doubly nonnegative cones, and between the dual cone of $\mathcal{CS}_+^n$ and trace positive non-commutative polynomials. By using the truncated tracial quadratic module as sufficient condition for trace positivity, one can define hierarchies of cones aiming to approximate the dual cone of $\mathcal{CS}_+^n$, which can then be used to construct hierarchies of semidefinite bounds approximating the quantum graph parameters. Their convergence properties are related to Connes' embedding conjecture in operator theory.
The talk is based on joint work with Monique Laurent.

Free function theory through matrix invariants

Špela Špenko

We use the theory of matrix invariants to study free maps, which enables us to consider free maps with involution. We present a characterization of polynomial free maps via properties of their finite-dimensional slices, and establish power series expansions for analytic free maps about scalar and non-scalar points; the latter are given by series of generalized polynomials.
This is a joint work with Igor Klep.

Implicit/inverse function theorems for free noncommutative functions

Dmitry Kaliuzhnyi-Verbovetskyi

Free noncommutative functions are mapping of matrices to matrices which preserve matrix size and respect direct sums and similarities. They have strong analyticity property: a mild assumption of local boundedness already implies analyticity. We prove implicit/inverse function theorems for free noncommutative functions and show that they are stronger than in the classical case.
The talk is based on joint work with Gulnara Abduvalieva.

Noncommutative (nc) integrability

Victor Vinnikov

Applying the nc difference-differential operator k times to a nc function yields a new object called a nc function of order k. I will discuss necessary (and perhaps also sufficient) conditions for a higher order nc function to be integrable, i.e., to be the result of repeatedly applying the nc difference-differential operator to a nc function.
This is a joint work with Dmitry Kaliuzhnyi-Verbovetskyi.

[1] D.S. Kaliuzhnyi-Verbovetzkyi and V. Vinnikov, *Foundations of Noncommutative Function Theory*, to be published in *Math. Surveys and Monographs*, Amer. Math. Society, preprint `arXiv:1212.6345`.

Partial differential operators and potential method (PDO)
Organizers: Roland Duduchava & Vladimir Rabinovich

Speaker	title	time	place
		Tuesday	P647
Roland Duduchava	Mixed boundary value problem for the Laplace-Beltrami equation on a hypersurface	13.30-13.55	
Jaime Cruz-Sampedro	Decay of eigenfunctions of Schrödinger operators	13.55-14.20	
Anatoli Merzon	Scattering of plane waves by two-dimensional wedges	14.20-14.45	
Sergey Mikhailov	Scaling of localized boundary-domain integral operators and their norm estimates	14.45-15.15	
Massimo Lanza de Cristoforis	Composition operators and analyticity of nonlinear integral operators	15.50-16.15	
Paolo Musolino	A quasi-linear heat transmission problem in a periodic two-phase dilute composite	16.15-16.40	
Joachim Toft	Harmonic oscillator and its inverse	16.40-17.05	
Victor Kovtunenko	Optimization theory in inverse problems of object identification: variational methods by inverse scattering	17.05-17.30	
		Friday	M 607
Vladimir Rabinovich	Transmission problems for conical and quasi-conical at infinity domains	13.30-13.55	
Bert-Wolfgang Schulze	Operators on manifolds with singularities	13.55-14.20	
Anton Selitskii	Maximal regularity of the 2-d mixed problem for parabolic differential-difference equations	14.20-14.45	
Mirela Kohr	Boundary value problems of Robin-transmission type for the nonlinear Darcy-Forchheimer-Brinkman and Navier-Stokes systems. Applications	14.55-15.10	

Partial differential operators and potential method

Mixed boundary value problem for the Laplace-Beltrami equation on a hypersurface

Roland Duduchava

We investigate the mixed Dirichlet-Neumann boundary value problems for the anisotropic Laplace-Beltrami equation $\mathrm{div}_{\mathcal{C}}\mathcal{A}\nabla_{\mathcal{C}}\varphi = f$ on a smooth hypersurface $\mathcal{C}$ with the boundary $\Gamma := \partial\mathcal{C}$ in $\mathbb{R}^n$. $\mathrm{div}_{\mathcal{C}}$ and $\nabla_{\mathcal{C}}$ are the surface divergence and the surface gradient written in Günter's derivatives (see [1,2]). $\mathcal{A}(x)$ is an $n \times n$ bounded measurable positive definite matrix function. The boundary is decomposed in two non-intersecting connected parts $\Gamma = \Gamma_D \cup \Gamma_N$ and on the part Γ_D the Dirichlet boundary conditions while on Γ_N the Neumann boundary condition are prescribed. The unique solvability of the mixed BVP is proved in the classical weak setting $\varphi \in \mathbb{H}^1(\mathcal{C})$ is proved, based upon the Green formulae and Lax-Milgram Lemma.
We also prove the invertibility of the perturbed operator in the Bessel potential spaces $\mathrm{div}_{\mathcal{S}}\mathcal{A}\nabla_{\mathcal{S}} + \mathcal{H}\,I\ :\ \mathbb{H}^s_p(\mathcal{S}) \to \mathbb{H}^{s-2}_p(\mathcal{S})$ for a smooth hypersurface $\mathcal{S}$ without boundary for arbitrary $1 < p < \infty$ and $-\infty < s < \infty$, provided $\mathcal{H}$ satisfies one of the following three conditions:
i. has non-negative real part $\inf_{t\in\mathcal{S}} \mathrm{Re}\,\mathcal{H}(t) \geqslant 0$ and $\mathrm{mes\,supp\,Re}\,\mathcal{H} \neq 0$;
ii. has constant complex part $\mathrm{Im}\,\mathcal{H}(t) = \mathrm{const} \neq 0$;
iii. $\mathrm{Re}\,\mathcal{H} = 0$, $\mathrm{mes\,supp\,Im}\,\mathcal{H} \neq 0$ and the complex part $\mathrm{Im}\,\mathcal{H}$ does not change the sign: either $\mathrm{Im}\,\mathcal{H}(t) \geqslant 0$ for all $t \in \mathcal{S}$ or $\mathrm{Im}\,\mathcal{H}(t) \leqslant 0$ for all $t \in \mathcal{S}$.
The existence of the fundamental solution to $\mathrm{div}_{\mathcal{S}}\mathcal{A}\nabla_{\mathcal{S}}$ is proved, which is interpreted as the invertibility of this operator in the setting $\mathbb{H}^s_{p,\#}(\mathcal{S}) \to \mathbb{H}^{s-2}_{p,\#}(\mathcal{S})$, where $\mathbb{H}^s_{p,\#}(\mathcal{S})$ is a subspace of the Bessel potential space and consists of functions with mean value zero (the spaces with detached constants).
The fundamental solution is used to define the single and double layer potentials, which allow to prove the representation formulae for a solution to the mixed boundary value problem and reduce the latter to a solution of equivalent boundary integral (pseudodifferential) equation (BIE) on a boundary curve. By investigating the BIE the solvability criteria of the mixed BVP in the non-classical weak setting $\varphi \in \mathbb{H}^s_p(\mathcal{C})$ for $s > 1/p$, $1 < p < \infty$, is proved, based upon solvability criteria for the BIE in the spaces $\mathbb{H}^s_p(\Gamma)$.
The BVP's for Laplace-Beltrami equation on hypersurfaces was investigated earlier either with Dirichlet or with Neumann boundary conditions (see [1] for details and survey).
The talk is based on joint papers with Medea Tsaava & Tamta Tsutsunava.
The research is supported by the Shota Rustaveli Science foundation in the framework of the research grant DI/10/5-101/12 (Contract GNSF 13/14).

[1] R. Duduchava, Partial differential equations on hypersurfaces, *M*emoirs on Differential Equations and Mathematical Physics **48**, 2009, 19-74.

[2] R. Duduchava, D.Mitrea, M.Mitrea, Differential operators and boundary value problems on surfaces. *M*athematische Nachrichten **279**, No. 9-10 (2006), 996-1023.

Decay of eigenfunctions of Schrödinger operators

Jaime Cruz-Sampedro

A Schrödinger operator is a differential operator of the form $H = -\Delta + V$, where Δ is the n-dimensional Laplace operator acting in $L^2(\mathbb{R}^n)$ and V a real-valued function called the potential. The eigenfunctions of H are the functions $u \in L^2(\mathbb{R}^n)$ that satisfy $Hu = Eu$. In this talk we present a brief survey of classical results and conjectures and then describe some results of the author about the fastest possible rate of decay of $u(x)$, as $|x| \to \infty$.

Scattering of plane waves by two-dimensional wedges

Anatoli Merzon

We consider *nonstationary* scattering of plane waves by a wedge $W := \{y = (y_1, y_2)\colon\ y_1 = \rho\cos\theta, y_2 = \rho\sin\theta, \rho > 0, 0 < \theta < \phi < \pi\}$. Let $u_{in}(y,t) := e^{-i\omega_0(t-n_0\cdot y)}f(t-n\cdot y)$, $t\in\mathbb{R}, y\in Q := \mathbb{R}^2\setminus W$; $\omega_0 > 0$, be the incident plane wave. We assume $n_0 = (\cos\alpha, \sin\alpha)$, where (for simplicity) $\max(0,\phi-\pi/2) < \alpha < \min(\pi/2,\phi)$. In this case $u_{in}(y,0) = 0$, $y \in \partial Q$. The profile $f \in \mathbb{C}^\infty(\mathbb{R})$, $f(s) = 0, s < 0$, and $f(s) = 1, s > \mu$, for some $\mu > 0$. Let Q_1, Q_2 be the sides of Q. The scattering is described by means of the following mixed wave problems in Q (depending on the properties of the wedge)

$$\left\{\begin{array}{ll} \Box u(y,t) = 0, & y\in Q \\ P_l u(y,t) = 0, & y \in Q_l \end{array}\right| \ t > 0, \quad \left\{\begin{array}{l} u(y,0) = u_{in}(y,0), \\ \dot u(y,0) = \dot u(y,0), \end{array}\right| \ y \in Q,\ l = 1,2 \tag{7}$$

where $P_l = 1$ or $P_l = \partial_{n_l}$ for the exterior normals n_l to Q_l (DD, NN or DN-problems). Denote by $\dot Q := \overline{Q}\setminus\{0\}$, $\{y\} := |y|/(1+|y|)$, $y\in\mathbb{R}^2$.
Definition For $\varepsilon, N \geq 0$, $\mathcal{E}_{\varepsilon,N}$ is the space of functions $u(t,y) \in C(\overline{Q}\times\overline{\mathbb{R}^+})$ with the finite norm $\|u\|_{\varepsilon,N} := \sup\limits_{t\geq 0}\Big[\sup\limits_{y\in\overline{Q}}|u(y,t)| + \sup\limits_{y\in\dot Q}(1+t)^{-N}\{y\}^\varepsilon|\nabla_y u(y,t)|\Big] < \infty$, $N \geq 0$. Let $\Phi := 2\pi - \phi$, $q := \pi/(2\Phi)$.
Theorem([3],[4],[6]) **i**) There exists a unique solution to the DD, NN, DN -problems (1) $u(y,t) \in \mathcal{E}_{1-2q,1-2q}$ in the case of DD and NN boundary conditions, and $u(y,t)\in\mathcal{E}_{1-q,1-q}$ in the case of DN boundary condition. The solution is expressed as the sum of the incident wave u_{in}, an optical wave u_r reflected by the sides of the wedge and a wave u_d diffracted by the edge of the wedge. The wave u_d admits a simple Sommerfeld-Malyuzhinetz type representation

$$u_d(y,t) = i\frac{e^{-i\omega_0 t}}{4\Phi}\int Z(\beta + i\theta)d\beta, \quad \theta \neq 2\phi-\alpha, \quad 2\pi-\alpha$$

with the Sommerfeld-Malyuzhinetz type kernel Z depending on the boundary conditions and the wave profile f.
ii) The Limiting Amplitude Principle holds: $u(y,t) - e^{-i\omega_0 t}A(y) \to 0$, $t\to\infty$, uniformly for $|y| \leq \rho_0$, where the limiting amplitude A is the Sommerfeld-Maljuzhinetz type solution to the classical stationary diffraction problem of a plane wave by the wedge [1].
We extend these results for the generalized incident wave using appropriate functional class of solutions [5]. In particular, we justify the classical Sobolev's and Keller- Balnk's solutions obtained for the pulse, i.e. for the incident wave which is the Heaviside step function. The theory uses the Method of Complex Characteristics [2].
The talk is based on joint work with Prof. Dr. Alexander Komech. The research was supported by CONACYT , CIC (UMSNH) and PROMEP (via el Proyecto RED) (México).

[1] V.M. Babich , M.A. Lyalinov , V.E. Gricurov: *Sommerfeld-Maljuzhinetz method in diffraction problems.* S.Petersburg University, 2003.

[2] A.I. Komech, A.E.Merzon, P.N. Zhevandrov: A method of complex characteristics for elliptic problems in angles and its applications, *American Mathematical Society Translation*, **206**(2) 2002, 125-159.

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[4] A.I.Komech, A.E.Merzon: Limiting Amplitude principle in the scattering by wedges, *Mathematical Methods in the Applied Sciences*, **29**(19) (2006), 1147-1185.

[5] A.I. Komech, A.E. Merzon, J.E. De la Paz Méndez. On justification of Sobolev formula for diffraction by a wedge (in consideration)

[6] A.E. Merzon, J.E. De la Paz Méndez. DN-scattering of a plane wave by wedges. *Mathematical Methods un the Applied Sciences* **34** (15) (2011), 1843-1872

Scaling of localized boundary-domain integral operators and their norm estimates

Sergey Mikhailov

The localized boundary-domain integral equation (LBDIE) systems, based on the parametrices localised by multiplication with a cut-off function, and associated with the Dirichlet and Neumann boundary value problems (BVP) for a scalar "Laplace" PDE with *variable* coefficient were introduced in [1]. Mapping and jump properties of the surface and volume integral potentials, based on a parametrix localised by multiplication with a radial localising function, and constituting the LBDIE systems were studied in the Sobolev (Bessel potential) spaces in [2] and the LBDIEs equivalence to the original variable-coefficient BVPs and the invertibility of the localised boundary-domain integral operators (LBDIOs) in the corresponding Sobolev spaces were also proved there.

In this talk, we give an overview of the LBDIEs and discuss how the LBDIO norms depend on a scaling parameter of the localising function. Even when the considered LBDIOs are invertible for any size of the characteristic domain of localization (i.e., for any positive value of the scaling parameter), it appears that the norms of the operators and of their inverse can grow as the scaling parameter decreases. This effect may be responsible for the deterioration of convergence of the mesh-based and mesh-less numerical methods of LBDIE solution, cf. [3], [4], observed for fine meshes (large number of collocation points).

Composition operators and analyticity of nonlinear integral operators

Massimo Lanza de Cristoforis

We prove an analyticity theorem for a class of nonlinear integral operators which involve a Nemytskij type composition operator. Such operators appear in the applications as pull-backs of layer potential operators.

To do so, we resort to a general result on composition operators acting in Banach algebras and we split our operators into nonlinear integral operators acting in Roumieu classes and composition operators.

The talk is based on joint work with Paolo Musolino.

[1] M. Lanza de Cristoforis.: Differentiability Properties of a Composition Operator, Rendiconti del Circolo Matematico di Palermo, Serie II, Suppl. **56**, pp. 157–165, 1998.

[2] M. Lanza de Cristoforis.: Differentiability Properties of an Abstract Autonomous Composition Operator, J. London Math. Soc. (2) **61**, no. 3, pp. 923–936, 2000.

[3] L. Preciso.: Regularity of the composition and of the inversion operator and perturbation analysis of the conformal sewing problem in Roumieu type spaces, Nat. Acad. Sci. Belarus, Proc. Inst. Math., **5**, (2000), pp. 99–104.

[4] M. Lanza de Cristoforis and P. Musolino.: A real analyticity result for a nonlinear integral operator, Journal of Integral Equations and Applications, **25**, (2013), pp. 21–46.

A quasi-linear heat transmission problem in a periodic two-phase dilute composite

Paolo Musolino

We consider a temperature transmission problem for a composite material which fills the n-dimensional Euclidean space. The composite has a periodic structure and consists of two materials. In each periodicity cell one material occupies a cavity of size ϵ, and the second material fills the remaining part of the cell. We assume that the thermal conductivities of the materials depend nonlinearly upon the temperature. We show that for ϵ small enough the problem has a solution, *i.e.*, a pair of functions which determine the temperature distribution in the two materials. Then we analyze the behavior of such a solution as ϵ approaches 0 by an approach which is alternative to those of asymptotic analysis. In particular we prove that if $n \geq 3$, the temperature can be

expanded into a convergent series expansion of powers of ϵ and that if $n = 2$ the temperature can be expanded into a convergent double series expansion of powers of ϵ and $\epsilon \log \epsilon$.
The talk is based on joint work with M. Lanza de Cristoforis.

Harmonic oscillator and its inverse

Joachim Toft

We study the inverse of the harmonic oscillator. In particular, we consider the Weyl symbol to this inverse. We establish precise expressions and convenient estimates. We also explain how such estimates can be used to ensure that this operator image analytical and super analytical functions to functions of the same class.
The talk is based on a paper which are obtained in collaboration with Marco Cappiello and Luigi Rodino at Turin university, Turin, Italy.

[1] M. Cappiello, R. Rodino, J. Toft *On the inverse to the harmonic oscillator*, preprint (arXiv:1306.6866).

Optimization theory in inverse problems of object identification: variational methods by inverse scattering

Victor Kovtunenko

The topology optimization theory is developing to inverse problems of identification from given boundary measurements of an unknown geometric object, which is illuminated by plane waves, under a-priori unknown boundary conditions. The identification problem has numerous applications to PDEs in the physical, geophysical, and biomedical sciences in the context of nondestructive testing with acoustic, elastic, and electromagnetic waves.

For the reference Helmholtz equation, the inverse scattering operator is approximated by combining variational techniques and methods of singular perturbations. Using optimality conditions it provides high precision identification of the test object of arbitrary shape in arbitrary spatial dimensions. The theoretical result is strengthened by numerical analysis based on a Petrov–Galerkin enrichment approach within generalized FEM.

The research is supported by the Austrian Science Fund (FWF) in the framework of the research projects P26147-N26.

[1] V.A. Kovtunenko: State-constrained optimization for identification of small inclusions, *Proc. Appl. Math. Mech*, 11 (2011), 721–722.

[2] V. A. Kovtunenko and K. Kunisch: High precision identification of an object: optimality conditions based concept of imaging, *SIAM J. Control Optim.*, to appear.

Transmission problems for conical and quasi-conical at infinity domains

Vladimir Rabinovich

Let $\mathcal{D}$ be a smooth unbounded domain in $\mathbb{R}^n, n \geq 2$ conical at infinity, $\mathcal{D}_1 = \mathcal{D}, \mathcal{D}_2 = \mathbb{R}^n \backslash \bar{\mathcal{D}}$. We consider general transmission problems defined by a differential equation

$$\sum_{|\alpha| \leq 2m} a_\alpha(x) D^\alpha u(x) = f(x), x \in \mathbb{R}^n \backslash \partial\mathcal{D}, \tag{8}$$

and transmission conditions on the boundary $\partial\mathcal{D}$

$$\left[\sum_{|\alpha| \leq m_j} b_{j\alpha}(x) D_x^\alpha u \right]_{\partial\mathcal{A}} (x') = \varphi_j(x'), x' \in \partial D, j = 1, ..., 2m, \tag{9}$$

where the coefficients $a_\alpha, b_{j\alpha}$ are discontinuous on ∂D functions, such that that $a_\alpha|_{\mathcal{D}_k}, b_{j\alpha}|_{\mathcal{D}_k} \in C_b^\infty(\overline{\mathcal{D}_k}), k = 1,2$ the space of infinitely differentiable functions in $\overline{\mathcal{D}_k}$ bounded with all derivatives, $[v]_{\partial\mathcal{D}}$ is a jump of the function v on $\partial\mathcal{D}$. We give a criterion for the operator

$$\mathbf{A} : H^s(\mathcal{D}_1) \oplus H^s(\mathcal{D}_2) \to H^{s-2m}(\mathcal{D}_1) \oplus H^{s-2m}(\mathcal{D}_2) \oplus_{j=1}^{2m} H^{s-m_j-1/2}(\partial\mathcal{D}),$$
$$s > \max_{1\leq j\leq 2m} \{m_j\} + 1/2.$$

of the transmission problem (8),(9) to be Fredholm. We also extend this result to more general quasi-conical at infinity domains. This criterion is applied to the anisotropic acoustic problem

$$\nabla \cdot a(x)\nabla u(x) + b(x)u(x) = f(x), x \in \mathbb{R}^n \backslash \partial\mathcal{D}, \quad (10)$$
$$[u]_{\partial\mathcal{D}} = 0, [Tu]_{\partial\mathcal{D}} = 0,$$

where $a(x) = (a_{p,q}(x))_{p,q=1}^n$ is a uniformly positive definite matrix on $\mathbb{R}^n$ with discontinuous on $\partial\mathcal{D}$ entries $a_{p,q}$ such that $a_{p,q} \mid_{\mathcal{D}_k} \in C_b^\infty(\overline{\mathcal{D}_k}), k = 1,2$, b is a discontinuous on $\partial\mathcal{D}$ function such that $b \mid_{\mathcal{D}_k} \in C_b^\infty(\overline{\mathcal{D}_k}), k = 1,2$, Tu is a conormal derivative. We prove that if the acoustic media are absorbed at infinity the problem (10) has an unique solution $u \in H^s(\mathcal{D}_1) \oplus H^s(\mathcal{D}_2)$ for every $f \in H^{s-2}(\mathcal{D}_1) \oplus H^{s-2}(\mathcal{D}_2), s \geq 2$.

Operators on manifolds with singularities

Bert-Wolfgang Schulze

We outline a number of new achievements of the pseudo-differential analysis on manifolds with edges and higher corners. Motivated by the general task of expressing parametrices of elliptic corner-degenerate differential operators within an algebra of pseudo-differential operators and characterising elliptic regularitiy in weighted spaces and subspaces with asymptotics we develop tools in terms of principal symbolic hierarchies which are contributed by the stratification of the underlying singular configuration. A basic model is the case of a manifold with edge. From the special case of a manifold with boundary we can read off typical elements of the edge calculus, although there are also essential differences to the general case. In our talk we focus on the composition behaviour of edge symbols which is crucial for the construction of parametrices, together with Green, Mellin, trace, and potential symbols.

Maximal regularity of the 2-d mixed problem for parabolic differential-difference equations

Anton Selitskii

Let Q be a bounded Lipschitz domain in $\mathbb{R}^n$, $n \in \mathbb{N}$. We consider the problem

$$\frac{du}{dt} + A_R u = f, \qquad u(0) = \varphi, \quad (1)$$

where A_R is a differential-difference operator (see [1]) with Neumann boundary condition, $f \in L_p(0,T; L_2(Q))$, $\varphi \in L_2(Q)$, $1 < p < \infty$, and $0 < T < \infty$.
Let us remind that a variational solution u to problem (1) is said to be a strong solution if $u \in D(A_R)$ for almost all $t \in (0,T)$ and $u_t \in L_p(0,T; L_2(Q))$. We prove that if the operator A_R is strongly elliptic, then a strong solution to problem (1) exists if and only if $\varphi \in B_{2,p}^{2-2/p}(Q)$, where $B_{q,\theta}^s$ denotes the Besov space. In order to prove this statement, we use the following equality prooved for $p = 2$ in [2]: $[L_2(Q), D(A_R)]_{1/2} = H^1(Q)$. This equality is equivalent to the Kato conjecture on the square root of the operator (see [3]). We note that smoothness of solutions of the equation $A_R v = F$ can be violated inside of the domain Q. Moreover, in case of incommensurable shifts, it can be violated on an almost everywhere dense set in Q. Thus, as in the case of strong elliptic differential operators with bounded measurable coefficients, a domain $D(A_R)$ can not be represented in an explicit form.

The work is partially supported by RFBR grants NN 13-01-00923 and 14-01-00265 and President grant for government support of the leading scientific schools of the Russian Federation No. 4479.2014.1.

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[2] A.L. Selitskii: Space of initial data of the second boundary-value problem for a parabolic differential-difference equation in Lipschitz domains, *Math. Notes*, 94 (2013), 135–138.

[3] T. Kato: Fractional powers of dissipative operators, *J. Math. Soc. Japan*, 13 (1961), 246–274.

Boundary value problems of Robin-transmission type for the nonlinear Darcy-Forchheimer-Brinkman and Navier-Stokes systems. Applications

Mirela Kohr

The purpose of this talk is to study a boundary value problem of Robin-transmission type for the nonlinear Darcy-Forchheimer-Brinkman and Navier-Stokes systems in two adjacent bounded Lipschitz domains in $\mathbb{R}^n$ ($n \in \{2, 3\}$), with linear transmission conditions on the internal Lipschitz interface and a linear Robin condition on the remaining part of the Lipschitz boundary. The case of nonlinear Robin and transmission conditions is also analyzed. For each of these problems we use layer potential theoretic methods combined with fixed point theorems, in order to show existence results in Sobolev spaces, when the given data are suitably small in L^2-based Sobolev spaces or in some Besov spaces. For the first mentioned problem, which corresponds to linear Robin and transmission conditions, we also present a uniqueness result. Note that the Brinkman-Forchheimer-extended Darcy equation is a nonlinear equation that describes saturated porous media fluid flows.
This talk is based on joint work with Massimo Lanza de Cristoforis (Padova) and Wolfgang L. Wendland (Stuttgart).

[1] M. Dindoš and M. Mitrea, The stationary Navier-Stokes system in nonsmooth manifolds: the Poisson problem in Lipschitz and C^1 domains, *Arch. Ration. Mech. Anal.*, **174** (2004), 1–47.

[2] M. Kohr, M. Lanza de Cristoforis and W.L. Wendland, Nonlinear Neumann-transmission problems for Stokes and Brinkman equations on Euclidean Lipschitz domains, *Potential Anal.*, **38** (2013), 1123–1171.

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[5] M. Kohr, I. Pop, Viscous Incompressible Flow for Low Reynolds Numbers, WIT Press, Southampton (UK) (2004).

Operators, matrices, and indefinite inner products. Special session in honor of Leiba Rodman. (OMII)
Organizers: Christian Mehl, Michał Wojtylak

Speaker	title	time	place
		Thursday	M607
Joseph Ball	The Takagi problem on the disk and the bidisk	13.30-13.55	
Harm Bart	Zero sums of projections and unusual Cantor sets	13.55-14.20	
André Ran	Stability of invariant maximal semidefinite subspaces	14.20-14.45	
Seppo Hassi	Completion, extension, factorization, and lifting of operators with a negative index	14.45-15.10	
Dawid Janse van Rensburg	Rank one perturbations of H-positive real matrices	15.50-16.15	
Leslie Leben	Algebraic eigenspaces of H-nonnegative matrices after a rank one perturbation	16.15-16.40	
Michał Wojtylak	On the Weyl function for operators, linear pencils and structured random matrices	16.40-17.05	
Henrik Winkler	Global and local behavior of zeros of negative type	17.05-17.30	
		Friday	P647
Marinus Kaashoek	On a class of matrix polynomial equations with degree constraints	13.30-13.55	
Peter Šemrl	Invertibility preservers	13.55-14.20	
Cristina Camara	Toeplitz operators, one sided invertibility of matrices and corona problems	14.20-14.45	
André Klein	The matrix resultant property of Fisher information matrices of stationary processes	14.45-15.10	

Operators, matrices, and indefinite inner products

Special session in honor of Leiba Rodman on the occasion of his 65th birthday

The Takagi problem on the disk and the bidisk

Joseph A. Ball

Given points $(\lambda_1, \ldots, \lambda_N)$ in the open unit disk $\mathbb{D}$ and complex numbers $(w_1, \ldots, w_N)$, a classical theorem of G. Pick assets that there exists a holomorphic function ϕ on $\mathbb{D}$ that interpolates the data (i.e., $\phi(\lambda_i) = w_i$ for $1 \leq i \leq N$) and, in addition maps the unit disk $\mathbb{D}$ into the closed unit disk $\overline{\mathbb{D}}$ if and only if the Pick matrix $\Gamma = \left[\frac{1-w_i\overline{w}_j}{1-\lambda_i\overline{\lambda}_j}\right]_{i,j=1}^N$ is positive semi-definite. Moreover, when Γ is positive sami-definite, ϕ can be chosen to be a Blaschke product with degree equal to the rank of Γ, and so extends to be meromorphic on the whole Riemann sphere $\mathbb{C}_\infty$ and to have modulus 1 on the unit circle $\mathbb{T}$. When Γ is allowed to have negative eigenvalues, it is known that one can still solve the interpolation conditions with a rational function ϕ with unimodular values on $\mathbb{T}$, but one must allow ϕ to have up to $N - \pi$ poles and $N - \nu$ zeros in $\mathbb{D}$, where π is the number of positive eigenvalues and ν is the number of negative eigenvalues of Γ. We discuss extensions of these ideas to the setting where the unit disk $\mathbb{D}$ is replaced by the unit bidisk $\mathbb{D}^2 = \mathbb{D} \times \mathbb{D}$. The idea of the proof is to adapt a lurking isometry argument to an indefinite metric setting specified by the problem data.
The talk is based on joint work with Jim Agler and John McCarthy.

1. J. Agler, J. A. Ball, and J.E. McCarthy: The Takagi problem on the disk and bidisk, *Acta Sci. Math. (Szeged)*, 79 (2013) no. 1-2, 63–78.

Zero sums of projections and unusual Cantor sets

Harm Bart

Sums of projections (more generally, idempotents) are of central importance in the analysis of spectral regularity issues for Banach algebras. In this talk, the focus is on the construction of non-trivial zero sums of projections. It is intriguing that we need only five projections in all known instances. The construction relates to deep problems concerning the geometry of Banach spaces and general topology. Along the way, a novel way to produce Cantor sets is discussed. It leads to instances of such sets of an unusual type,
The talk is based on joint work with Torsten Ehrhardt (Santa Cruz) and Bernd Silbermann (Chemnitz).

Stability of invariant maximal semidefinite subspaces

André Ran

For matrices that have structure in an indefinite inner product one is often interested in invariant maximal semidefinite subspaces. Finding such subspaces plays a role in linear-quadratic optimal control theory for instance, and also in transport theory. It is then a natural question to ask whether such subspaces can be computed in a numerically reliable way. In this talk we shall address this question, discuss old results obtained in the eighties of the twenties century by the speaker and L. Rodman, and also discuss some more recent results.
The talk is based on joint papers with: L. Rodman, Chr. Mehl, V. Mehrmann, D.B. Janse van Rensburg, J.H. Fourie and G.J. Groenewald.

Completion, extension, factorization, and lifting of operators with a negative index

Seppo Hassi

The well-known results of M.G. Kreĭn on selfadjoint contractive extensions of a Hermitian contraction T_1 and the characterization of all nonnegative selfadjoint extensions $\widetilde{A}$ of a nonnegative operator A via the inequalities $A_K \leq \widetilde{A} \leq A_F$, where A_K and A_F are the Kreĭn-von Neumann extension and the Friedrichs extension of A, are extended to the situation, where $\widetilde{A}$ is allowed to have a fixed number of negative eigenvalues. The basic tools for this purpose include an indefinite generalization of an old result due to Yu.L. Shmul'yan on completions of 2×2 nonnegative block operators, some lifting results on J-contractive operators in Hilbert, Pontryagin and Kreĭn spaces, some Douglas type factorization results in indefinite setting, and so-called antitonicity results concerning inequalities between semibounded selfadjoint operators and their inverses with finite negative indices.
The talk is based on joint work with D. Baidiuk (Vaasa).

Rank one perturbations of H-positive real matrices

Dawid Janse van Rensburg

We consider a generic rank-one structured perturbation on H-positive real matrices. The complex case is treated in general, but the main focus for this article is the real case where something interesting occurs at eigenvalue zero and even size Jordan blocks. Generic Jordan structures of perturbed matrices are identified.
The talk is based on joint work with J.H. Fourie, G.J. Groenewald and A.C.M. Ran.

Algebraic eigenspaces of H-nonnegative matrices after a rank one perturbation

Leslie Leben

Let the space $(\mathbb{C}^n, [\cdot,\cdot])$ be equipped with the indefinite inner product $[x,y] = \langle Hx, y\rangle$, $H \in \mathbb{C}^{n\times n}$ being an invertible symmetric matrix. We consider rank one perturbations of an H-nonnegative matrix $A \in \mathbb{C}^{n\times n}$ (i.e. $[Ax,x] \geq 0$). In the case of so-called generic perturbations, it is known that at an eigenvalue $\lambda \in \mathbb{R}$ of A, one of the largest Jordan blocks of A at λ is destroyed under the perturbation. In this talk, we investigate how the algebraic eigenspace $\mathfrak{L}_\lambda(A)$ of A at λ behaves under a *general* rank one perturbation. If B is a symmetric matrix with respect to $[\cdot,\cdot]$, such that $B - A$ is of rank one, we determine all possible structures of the algebraic eigenspace $\mathfrak{L}_\lambda(B)$ of B at λ: It turns out, that there are at most 15 possible cases for $\mathfrak{L}_0(B)$ and at most 5 possible cases for $\mathfrak{L}_\lambda(B)$, $\lambda \neq 0$. In particular, at zero, we show the estimate

$$|\dim \mathfrak{L}_0(A) - \dim \mathfrak{L}_0(B)| \leq 2,$$

and for non-zero λ we show the estimate

$$|\dim \mathfrak{L}_\lambda(A) - \dim \mathfrak{L}_\lambda(B)| \leq 3.$$

The talk is based on a joint work with J. Behrndt (Graz), R. Möws (Berlin), F. Martínez Pería (Buenos Aires), and C. Trunk (Ilmenau).

On the Weyl function for operators, linear pencils and structured random matrices

Michał Wojtylak

The function

$$Q(\lambda) = \langle R(\lambda)e, e\rangle,$$

where $R(\lambda)$ denotes the resolvent and e is a fixed vector, is well known in the litarature under many names. It is a powerful tool in the analysis of spectra, especially in studying rank one perturbations. We will consider the following instances:

(i) A is an H-selfadjoint operator, i.e, HA is selfadjoint in a Hilbert space, $R(\lambda) = (A - \lambda I)^{-1}$,

(ii) A is a large H-selfadjoint random matrix, $R(\lambda) = (A - \lambda I)^{-1}$,

(iii) $A + \lambda E$ is a linear pencil, with A, E being Hermitian matrices, $R(\lambda) = (A - \lambda E)^{-1}$.

In all three cases the Weyl function allows to derive results on spectral properties of the underlying objects, like localization of the spectrum (i,ii) and distance to a singular pencil (iii).
The talk is based on joint projects with Henk de Snoo and Henrik Winkler (i), Patryk Pagacz (ii), and with Christian Mehl and Volker Mehrmann (iii).

Global and local behavior of zeros of negative type

Henrik Winkler

A generalized Nevanlinna function $Q(z)$ with one negative square has precisely one generalized zero of negative type in the closed extended upper halfplane. The fractional linear transformation defined by $Q_\tau(z) = (Q(z) - \tau)/(1 + \tau Q(z))$, $\tau \in R \cup \{\infty\}$, is a generalized Nevanlinna function with one negative square. Its generalized zero of nonpositive type $\alpha(\tau)$ as a function of τ is being studies. In particular, it is shown that it is continuous and its behavior in the points where the function extends through the real line is investigated. The talk is based on joint work with H.S.V. de Snoo and M. Wojtylak.

[1] H.S.V. de Snoo, H. Winkler, M. Wojtylak: Zeros of nonpositive type of generalized Nevanlinna functions with one negative square, *J. Math. Anal. Appl.* 382 (1) (2011), 399–417.

[2] H.S.V. de Snoo, H. Winkler, M. Wojtylak: Global and local behavior of zeros of nonpositive type, *J. Math. Anal. Appl.* 414 (2014), 273-284.

On a class of matrix polynomial equations with degree constraints

Marinus Kaashoek

In this talk we deal with a class of matrix polynomial equations with degree constraints on the unknowns. Such equations appear in a natural way in the study of Szegő-Kreĭn orthogonal matrix polynomials [1]. Necessary and sufficient conditions for solvability are given in terms of left and right root functions of the coefficients. The main result is a discrete version of the main theorem in [2] and extends an earlier result of H. K. Wimmer in [4].

The talk is based on joint work [3] with Leonia Lerer (Z"L).

[1] I. Gohberg, and L. Lerer, Matrix generalizations of M.G. Krein theorems on matrix polynomials, in *Orthogonal Matrix-Valued Polynomials and Applications* (I. Gohberg, ed.), OT **34**, Birkhäuser Verlag, Basel, 1988, pp. 137–202.

[2] I. Gohberg, M.A. Kaashoek, and L. Lerer, On a class of entire matrix function equations. *Lin. Alg. Appl.* **425** (2007), 434–442.

[3] M.A. Kaashoek, and L. Lerer, On a class of matrix polynomial equations, *Lin. Alg. Appl.* **439** (2013), 613–620.

[4] H.K. Wimmer, The generalized Sylvester equation in polynomial matrices, *EEE Trans. Automat. Control* **41** (1996), 1372–1376.

Invertibility preservers

Peter Šemrl

We will present a structural result for linear preservers of invertibility on central simple algebras. The main tool in the proof is the localization technique for linear preservers on matrix spaces which has been recently developed in a joint paper with Leiba Rodman.

Toeplitz operators, one sided invertibility of matrices and corona problems

Cristina Câmara

When is a matricial Toeplitz operator Fredholm-equivalent to a scalar Toeplitz operator? When does it have Coburn's property? Can left invertibility of an $n \times m$ ($m \leq n$) matrix over a unital commutative ring be studied in terms of an associated scalar corona problem? These apparently independent questions are addressed taking an algebraic approach which, moreover, provides a good illustration of how the study of Toeplitz operators knits together different areas of mathematics.
The talk is based on joint work with L. Rodman and I. M. Spitkovsky.

The matrix resultant property of Fisher information matrices of stationary processes

André Klein

In this talk, proofs are included to confirm the matrix resultant property of two asymptotic Fisher information matrices of stationary processes. The proofs developed are different to the ones presented in Klein, Spreij 1996 and Klein, Mélard, Spreij 2005. The proofs are mainly based on the null space of two Fisher information matrices expressed in terms of the null space of the Sylvester resultant matrices and the tensor Sylvester matrix. It is shown that the Fisher information matrices of stationary processes fulfill the matrix resultant property. The Fisher information matrix is an ingredient of the Cramèr-Rao inequality, and belongs to the basics of asymptotic estimation theory in mathematical statistics.

[1] A. Klein, P. Spreij, On Fisher's information matrix of an ARMAX process and Sylvester's resultant matrices, *Linear Algebra Appl.* 237/238 (1996), 579–590.

[2] A. Klein, G. Mélard, P. Spreij, On the Resultant Property of the Fisher Information Matrix of a Vector ARMA process, *Linear Algebra Appl.* 403 (2005), 291–313.

Infinite dimensional systems (IDS)
Organizers: B. Jacob and H. Zwart

Speaker	title	time	place
		Thursday	M623
Birgit Jacob	The weighted Weiss conjecture for admissible observation operators	13.30-13.55	
Christian Le Merdy	Admissibility for Ritt operators	13.55-14.20	
Dmitry Yakubovich	Which linear optimal regulators do well their job?	14.20-14.45	
Hans Zwart	The zero-two law for cosine families	14.45-15.10	

Infinite dimensional systems

The weighted Weiss conjecture for admissible observation operators

Birgit Jacob

The weighted Weiss conjecture states that the system theoretic property of weighted admissibility can be characterised by a resolvent growth condition. For positive weights, it is known that the conjecture is true if the system is governed by a normal operator; however, the conjecture fails if the system operator is the unilateral shift on the Hardy space $H^2(\mathbb{D})$ (discrete time) or the right-shift semigroup on $L^2(\mathbb{R}_+)$ (continuous time). To contrast and complement these counterexamples, in this talk positive results are presented characterising weighted admissibility of linear systems governed by shift operators and shift semigroups. These results are shown to be equivalent to the question of whether certain generalized Hankel operators satisfy a reproducing kernel thesis.
The talk is based on joint work with E. Rydhe and A. Wynn.

Admissibility for Ritt operators

Christian Le Merdy

Let $T\colon H \to H$ be a power bounded operator on Hilbert space H and let K be another Hilbert space. An operator $C\colon H \to K$ is called admissible for T if it satisfies an estimate $\sum_k \|CT^k(x)\|^2 \leq M^2\|x\|^2$. This talk deals with the validity of a certain Weiss conjecture in this discrete setting. The main result states that when T is a Ritt operator satisfying a square function estimate $\sum_k k\|T^{k+1}(x) - T^k(x)\|^2 \leq K^2\|x\|^2$, then C is admissible for T if and only if it satisfies a uniform estimate $(1-|\omega|^2)^{\frac{1}{2}}\|C(I-\omega T)^{-1}\| \leq c$ for complex numbers ω with $|\omega| < 1$. If time permits, we will discuss extensions to the more general setting of α-admissibility and/or to non Hilbertian Banach spaces, using a natural variant of admissibility involving R-boundedness.

Which linear optimal regulators do well their job?

Dmitry Yakubovich
Let $x' = Ax + Bu$ be a linear control system. The linear quadratic regulator (LQR) $u(t) = -Fx(t)$ gives a solution to the problem of finding the control u that minimizes the energy functional $J = \int_0^\infty \|x(t)\|^2 + \|u(t)\|^2\, dt$. The closed-loop system obtained by the LQR is optimal in this sense, but there are many other important characteristics of the closed-loop system measuring its quality. In this work (published recently in [1]), we investigate the rate of exponential decay of the closed-loop system for linear quadratic regulators, which shows how deep inside the left half-plane is the spectrum of the matrix $A - BF$. Using different complex variable techniques, we give several upper and lower estimates of this number and show that for a class of cases, these estimates are sharp.
In many practical applications, a control engineer may have a wide range of control matrices B to choose from, or the matrix may depend on certain parameters. Our results lead to an efficient algorithm that enables the engineer to choose a control B that also performs well in the sense of the decay rate.
This is joint work with Daniel Estévez (Autónoma Univ. of Madrid).

[1] D. Estévez, D.V. Yakubovich, Decay rate estimations for linear quadratic optimal regulators. Linear Algebra Appl. 439 (2013), no. 11, 3332–3358.

The zero-two law for cosine families

Hans Zwart

For $(C(t))_{t\geq 0}$ being a strongly continuous cosine family on a Banach space, we show that the estimate $\limsup_{t\to 0+} \|C(t) - I\| < 2$ implies that $C(t)$ converges to I in the operator norm. This implication has become known as the zero-two law. We further prove that the stronger

assumption of $\sup_{t\geq 0}\|C(t)-I\| < 2$ yields that $C(t)=I$ for all $t\geq 0$. By considering the cosine family $(\cos(t))_{t\geq 0}$ it is easy to see that this result is optimal.
The corresponding results for C_0-semigroups, such as: $\sup_{t\geq 0}\|T(t)-I\| < 1 \Rightarrow T(t)=I$, hold without the assumption of strong continuity. However, our proof needs the strong continuity, and it is an open question whether this is necessary.
The talk is based on joint work with F. Schwenninger.

Dynamics of linear operators (DLO)

Catalin Badea, Frédéric Bayart, Sophie Grivaux

Speaker	title	time	place
		Thursday	P647
Étienne Matheron	Baire category and ergodic measures for linear operators	13.30-13.55	
Alfred Peris	Dynamics on invariant sets of linear operators	13.55-14.20	
Andrei Lishanskii	On S. Grivaux' example of a hypercyclic rank one perturbation of a unitary operator	14.20-14.45	
Hubert Klaja	Rank one perturbations of diagonal operators without eigenvalues	14.45-15.10	
Yuri Tomilov	On fine scales of decay of operator semigroups	15.50-16.15	
J. Alberto Conejero	An extension of hypercyclicity for N-linear operators	16.15-16.40	
Quentin Menet	Existence of common hypercyclic subspaces	16.40-17.05	
George Costakis	Common hypercyclic vectors and sparse orbits	17.05-17.30	

Dynamics of linear operators

Baire category and ergodic measures for linear operators

Étienne Matheron

In this talk, I will try to explain how it is possible to prove by simple Baire category arguments that if a linear operator acting on a Polish topological vector space has a perfectly spanning set of unimodular eigenvectors, then it admits an ergodic measure with full support.
The talk is based on joint work with Sophie Grivaux.

[1] S. Grivaux and É. Matheron : Invariant measures for frequently hypercyclic operators, *arXiv preprint, http://arxiv.org/abs/1403.4555.*

Dynamics on invariant sets of linear operators

Alfred Peris

We study some notions of topological dynamics (hypercyclicity, mixing properties, etc.) and of measure-theoretic dynamics (strong mixing) for operators on topological vector spaces with invariant sets. More precisely, we will show some links between the fact of satisfying any of our dynamical properties on certain invariant sets, and the corresponding property on the closed linear span of the invariant set. Particular attention is given to the case of positive operators and semigroups on Fréchet lattices, and the (invariant) positive cone.
This talk is based on joint work with M. Murillo-Arcila.

[1] F. Bayart and É. Matheron, *Dynamics of linear operators*, Cambridge University Press, Cambridge, 2009.

[2] K. G. Grosse-Erdmann and A. Peris Manguillot, *Linear chaos* Universitext, Springer-Verlag, London, 2011.

[3] M. Murillo-Arcila, A. Peris, Strong mixing measures for linear operators and frequent hypercyclicity, J. Math. Anal. Appl. 398 (2013), 462-465.

On S. Grivaux' example of a hypercyclic rank one perturbation of a unitary operator

Andrei Lishanskii

Recently, Sophie Grivaux showed that there exists a rank one perturbation of a unitary operator in a Hilbert space which is hypercyclic [1]. We give a different proof of this theorem using a functional model for rank one perturbations of singular unitary operators [2]. This model (which is due to V. Kapustin, A. Baranov and D. Yakubovich) reduces the problem to the construction of a special family of functions in a shift-coinvariant subspace K_θ in the unit disc.
The talk is based on joint work with Anton Baranov.

[1] S. Grivaux: A hypercyclic rank one perturbation of a unitary operator, *Math. Nachr.*, **285** (2012), 5-6, 533–544.

[2] A. Baranov, A. Lishanskii: On S. Grivaux' example of a hypercyclic rank one perturbation of a unitary operator, http://arxiv.org/abs/1401.1934.

Rank one perturbations of diagonal operators without eigenvalues

Hubert Klaja

In 1984, Stampfli built a Hilbert space diagonal operator which admits a rank one perturbation without any eigenvalue, i.e. he constructed a diagonal operator D and two vectors u and v such that $\sigma_p(D + u \otimes v) = \emptyset$. In 2001, Ionascu asked when does a rank one perturbation of a diagonal

operator without eigenvalues exists. On the other hand, in 2012 Sophie Grivaux built a hypercyclic rank one perturbation of a unitary diagonal operator with uncountably many eigenvalues.
In this talk we will discuss the existence of rank one perturbation of diagonal operators without eigenvalues and give a solution to Ionascu's problem.

On fine scales of decay of operator semigroups

Yuri Tomilov

The talk will concern operator dynamics which is regular in a sense of convergence to zero with a certain rate. Motivated by potential applications to partial differential equations, we developed a theory of fine scales of decay rates for operator semigroups. The theory contains, unifies, and extends several notable results on asymptotics of of operator semigroups and yields a number of new ones. Its core is a new operator-theoretical method of deriving rates of decay combining ingredients from functional calculus, and complex, real and harmonic analysis. In this talk, we will present a glimpse at the theory.
This is joint work with Charles Batty and Ralph Chill to appear in J. Europ. Math. Soc.

An extension of hypercyclicity for N-linear operators

J. Alberto Conejero

Grosse-Erdmann and Kim recently introduced the notion of bihypercyclicity for studying the existence of dense orbits under bilinear operators [2]. We propose an alternative notion of orbit for N-linear operators that is inspired by difference equations. Under this new notion, every separable infinite dimensional Fréchet space supports supercyclic N-linear operators, for each $N \geq 2$. Indeed, the non-normable spaces of entire functions and the countable product of lines support N-linear operators with residual sets of hypercyclic vectors, for N=2.
The talk is based on joint work with Juan Bès [1].

[1] J. Bès and J.A. Conejero: An extension of hypercyclicity for N-linear operators, To appear in *Abstr. Appl. Anal.*

[2] K.-G. Grosse-Erdmann and S. G. Kim: Bihypercyclic bilinear mappings, *J. Math. Anal. Appl.*, 399, no. 2, (2013), 701–708.

Existence of common hypercyclic subspaces

Quentin Menet

A sequence $(T_n)_{n\geq 0}$ of operators from X to Y is said to be *hypercyclic* if there exists a vector $x \in X$ (also called hypercyclic) such that the set $\{T_n x : n \geq 0\}$ is dense in Y. An important question about hypercyclic operators concerns the existence of *hypercyclic subspaces* i.e. infinite-dimensional closed subspaces in which every non-zero vector is hypercyclic. These notions can be extended to families $(T_{n,\lambda})_{n\geq 0,\lambda\in\Lambda}$ of sequences of operators by saying that a vector $x \in X$ is a *common hypercyclic vector* for the family $(T_{n,\lambda})$ if x is a hypercyclic vector for each sequence $(T_{n,\lambda})_{n\geq 0}$.
In the case of hypercyclic subspaces, we know two very useful criteria for the existence of such subspaces: Criterion M_0 and Criterion (M_k). While Criterion M_0 was generalized by F. Bayart to common hypercyclic subspaces, no version of Criterion (M_k) was so far known for the existence of common hypercyclic subspaces.
In this talk, we will develop the link between Criterion M_0 and Criterion (M_k) in order to generalize Criterion (M_k) to common hypercyclic subspaces and we will then introduce several consequences of this generalization.
The talk is based on joint work with Juan Bès.

Common hypercyclic vectors and sparse orbits

George Costakis

I shall discuss some recent advances on the subject of common hypercyclic vectors. In particular, for a large class of operators we establish the existence of common hypercyclic vectors for "sparse" orbits, say of polynomial type. A basic ingredient of the proof is the concept of uniformly distributed sequences. On the other hand, for "very sparse" orbits, say of exponential type, the above result fails to hold. The talk is based on joint with N. Tsirivas.

Operator theory and harmonic analysis (OTHA)
Organizers: Alfonso Montes, Haakan Hedenhalm and Manuel Cepedello

Speaker	title	time	place
		Thursday	M655
Alexandru Aleman	Residual subspaces and spectral synthesis for differentiation on C^∞	13.30-13.55	
Nicola Arcozzi	The Hardy space of regular functions on the quaternions	13.55-14.20	
Anton Baranov	Strong M-bases of reproducing kernels and spectal theory of rank-one perturbations of selfadjoint operators	14.20-14.45	
Isabelle Chalendar	Weighted composition operators on the Direichlet space: Boundedness and spectral properties	14.45-15.10	
Kristian Seip	Inequalities on the polydisc	15.50-16.15	
Nikolay Vasilevski	Commutative C*-algebras of Toeplitz operators on the Bergman space	16.15-16.40	
Vasily Vasyunin	Extremal problems on BMO and developable surfaces	16.40-17.05	

Operator theory and harmonic analysis

Residual subspaces and spectral synthesis for differentiation on C^{∞}

Alexandru Aleman

If I is an interval on the real line, the differentiation operator $\frac{d}{dt}$ restricted to its invariant subspaces of $C^{\infty}(I)$ shows a fairly intricated behavior. The spectrum of such a restriction, may be void, or equal to the whole complex plane, or consist of a countable set of eigenvalues. Arbitrary invariant subspaces may contain a nontrivial "residual part" where the spectrum of the restriction is void.
The talk is focused on two topics:
1) The structure of residual subspaces,
2) The appropriate spectral synthesis in invariant subspaces with a countable spectrum.
The material is based on earlier joint work with B. Korenblum and recent results with A. Baranov and Y. Belov.

The Hardy space of regular functions on the quaternions

Nicola Arcozzi

Graziano Gentili and Daniele Struppa developed in the past few years an analog of complex function theory on the quaternions. We discuss here some features of the corresponding Hardy space theory, with an emphasis on differences, rather than on analogies. Some open problems will be also discussed.
Work in collaboration with Giulia Sarfatti.

Strong M-bases of reproducing kernels and spectal theory of rank-one perturbations of selfadjoint operators

Anton Baranov

Let $\{x_n\}_{n\in N}$ be a complete and minimal system in a separable Hilbert space H, and let $\{y_n\}$ be its biorthogonal system. The system $\{x_n\}$ is said to be hereditarily complete (or a strong M-basis) if for any $x \in H$ we have $x \in \mathrm{Span}\{(x, y_n)x_n\}$. This property may be understood as a very weak form of the reconstruction of a vector x from its (formal) Fourier series $\sum_n (x, y_n)x_n$. By the results of A.S. Markus hereditary completeness of a system of eigenvectors (root vectors) of a compact operator is equivalent to the spectral synthesis property for this operator.
In [1] the hereditary completeness problem for exponential systems in $L^2(-a, a)$ (equivalently, reproducing kernels of the Paley-Wiener space PW_a^2) was solved. It turned out that the non-hereditary completeness may occur even in the case of exponential systems, though the defect of incompleteness is always at most one.
In [2] we study the hereditary completeness for the reproducing kernels in Hilbert spaces of entire functions introduced by L. de Branges. One of our motivations is a question by N.K. Nikolski. Another motivation is in the relation (via a functional model) between this problem and the spectral synthesis for rank one perturbations of compact selfadjoint operators. We give a complete description of de Branges spaces where nonhereditarily complete systems of reproducing kernels exist, in terms of their spectral measures. As a corollary, we obtain a series of striking examples of rank one perturbations of compact selfadjoint operators for which the spectral synthesis fails up to finite- or even infinite-dimensional defect.
The talk is based on joint work with Yurii Belov and Alexander Borichev.

[1] A. Baranov, Y. Belov, A. Borichev, Hereditary completeness for systems of exponentials and reproducing kernels, *Advances in Mathematics*, 235 (2013), 525–554.

[2] A. Baranov, Y. Belov, A. Borichev, Strong M-basis property for systems of reproducing kernels in de Branges spaces, arXiv:1309.6915.

Weighted composition operators in the Dirichlet space: boundedness and spectral properties

Isabelle Chalendar

Boundedness of weighted composition operators $W_{u,\varphi}$ acting on the classical Dirichlet space $\mathcal{D}$ as $W_{u,\varphi}(f) = uf \circ \varphi$ is studied in terms of the multiplier space associated to the symbol φ. A prominent role is played by the multipliers of the Dirichlet space. As a consequence, we determine the spectrum of $W_{u,\varphi}$ in $\mathcal{D}$ whenever φ is an automorphism of the unit disc.
Joint work with E. Gallardo and J. Partington.

Inequalities on the polydisc

Kristian Seip

We will discuss some inequalities that appear naturally in the function theory of the polydisc. We will discuss an idea of H. Helson showing the importance of contractivity when "lifting" one-dimensional inequalities to several variables and see how it leads to certain Hardy–Littlewood-type inequalities. We will also discuss inequalities with no counterpart in the one-dimensional setting, such as the Bohnenblust–Hille inequality. Applications to estimates for Dirichlet polynomials will be mentioned.

Commutative C^*-algebras of Toeplitz operators on the Bergman space

Nikolay Vasilevski

We give a complete characterization of all commutative C^*-algebras generated by Toeplitz operators acting on the weighted Bergman spaces over the unit disk. Each such algebra is generated by Toeplitz operators whose bounded measurable symbols are invariant under the action of a maximal abelian subgroup of Moebius transformation of the unit disk, and is isometrically isomorphic to the algebra of sequences (or functions) that are slowly oscillate in a certain specific sense.
As a byproduct we give a solution of a weighted extension of the classical Hausdorff moment problem.

Extremal problems on BMO and developable surfaces

Vasily Vasyunin

Introduction to the Bellman function method will be presented. The presentation will be based on examples of some classical inequalities for BMO functions (such as the well-known John–Nirenberg inequality). A prove of such inequality with sharp constants can be obtained by solving some boundary value problem for homogeneous Monge–Ampère equation. The graph of any solution of such equation is a so-called developable surface. Geometrical construction that foliate the domain of solution by special straight line segments is a corner stone of the method of constructing the required developable surface and therefore of finding a solution of the boundary value problem. The found Bellman function immediately supplies us with the desired inequality with the sharp constants.

Abstracts of Contributed Talks

Added after PDO-session on Friday afternoon

Bounds on variation of spectral subspaces

Alexander K. Motovilov

We give a survey of recent results concerning the bounds on variation of the spectral subspace of a self-adjoint operator under an additive perturbation. A particular attention is payed to the bounds obtained in [1-4]. Furthermore, we present a new estimate on the maximal angle between unperturbed and perturbed spectral subspaces in the generic off-diagonal subspace perturbation problem that is stronger than the best previously known bound from [3].

[1] S. Albeverio and A. K. Motovilov: Sharpening the norm bound in the subspace perturbation theory, *Complex Analysis and Operator Theory*, 7 (2013), 1389–1416.

[2] S. Albeverio and A. K. Motovilov: The a priori $\tan\theta$ theorem for spectral subspaces, *Integral Equations and Operator Theory*, 73 (2012), 413–430.

[3] K. A. Makarov and A. Seelmann: The length metric on the set of orthogonal projections and new estimates in the subspace perturbation problem, *Journal für die reine und angewandte Mathematik*, DOI: 10.1515/crelle-2013-0099 (2013).

[4] A. Seelmann: On an estimate in the subspace perturbation problem, *arXiv*: 1310.4360v1 (2013).

A regular version of Smilansky model

Diana Barseghyan

In the paper of Uzy Smilansky one discussed a simple example of quantum dynamics which could exhibit a behavior of two substantial different types controlled by the value of the coupling constant. The model in which it can be demonstrated allows for interpretation as a simple quantum graph to the vertices of which one or more one-dimensional harmonic oscillators are attached with a position-dependent coupling strength. We discuss a modification of Smilansky model in which a singular potential is replaced by a regular, below unbounded potential. We demonstrate that, similarly to the original model, such a system exhibits a spectral transition with respect to the coupling constant, and determine the critical value above which a new spectral branch opens.

Added after session SL on Friday afternoon

"Bulk" states and "surface" states for chain-type nanostructures

Igor Popov

Nanosystems demonstrates interesting physical properties in situations when its Hamiltonians have spectral bands corresponding to the surface states which are fully imbedded in the gaps of the bulk states spectrum. One of the most popular example of such system is topological insulator where this phenomenon takes place due to strong spin-orbit interaction. We consider other systems with this property. In 3D we compare the spectrum for infinite chain (straight, bent or branched) of touching identical balls with the corresponding spectrum of the analogous chain of spheres. The Hamiltonians are constructed by the operator extensions theory approach from the orthogonal sum of free Schrödinger Hamiltonians, i.e. the Laplacians for the balls (the Laplace-Beltrami operators for spheres). Namely, we restrict the initial self-adjoint operator on the set of functions vanishing at the touching points and then extend the obtained symmetric operator to a self-adjoint one. "Surface" bands which are separated from the "bulk" bands are studied. In 2D case we deal with similar systems (chains of discs and chain of rings). The influence of the magnetic field orthogonal to the system plane is studied. We assume that there are $\delta-$like potentials at the touching points. For the bent and branched chains we investigate the discrete spectrum of the Hamiltonians also.

[1] J.Cayssol, B.Dora, F.Simon,R.Moessner: Floquet topological insulator, *Phys. Status Solidi RRL*, 7 (2013), 101-108.

[2] I.Yu.Popov, A.N.Skorynina, I.V.Blinova: On the existence of point spectrum for branching strips quantum graph, *Journal of Mathematical Physics*, 55 (2014), 033504/1-20.

[3] P. Duclos, P. Exner, O. Turek: On the spectrum of a bent chain graph. *J. Phys. A: Math. Theor.*451 (2008), 415206.

Toeplitz Operators

Friday afternoon

Inversion of centrosymmetric Toeplitz-plus-Hankel Bezoutians

Karla Rost

The main aim is to discuss how to compute the inverse of a nonsingular, centrosymmetric Toeplitz-plus-Hankel Bezoutian B of order n and how to find a representation of B^{-1} as a sum of a Toeplitz and a Hankel matrix. Besides the known splitting property of B as a sum of two split-Bezoutians, the connection of the latter to Hankel Bezoutians of about half size is used. Results on fast inversion of Hankel Bezoutians together with a corresponding inversion formula lead to the desired representation for B^{-1} as a Toeplitz-plus-Hankel matrix and make it possible to design an $O(n^2)$ inversion algorithm.
The talk is based on joint work with Torsten Ehrhardt.

Toeplitz Operators with non-commuting symbols

Stephen Sontz

Toeplitz operators are defined for symbols in the quantum plane (which is a non-commutative algebra), in other quite general non-commutative algebras and even in some vector spaces which have no product at all. In all cases one obtains a Toeplitz quantization which does not necessarily come from a measure and whose symbols are not functions. We then introduce creation and annihilation operators as special cases of these Toeplitz operators and discuss their commutation relations. Since we are quantizing a 'classical' space that is not commutative (that is, it is a 'quantum' space), this is a type of second quantization. Five recent papers of mine on this topic can be found at arxiv.org.

Some good news on limit operators

Markus Seidel

The Limit Operator Method is an indispensable tool for the study of Fredholm properties of e.g. band-dominated operators, as well as for the characterization of the applicability of approximation methods to various classes of operators. It has left its imprint on or is even the topic of the monographs by Hagen/Roch/Silbermann, Rabinovich/Roch/Silbermann and Lindner. In the booklet [1], Chandler-Wilde and Lindner pushed these tools forward, and considered applications to various more concrete classes of band-dominated operators. Moreover, they collected and raised eight open problems in that field. Recently, five of them could be answered, and among them there is an affirmative answer to the question whether the invertibility of all limit operators already imply their uniform invertibility. This problem is known in the literature as the "Big Question in Limit Operator Business".
The aim of this talk is to give an overview on the limit operator concept and its main results and, of course, to address (some of) the solutions to the mentioned open problems.
The talk is based on joint work with Marko Lindner.

[1] S. N. Chandler-Wilde, M. Lindner, *Limit Operators, Collective Compactness and the Spectral Theory of Infinite Matrices*, Mem. Amer. Math. Soc. **210** (2011), No. 989.

Toeplitz generated matrices: Asymptotics, norms, singular values

Hermann Rabe

In the research described here, Toeplitz-generated (T-gen) matrices are matrices of the form $X_n = T_n + f_n(T_n^{-1})^*$, where T_n denotes a banded $n \times n$ Toeplitz matrix and f_n a sequence of positive real numbers converging to zero. We will give an overview of the properties of these T-gen matrices

with respect to their norms, norms of inverses and singular values - all of which will be studied as n tends to infinity. Certain finite rank perturbations of these matrices will also be considered.
The talk is based on joint work with André C.M. Ran.

[1] H. Rabe, A.C.M. Ran. Asymptotics of the smallest singular value of a class of Toeplitz-generated matrices and related finite rank perturbations. *Integral Equations and Operator Theory*, 77 (2013), 385–396.

[2] H. Rabe, A.C.M. Ran. Asymptotics of the smallest singular value of a class of Toeplitz-generated matrices II. *Integral Equations and Operator Theory*, online first article, (DOI: 10.1007/s00020-014-2134-0).

Quasi-banded operators and approximations of convolutions with almost periodic or quasi-continuous symbol.

Pedro A. Santos

We present the stability and Fredholm property of the finite sections of quasi-banded operators acting on L^p spaces over the real line. This family is significantly larger than the set of band-dominated operators, but still permits to derive criteria for the stability and results on the splitting property, as well as an index formula in the form as it is known for the classical cases. In particular, this class covers convolution type operators with semi-almost periodic and quasicontinuous symbols, and operators of multiplication by slowly oscillating, almost periodic or even more general coefficients.
The talk is based on joint work with Helena Mascarenhas and Markus Seidel.

[1] H. Mascarenhas, P.A. Santos, M. Seidel: Quasi-banded operators, convolutions with almost periodic or quasi-continuous data, and their approximations *Journal of Mathematical Analysis and Applications*, (2014), http://dx.doi.org/10.1016/j.jmaa.2014.03.079.

Singular values of variable-coefficient Toeplitz matrices

Helena Mascarenhas

In this talk we aim to describe asymptotic spectral properties of sequences of variable-coefficient Toeplitz matrices. These sequences, $A_N(a)$, with a being in a Wiener type algebra and defined on a cylinder $([0,1]^2 \times \mathbb{T})$, widely generalizes the sequences of finite sections of a Toeplitz operator. We prove that if $a(x,x,t)$ does not vanish for every $(x,t) \in [0,1] \times \mathbb{T}$ then the singular values of $A_N(a)$ have the k-splliting property, which means that, there exist an integer k such that, for N large enough, the first k_{th}-singular values of $A_N(a)$ converge to zero as $N \to \infty$, while the others are bounded away from zero, with $k = \dim\ker T(a(0,0,0)) + \dim\ker T(a(0,0,t^{-1}))$, the sum of kernel dimension of two Toeplitz operators.
The talk is based on joint work with B. Silbermann.

Operator Algebras

Thursday afternoon first half

Operator algebras associated with multi-valued mappings

Alla Kuznetsova

I am going to talk about a new approach of construction of operator algebras generated by multi-valued mappings on countable sets. The starting point is a countable set X being $\mathbb{Z}$-*graded* in the following sense: $X = \cup_{n\in\mathbb{Z}} X_n$, where all X_n are countable or finite and mutually disjoint for different n.
Multi-valued mapping is a mapping ω from X to the set of all finite subsets of X. The composition $\omega = \omega_1 \circ \omega_2$ of two multi-valued mappings is defined as $\omega(x) = \{t : \ t \in \omega_2(y), \text{ for some } y \in \omega_1(x)\}$. The conjugate mapping ω^* is defined as $\omega^*(x) = \{y \in X : x \in \omega(y)\}$. Evidently, the mapping $^* : \omega \to \omega^*$ is an involution.
A mapping ω is *directed* if $\omega(X_n) \subset X_{n+k}$ for an integer k and all $n \in \mathbb{Z}$.
We consider on the Hilbert space $l^2(X)$ the standard basis $\{e_x,\, x \in X\}$, $e_x(y) = \delta_{x,y}$. A directed mapping ω induces a linear operator T_ω defined on the basis as $T_\omega e_x = \sum\limits_{y\in\omega(x)} e_y$. Obviously, $T_\omega^* = T_{\omega^*}$, and $T_{\omega_1 \circ \omega_2} = T_{\omega_1} T_{\omega_2}$.
We say a mapping ω is of *bounded type* if $\sup\limits_{x\in X} (\operatorname{card} \omega(x) + \operatorname{card} \omega^*(x)) < \infty$.
We consider the algebra $\mathfrak{A}_\Omega(X)$, the uniformly closed $*$-subalgebra of $\mathcal{B}(l^2(X))$ generated by the operators T_ω where ω are taken from a finite or countable set Ω of directed mappings of bounded type.
I will discuss various results concerning the structure and some properties of operator algebra $\mathfrak{A}_\Omega(X)$. Some illustrative examples will be presented. The talk is based on a joint work with Victor Arzumanian and Suren Grigoryan.

[1] S. Grigoryan, A. Kuznetsova: C*-algebras Generated by Mappings, *Math. Notes*, **87**(5) (2010), 663-671.

[2] S. Grigoryan, A. Kuznetsova: On a class of nuclear C*-algebras *Proceedings of the 23rd International Conference on operator theory (Timisoara, Romania, 2010)*, 39-50.

[3] V. Arzumanian, S. Grigoryan, A. Kuznetsova: Operator algebras associated with multi-valued mappings, *Lobachevskii J. of Math.*, in print.

Strong algebras

Guy Salomon

In this talk I will introduce algebras which are inductive limits of Banach spaces and carry inequalities which are counterparts of the inequality for the norm in a Banach algebra. The case where the inductive limit consists of one Banach space gives a Banach algebra, while the case where the inductive limit is of infinite number of Banach spaces gives some other “well behaved” topological algebras. I will then show that the well-known Wiener theorem can be generalized to the setting of these algebras, and also consider factorization theory. Finally, I will focus on the case where the multiplication is a convolution of measurable functions on a locally compact group.
This talk is based on joint work with Daniel Alpay.

[1] D. Alpay and G. Salomon. On algebras which are inductive limit of Banach algebras. *p*reprint on arXiv, 2013.

[2] D. Alpay and G. Salomon. Topological convolution algebras. *Journal of Functional Analysis*, 2013.

[3] D. Alpay and G. Salomon. Non-commutative stochastic distributions and applications to linear systems theory. *Stochastic Processes and their Applications*, 2013.

Projective spectrum in Banach algebras

Rongwei Yang

For a tuple $A = (A_1,\ A_2,\ ...,\ A_n)$ of elements in a unital algebra $\mathcal{B}$ over $\mathbb{C}$, its *projective spectrum* $P(A)$ (or $p(A)$) is the collection of $z \in \mathbb{C}^n$ (or respectively $z \in \mathbb{P}^{n-1}$) such that $A(z) = z_1 A_1 + z_2 A_2 + \cdots + z_n A_n$ is not invertible in $\mathcal{B}$. In finite dimensional case, projective spectrum is a projective hypersurface. When A is commuting, $P(A)$ is a union of hyperplanes that looks like a bundle over the Taylor spectrum of A. The *projective resolvent set* $P^c(A) := \mathbb{C}^n \setminus P(A)$ can be identified with $\mathcal{B}^{-1} \cap span\{A_1,\ A_2,\ ...,\ A_n\}$. For every Banach algebra $\mathcal{B}$, $P^c(A)$ is a domain of holomorphy. $\mathcal{B}$-valued Maurer-Cartan type 1-form $A^{-1}(z)dA(z)$ reveals the topology of $P^c(A)$. There is a map from multilinear functionals on $\mathcal{B}$ to the de Rham cohomology $H^*_d(P^c(A),\ \mathbb{C})$ ([1]). In finite dimensional commutative case, this map is a surjective homomorphism by a theorem of Brieskon and Arnold. In noncummutative case, this map links the cyclic cohomology of $\mathcal{B}$ to $H^*_d(P^c(A),\ \mathbb{C})$. Further, there exists a higher order form of the classical Jacobi's formula ([2]).

[1] R. Yang: *Projective spectrum in Banach algebras.* J. of Topology and Analysis Vol. 1 No. 3 (2009), 289–306.

[2] P. Cade and R. Yang: *Projective spectrum and cyclic cohomology.* J. Funct. Anal. Vol. 265 No. 9 (2013), 1916-1933.

A factorization theorem for $\mathfrak{K}$-families and CPD-H-extendable families

Harsh Trivedi

Let E and F be Hilbert C^*-modules over unital C^*- algebras $\mathcal{B}$ and $\mathcal{C}$ respectively. Let S be a set and let $\mathfrak{K} : S \times S \to B(\mathcal{B}, \mathcal{C})$ be a kernel over S with values in the bounded maps from $\mathcal{B}$ to $\mathcal{C}$. For each $\sigma \in S$, $\mathcal{K}^\sigma$ is a map from E to F. The family $\{\mathcal{K}^\sigma\}_{\sigma \in S}$ is called $\mathfrak{K}$-*family* if

$$\langle \mathcal{K}^\sigma(x), \mathcal{K}^{\sigma'}(x')\rangle = \mathfrak{K}^{\sigma,\sigma'}(\langle x, x'\rangle) \text{ for } x, x' \in E,\ \sigma, \sigma' \in S.$$

We show that if $\mathfrak{K} : S \times S \to B(\mathcal{B}, \mathcal{C})$ is a CPD-kernel in the sense of Barreto, Bhat, Liebscher and Skeide, then each map $\mathcal{K}^\sigma$ in the family $\{\mathcal{K}^\sigma\}_{\sigma \in S}$ factors through an isometry. Using a covariant version of this theorem we show that any $\mathfrak{K}$-family $\{\mathcal{K}^\sigma\}_{\sigma \in S}$ which is covariant with respect to a dynamical system on E extends to a $\widetilde{\mathfrak{K}}$-family where $\widetilde{\mathfrak{K}}$ is a CPD-kernel on S from the crossed product of $\mathcal{B}$ to $\mathcal{C}$.
We discuss certain dilation theory of CPD-kernels and its relation with $\mathfrak{K}$-family. Under the assumption E is full, we obtain several characterizations of these maps and give a covariant version by considering a C^*-dynamical system on the (extended) linking algebra of E. One of these characterizations says that these families extend as CPD-kernels on (extended) linking algebra whose $(2, 2)$-corner is a homomorphism.

General Semigroups

Thursday afternoon second half

Duality between compact semigroups and Abelian semigroups

Marat Aukhadiev

An approach of compact quantum semigroups to the problem of semigroup duality is considered. For any Abelian semigroup S satisfying cancellation law we associate a compact quantum semigroup (see [1,2]) $\mathcal{QS}_{red}$. This process is based on noncommutative deformation of the C^*-algebra $C(G)$ of continuous functions on Pontryagin dual group G to the Grothendieck group Γ of S. Such deformation gives a C^*-algebra $C^*_{red}(S)$, which is often called the Toeplitz algebra of S. Using multiplication in G, we endow $C^*_{red}(S)$ with a unital *-homomorphism comultiplication. Hence, we obtain a compact quantum semigroup, associated with S, denoted by $\mathcal{QS}_{red}$. In a special case, $S = \Gamma$, the corresponding quantum semigroup is in fact the compact group G. Hence, this passage extends Pontryagin duality.
Natural question arises here: whether there is a way back, that is a method that allows one to associate an Abelian semigroup S to any compact quantum semigroup $\mathcal{QS}_{red}$. And moreover, does this process give a full duality between a category of Abelian cancellative semigroups and a category of compact quantum semigroups, generalizing Pontryagin duality?
The answers to these questions are given in the talk.

[1] M. A. Aukhadiev, S. A. Grigoryan, E. V. Lipacheva: Topology-preserving quantum deformation with non-numerical parameter, *J. Phys.: Conf. Ser.*, 474 012007 (2013).

[2] M. A. Aukhadiev, S. A. Grigoryan, E. V. Lipacheva: Operator approach to quantization of semigroups, *Mat. Sb.* 205:3 (2014), 15-40.

C^*-algebras generated by representations of elementary inverse semigroup

Ekaterina Lipacheva

A category of C^*-algebras generated by representations of elementary inverse semigroup is considered, their properties and irreducible representations are described.
We show that an infinite set of objects in this category can be given structure of infinite-dimensional compact quantum semigroups. We prove existence of universally repelling object, which is endowed with structure of infinite-dimensional compact quantum semigroup.

C^*-algebras generated by semigroup of maps

Tamara Grigorian and Marat Aukhadiev

The talk is devoted to research on C^*-algebras generated by a semigroup of mappings on a countable set. We prove that in case this semigroup is commutative, the C^*-algebra can be endowed with $\mathbb{Z}$-grading. This grading allows one to construct Hilbert modules generated by a map.

On the standard K-loop structure of positive invertible elements in a C^*-algebra

Lajos Molnár

We investigate the algebraic properties of the standard K-loop operation $a \circ b = \sqrt{a} b \sqrt{a}$ on the set $\mathcal{A}_+^{-1}$ of all positive invertible elements of a C^*-algebra $\mathcal{A}$. We show that its commutativity, associativity and distributivity are each equivalent to the commutativity of $\mathcal{A}$. We present abstract characterizations of the operation $\circ$ and a few related ones, too. For example, in the case where $\mathcal{A}$ has a faithful trace τ, we show that $\circ$ can be characterized as the one and only binary relation $\bullet$ on $\mathcal{A}_+^{-1}$ with the following properties:

(i) for every pair a, b of elements of $\mathcal{A}_+^{-1}$, the equation $a \bullet x = b$ has a unique solution $x \in \mathcal{A}_+^{-1}$;

(ii) $a \bullet 1 = a$ and $a \bullet a = a^2$ for all $a \in \mathcal{A}_+^{-1}$;

(biii) $(a \bullet a) \bullet b = a \bullet (a \bullet b)$ for all $a, b \in \mathcal{A}_+^{-1}$;

(iv) $\tau((a \bullet b) \bullet c) = \tau(b \bullet (a \bullet c))$ for all $a, b, c \in \mathcal{A}_+^{-1}$;

(v) $\tau(a \bullet b) = \tau(ab)$ for all $a, b \in \mathcal{A}_+^{-1}$.

The talk is based on joint work with R. Beneduci.

[1] R. Beneduci and L. Molnár: On the standard K-loop structure of positive invertible elements in a C^*-algebra, *J. Math. Anal. Appl.,* to appear.

Function Spaces

Thursday afternoon second half

Isometries on non-commutative function spaces

Pierre de Jager

The classical Banach-Stone theorem, characterizing isometries between spaces of continuous functions, was generalized to the non-commutative setting by Kadison ([3]) in 1951. Similarly, Lamperti's theorem characterizing isometries between commutative L^p-spaces has been generalized to the non-commutative setting by Yeadon ([7]). Extensions of Lamperti's results to more general Banach function spaces are available in the commutative context ([2]). For a number of these non-commutative analogues exist (see for example [1] and [6]). Merlo ([4]) has characterized isometries between commutative L^p and L^q-spaces (for $p \neq q$) using Bochner kernels. We use Neuhardt's description ([5]) of kernel operators on non-commutative function spaces to investigate the possibility of developing a non-commutative analogue of Merlo's result.

[1] Chilin, V.I., Medzhitov, A.M. and Sukochev, P. A., *Isometries of non-commutative Lorentz spaces*, Math. Z. **200** (1989), 527-545.

[2] Fleming, R.J. and Jamison, J.E., *Isometries on Banach function spaces: function spaces*, Chapman and Hall/CRC, 2003.

[3] Kadison, R.V., *Isometries of operator algebras*, Ann. Math., **54** (1951), 325-338.

[4] Merlo, J., *On isometries of L^p-spaces*, Advances in Mathematics **15** (1975), 194-197.

[5] Neuhardt, E., *Kernel operators on L^p-spaces associated with semifinite von Neumann algebras*, Math. Z. **204** (1990), 297-308.

[6] Sukochev, F.A., *Isometries of symmetric operator spaces associated with AFD factors of type II and symmetric vector-valued spaces*, Integr. Equat. Oper. Th. **26** (1996) 102-124.

[7] Yeadon, F.J., *Isometries of non-commutative L^p-spaces*, Math. Proc. Camb. Phil. Soc., **90** (1981), 41-50.

The spaces of bilinear multipliers of weighted Lorentz type modulation spaces

A. Turan Gürkanlı

Fix a nonzero window $g \in \mathcal{S}(\mathbb{R}^n)$, a weight function w on $\mathbb{R}^{2n}$and $1 \le p, q \le \infty$. The weighted Lorentz type modulation space $M(p,q,w)(\mathbb{R}^n)$ consists of all tempered distributions $f \in \mathcal{S}'(\mathbb{R}^n)$ such that the short time Fourier transform $V_g f$ is in the weighted Lorentz space $L(p,q,wd\mu)\left(\mathbb{R}^{2n}\right)$. The norm on $M(p,q,w)(\mathbb{R}^n)$ is $\|f\|_{M(p,q,w)} = \|V_g f\|_{pq,w}$. This space was firstly defined and investigated some properties for unweighted case by Gürkanlı in [4] and generalized to the weighted case by Sandıkçı and Gürkanlı in [6].

Let $1 < p_1, p_2 < \infty$, $1 \le q_1, q_2 < \infty$, $1 \le p_3, q_3 \le \infty$, ω_1, ω_2 be polynomial weights and ω_3 be a weight function on $\mathbb{R}^{2n}$. In the present paper we define the bilinear multiplier operator from $M(p_1,q_1,\omega_1)(\mathbb{R}^n) \times M(p_2,q_2,\omega_2)(\mathbb{R}^n)$ to $M(p_3,q_3,\omega_3)(\mathbb{R}^n)$ in the following way: Assume that $m(\xi,\eta)$ is a bounded function on $\mathbb{R}^{2n}$. Define

$$B_m(f,g)(x) = \int\limits_{\mathbb{R}^n}\int\limits_{\mathbb{R}^n} \hat{f}(\xi)\,\hat{g}(\eta)\,m(\xi,\eta)\,e^{2\pi i\langle \xi+\eta, x\rangle} d\xi d\eta$$

for all $f, g \in \mathcal{S}(\mathbb{R}^n)$. m is said to be a bilinear multiplier on $\mathbb{R}^n$ of type $(p_1,q_1,\omega_1;p_2,q_2,\omega_2)$, if B_m is a bounded bilinear operator from $M(p_1,q_1,\omega_1)(\mathbb{R}^n) \times M(p_2,q_2,\omega_2)(\mathbb{R}^n)$ to $M(p_3,q_3,\omega_3)(\mathbb{R}^n)$. We denote by $BM(p_1,q_1,\omega_1;p_2,q_2,\omega_2)(\mathbb{R}^n)$ the space of all bilinear multipliers of type

$(p_1,q_1,\omega_1;p_2,q_2,\omega_2)$ and $\|m\|_{(p_1,q_1,\omega_1;p_2,q_2,\omega_2)} = \|B_m\|$. We discuss the necessary and sufficient conditions to be B_m is bounded. Later we investigate properties of this space and we give some examples.
The talk is based on joint work A. Sandıkçı and Ö. Kulak.
Some key references are given below.

[1] O. Blasco: Notes on the spaces of bilinear multipliers, *Revista de la Union Matematica Argentina*, 50(2) (2009), 23–37.

[2] H.G. Feichtinger and A.T. Gürkanlı: On a family of weighted convolution algebras, *Internat. J. Math. Sci*, 13 (1990), 517–526.

[3] R.H. Fischer, A.T. Gürkanlı and T.S. Liu: On a Family of weighted space, *Mat. Slovaca*, 46 (1996), 71–82.

[4] A.T. Gürkanlı: Time-frequency analysis and multipliers of the spaces $M(p,q)\left(\mathbb{R}^d\right)$ and $M(p,q)\left(\mathbb{R}^d\right)$, *J. Math. Kyoto Univ.*, 46(3) (2006), 595–616.

[5] Ö. Kulak and A.T. Gürkanlı: Bilinear multipliers of weighted Lebesgue spaces and variable exponent Lebesgue spaces, *Journal of inequalities and Applications*, (2013), 259.

[6] A. Sandıkçı and A.T. Gürkanlı: Gabor Analysis of the spaces $M\left(p,q,w\right)\left(\mathbb{R}^d\right)$ and $S\left(p,q,r,w,\omega\right)\left(\mathbb{R}^d\right)$. *Acta Math. Sci.,* 31B (2011), 141–158.

Continuity of Wigner-type operators on Lorentz spaces and Lorentz mixed normed modulation spaces

Ayse Sandikci

Let $\tau \in [0,1]$. For $f,g \in S(\mathbb{R}^d)$, the $\tau-$Wigner transform is defined as

$$W_\tau(f,g)(x,w) = \int_{\mathbb{R}^d} f(x+\tau t)\overline{g(x-(1-\tau)t)}e^{-2\pi i t w}dt.$$

If $\tau = \frac{1}{2}$, then the $\tau-$Wigner transform is the cross-Wigner distribution. Also let $a \in S(\mathbb{R}^{2d})$. Then, for $\tau \in [0,1]$, the $\tau-$ Weyl pseudo-differential operators with $\tau-$ symbol a

$$W_\tau^a : f \to W_\tau^a f(x) = \int_{\mathbb{R}^{2d}} e^{2\pi i(x-y)w} a((1-\tau)x+\tau y, w)f(y)dydw$$

is defined as continuous map from $S(\mathbb{R}^d)$ to itself (see $[1,2]$).
In the present paper we study various continuity properties for $\tau-$Wigner transform on Lorentz spaces and $\tau-$ Weyl operators W_τ^a with symbols belonging to appropriate Lorentz spaces. These extend the results in $[1,2]$ to the Lorentz spaces. We also study the action of $\tau-$Wigner transform on Lorentz mixed normed modulation spaces.
Some key references are given below.

[1] P. Boggiatto, G. De Donno, A. Oliaro: A class of quadratic time-frequency representations based on the short-time Fourier transform, *Oper Theor*, 172 (2007), 235-249.

[2] P. Boggiatto, G. De Donno, A. Oliaro: Time-frequency representations of Wigner type and pseudo-differential operators, *Trans Amer Math Soc*, 362 (2010), 4955-4981.

[3] K. Grochenig: Foundations of Time-Frequency Analysis, *Birkhauser*, 2001.

[4] A.T. Gurkanli: Time-frequency analysis and multipliers of the spaces $M(p,q)(\mathbb{R}^d)$ and $S(p,q)(\mathbb{R}^d)$, *J Math Kyoto Univ*, 46(3) (2006), 595-616.

[5] R. O'Neil: Integral transforms and tensor products on Orlicz spaces and $L(p,q)$ spaces, *J d'Analyse Math*, 21 (1968), 1-276.

[6] A. Sandikci: On Lorentz mixed normed modulation spaces, *J Pseudo- Differ Oper Appl*, 3 (2012), 263-281.

[7] A. Sandikci, A.T. Gurkanli: Generalized Sobolev-Shubin spaces, bound- edness and Schatten class properties of Toeplitz operators, *Turk J Math*, 37 (2013), 676-692.

Paley-Wiener theorems for function spaces of polyanalytic functions

Ana Moura Santos

In a earlier work, we established Paley-Wiener theorems for the true poly-Bergman and poly-Bergman spaces based on properties of the compression of the Beurling-Alhfors transform to the upper half-plane. Moreover, we were able to use the classical Paley-Wiener theorems to show that the poly-Bergman space of order j is isometrically isomorphic to j copies of the Hardy space. Now, reasoning in a analogous way, we want to present Paley-Wiener type theorems for polyharmonic Bergman and true polyharmonic Bergman spaces.
The talk is based on joint work with Luís V. Pessoa.

[1] L.V. Pessoa, A. Moura Santos: Theorems of Paley-Wiener type for spaces of polyanalytic functions, (2014), accepted to publication in the Proceedings of the 9th ISAAC Congress.

[2] L.V. Pessoa, Planar Beurling transform and Bergman type spaces, (2013), to appear in Complex Anal. Oper. Theory (DOI 10.1007/s11785-012-0268-0).

Sturm Liouville and differential operators

Tuesday afternoon first half

On the Sturm-Liouville operator with degenerate boundary conditions

Alexander Makin

Consider the Sturm-Liouville equation

$$u'' - q(x)u + \lambda u = 0 \tag{1}$$

with degenerate boundary conditions

$$u'(0) + du'(\pi) = 0, \quad u(0) - du(\pi) = 0, \tag{2}$$

where $d \neq 0$ and $q(x)$ is an arbitrary complex-valued function of class $L_1(0, \pi)$.
Completeness of the root function system of problem (1), (2) was investigated in [1]. Denote $Q(x) = q(x) - q(\pi - x)$. In particular, it was shown that if $q(x) \in C^k[0, \pi]$ for some $k \geq 0$, and $Q^{(k)}(\pi) \neq 0$, then the root function system is complete in $L_p(0, \pi)$ $(1 \leq p < \infty)$. Here we obtain a more general result.
Theorem. *If for a number $\rho > 0$*

$$\lim_{h \to 0} \frac{\int_{\pi-h}^{\pi} Q(x)dx}{h^\rho} = \nu,$$

and $\nu \neq 0$, then the root function system of problem (1), (2) is complete in the space $L_p(0, \pi)$ $(1 \leq p < \infty)$.
We also study the structure of the spectrum of problem (1), (2), corresponding inverse problem, and construct examples with nonclassical asymptotics of the spectrum.

[1] M. M. Malamud: On the Completeness of the System of Root Vectors of the Sturm–Liouville Operator with General Boundary Conditions, *Functional Analysis and Its Applications*, 42 (2008), 198-204.

Inverse spectral problems with varying transmission conditions

Casey Bartels

The inverse spectral problem of determining the potential in a Sturm-Liouville operator from two spectra generated by the same potential but with varied terminal condition has been well studied. However the inverse problem of determining the potential from two spectra for a Sturm-Liouville operators and the operator with the same boundary conditions but with potential differing from the original by a known perturbation has only recently been considered, see [1]. Related to this inverse problem is that of determining the potential for a Sturm-Liouville with transmission condition from two spectra generated by varying the transmission condition, which is the focus of this talk.
The talk is based on joint work with Bruce A Watson.

[1] W. Rundell, P. Sacks: An inverse eigenvalue problem for a vibrating string with two Dirichlet spectra, *SIAM J. Appl. Math.*, 73 (2013), 1020–1037.

Spectral problems in a domain with "trap"-like geometry of the boundary

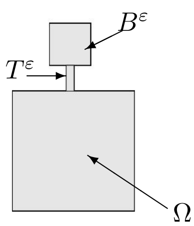
Andrii Khrabustovskyi

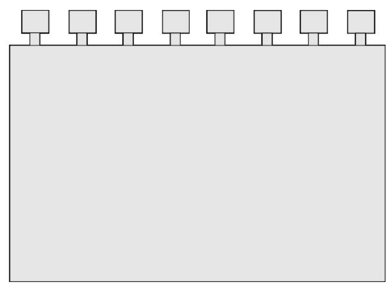
It is well known that under smooth perturbation of a domain, the eigenvalues of the Neumann Laplacian vary continuously. If the perturbation is only C^0, then, in general, this is not true.

The following example demonstrating this was considered in the classical book [1]. Let $\varepsilon > 0$ be a small parameter. Let Ω^ε be a domain consisting of a fixed domain Ω and a small domain (in what follows we will call such small domains as "traps"), which is a union a small square B^ε with a side length b^ε and a thin rectangle T^ε of the width d^ε and height h^ε (see the left figure). Here $d^\varepsilon = \varepsilon^4$, $h^\varepsilon = b^\varepsilon = \varepsilon$. The domain Ω^ε can be viewed as a C^0 perturbation of Ω. It was shown in [1] that the first principal eigenvalue of the Neumann Laplacian $-\Delta_{\Omega^\varepsilon}$ in Ω^ε goes to zero as $\varepsilon \to 0$, although the first principal eigenvalue of the Neumann Laplacian in Ω is positive.
In the present talk we consider the domain Ω^ε obtained by attaching to Ω many "traps" (see the right figure). Their number is finite for a fixed ε and goes to ∞ as $\varepsilon \to 0$. The traps are attached along a flat part of $\partial\Omega$ (we denote it Γ). We consider the operator

$$\mathcal{A}^\varepsilon = -\frac{1}{\rho^\varepsilon}\Delta_{\Omega^\varepsilon},$$

where the weight $\rho^\varepsilon(x)$ is positive and equal to 1 in Ω. Our goal is to study the behaviour of its spectrum as $\varepsilon \to 0$.





For a wide range of values of $d^\varepsilon,\ b^\varepsilon,\ h^\varepsilon$ and $\rho^\varepsilon|_{\Omega^\varepsilon\setminus\Omega}$ we prove that the spectrum of the operator $\mathcal{A}^\varepsilon$ converges as $\varepsilon \to 0$ to the spectrum of some operator $\mathcal{A}$ acting either in $L_2(\Omega)$ or in $L_2(\Omega)\oplus L_2(\Gamma)$. The form of the operator $\mathcal{A}$ depends on some relations between d^ε, b^ε, h^ε and $\rho^\varepsilon|_{\Omega^\varepsilon\setminus\Omega}$. In particular, in some cases $\mathcal{A}$ may have nonempty essential spectrum.
This is a joint work with Giuseppe Cardone (University of Sannio, Benevento, Italy). The work is supported by DFG via GRK 1294.

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Spectral analysis of integro-differential operators arising in the theory of viscoelasticity

Victor V. Vlasov and Nadezhda A. Rautian
The main purpose of our research is to study the qualitative and asymptotic behavior of the solutions of integro-differential equations arising in the theory of viscoelasticity on the base of spectral analysis of corresponding operator-valued functions. We obtain the correct solvability of the initial value problems for given integro-differential equations in the weighted Sobolev spaces on the positive semiaxis. We study the spectrum of the operator-valued functions which are the symbols of these integro-differential equations.

Differential operators

Monday afternoon

Self-adjoint quadratic operator pencils and applications

Manfred Möller

Let A, K, and M be self-adjoint operators in a Hilbert space H. We will give an overview of results on the spectral theory of the quadratic operator pencil

$$L(\lambda) = \lambda^2 M - i\lambda K - A,\ \lambda \in \mathbb{C}.$$

Due to the occurrence of the operator K, the spectra of such operators are no more located on the real and imaginary axes, in general, but still obey certain rules. Here we will assume that K and M are non-negative bounded operators, that $M + K$ is strictly positive, and that A is bounded below with compact resolvent. Then the spectrum of L is symmetric with respect to the imaginary axis and lies in the closed upper half-plane and on the imaginary axis. Particular properties of these eigenvalues will be discussed when K is a rank one operator.
Such quadratic operator pencils occur in mathematical models for problem in Mathematical Physics and Engineering, with the λ-quadratic term appearing in the differential equation and the λ-linear term appearing in the boundary conditions. Prime examples are are the Regge problem

$$\begin{gathered} -y'' + q(x)y = \lambda^2 y, \\ y(0) = 0, \\ y'(a) + i\lambda y(a) = 0, \end{gathered}$$

and a fourth order differential equation for damped vibrating beams

$$\begin{gathered} y^{(4)} - (gy')' = \lambda^2 y, \\ y(0) = 0, \quad y''(0) = 0, \quad y(a) = 0, \\ y''(a) + i\alpha\lambda y'(a) = 0. \end{gathered}$$

A comprehensive account of the theory and applications will be given in the forthcoming monograph [1].
The talk is based on joint work with V. Pivovarchik and B. Zinsou.

[1] M. Möller, V. Pivovarchik: *Spectral theory of operator pencils, Hermite-Biehler functions, and their applications.* Birkhäuser Verlag, in preparation.

[2] M. Möller, B. Zinsou: Self-adjoint fourth order differential operators with eigenvalue parameter dependent boundary conditions, *Quaest. Math.*, 34 (2011), 393–406.

[3] M. Möller, B. Zinsou: Sixth order differential operators with eigenvalue dependent boundary conditions, *Appl. Anal. Discrete Math.* 7 (2013), 378–389.

[4] V. Pivovarchik, C. van der Mee: The inverse generalized Regge problem, *Inverse Problems*, 17 (2001), 1831–1845.

Solvability of boundary value problems for kinetic operator-differential equations and related questions

Sergey Pyatkov
We study boundary value problems for the operator-differential equations

$$Mu \equiv B(t)u_t - L(t)u = f(t), \quad t \in (0,T), \quad T \leq \infty \tag{1}$$

where $L(t) : E \to E$ and $B(t) : E \to E$ are families of linear operators defined in a complex Hilbert space E. We do not assume that B is invertible and it is possible that the spectrum of the pencil $L - \lambda B$ includes infinite subsets of the left and right half-planes simultaneously. Equations of this type arise in physics (in particular, in problems of neutron transport, radiative transfer, and rarefied gas dynamics), geometry, population dynamics, hydrodynamics, and in some other fields. The equations (1) are sometimes called kinetic equations and the operator L the collision operator. It is often the case when the operators L, B are assumed to be selfadjoint. We assume that the operators $B(0) : E \to E$ and $B(T) : E \to E$ (if $T < \infty$) are selfadjoint. In this case we can define the spectral projection $E^{\pm}(0)$, $E^{\pm}(T)$ of these operators corresponding to the positive and negative parts of the spectrum of $B(0)$ and $B(T)$, respectively. Thus, we have $E_0^{\pm} B(0) = B(0) E_0^{\pm}$, $(E^+ - E^-)B(0) = |B(0)|$. We supplement the equation (1) with the boundary conditions

$$E^+(0)u(0) = u_0^+, \quad \lim_{t \to \infty} u(t) = 0 \quad (T = \infty). \tag{2}$$

$$\begin{aligned} E^+(0)u(0) &= h_{11}E^-(0)u(0) + h_{12}E^+(T)u(T) + u_0^+, \\ E^-(T)u(T) &= h_{21}E^-(0)u(0) + h_{22}E^+(T)u(T) + u_T^- \quad (T < \infty), \end{aligned} \tag{3}$$

where h_{ij} are linear operators. Under certain conditions on the operators L, B, h_{ij} (the operator L is assumed to be dissipative), we demonstrate that the problems (1), (2) and (1), (3) are solvable and present the conditions ensuring the uniqueness of solutions and their smoothness in t. Moreover, we present some examples which are forward-backward parabolic equations.

Borg's periodicity theorem for Hermitian systems in $\mathbb{C}^2$

Thomas Roth

The self-adjoint canonical system with Hermitian π-periodic potential $Q(z)$ is considered integrable on $[0, \pi)$. It is shown that all zeros of $\Delta + 2$ are double zeros if and only if this Hermitian system is unitarily equivalent to one in which $Q(z)$ is $\frac{\pi}{2}$-periodic. Furthermore, the zeros of $\Delta - 2$ are all double if and only if its Hermitian system is unitarily equivalent to one in which $Q(z)$ is $\frac{\pi}{2}$-σ_2-similar. Finally, it is shown that all instability intervals vanish if and only if $Q = p\sigma_0 + r\sigma_2$. This talk is based on joint work with Sonja Currie, Bruce A. Watson.

[1] H. Hochstadt, On a Hill's Equation with double Eigenvalues, *Proceedings of the American Math. Soc.*, 65 (1977) 373–374.

[2] S. Currie, B. A. Watson, T. T. Roth Canonical systems in $\mathbb{R}^2$ with vanishing instability intervals, (not yet Accepted).

[3] E. A. Coddington, N. Levinson, *Theory of ordinary differential equations*, McGraw-Hill Publishing, 1955.

[4] E. J. McShane, *Integration*, Princeton University Press, 1944.

[5] L. Hörmander, *Lectures on nonlinear hyperbolic differential equations*, Mathématiques & Applications 26, Springer Verlag, 1997.

[6] G. Borg, Eine Umkehrung der Sturm-Liouvilleschen Eigenwertaufgabe, Bestimmung der Differentialgleichung durch die Eigenwerte, *Acta Math.*, 78 (1946) 1–96.

Basis properties in a problem of a nonhomogeneous string with damping at the end

Lukasz Rzepnicki

This talk is concerned with the equation of a nonhomogeneous string of length one, which is fixed at the one end and damped into another with a parameter $h \in \mathbb{C}$. This problem can be rewritten as an abstract Cauchy problem for a densely defined, closed, non-selfadjoint operator A_h acting on an appropriate energy Hilbert space H. Under assumptions that the density function of the string $\rho \in W_2^1[0,1]$ is strictly positive and has $\rho(1) \neq h^2$ (if $h \in \mathbb{R}$), we prove that the set of root vectors of A_h form basis with parentheses in H. With the additional condition

$$\int_0^1 \frac{\omega_1^2(\rho', \tau)}{\tau^2}\, d\tau \, < \, \infty,$$

where ω_1 is the integral modulus of continuity, we show that the root vectors of the operator A_h form Riesz basis in H.
The talk is based on joint work with Alexander Gomilko.

Asymptotics of the eigenvalues of a self-adjoint fourth order boundary value problem with four eigenvalue parameter dependent boundary conditions

Bertin Zinsou

Considered is a regular fourth order differential equation which depends quadratically on the eigenvalue parameter λ and which has separable boundary conditions depending linearly on λ. It is shown that the eigenvalues lie in the closed upper half plane or on the imaginary axis and are symmetric with respect to the imaginary axis. The first four terms in the asymptotic expansion of the eigenvalues are provided. The talk is based on joint work with Manfred Möller.

[1] M. Möller and B. Zinsou: "Asymptotics of the eigenvalues of a self-adjoint fourth order boundary value problem with four eigenvalue parameter dependent boundary conditions". *J. Funct. Spaces Appl.* (2013), dx. doi.org/10.1155/2013/280970.

[2] M. Möller and B. Zinsou: "Spectral asymptotics of self-adjoint fourth order boundary value problem with eigenvalue parameter dependent boundary conditions". *Boundary Value Problems*, doi: 10.1186/1687-2770-2012-106.

[3] M. Möller and B. Zinsou: "Self-adjoint Fourth Order Differential Operators With Eigenvalue Parameter Dependent Boundary Conditions". *Quaestiones Math.*, **34**, 393–406 (2011), doi: 10.2989/16073606.2011.622913.

[4] M. Möller and V. Pivovarchik: Spectral properties of a fourth order differential equation. *J. Anal. Appl.*, Vol. **25**, 341–366 (2006).

Asymptotics of eigenvalues of some high-order differential operator with discrete self-similar weight

Igor Sheipak

We study the spectral boundary problem

$$(-1)^n \, y^{(2n)} = \lambda \rho y,$$
$$y^{(k)}(0) = y^{(k)}(1) = 0, \qquad 0 \leq k < n$$

where weight function ρ is a distribution from Sobolev space with negative smoothness. Under the assumption that generalized primitive of ρ is a function with degenerate self-similarity it is proved that the set of eigenvalues can be represented by several series with exponential growth. The order of growth and number of series are calculated via parameters of weight ρ.
By the same technique the asymptotics of counting function of eigenvalues is obtained for wider class of self-adjoint problems

$$(-1)^n y^{(2n)} + \left(p_{n-1} y^{(n-1)}\right)^{(n-1)} + \cdots + p_0 y = \lambda \rho y$$

with suitable boundary conditions.
The talk is based on joint works with A.Vladimirov and A.Nazarov.

Weyl function for sum of operator tensor product

Anton Boitsev

Spectral theory of differential operators is very important for mathematics and has many applications in quantum physics. The theory of self-adjoint operators and especially of self-adjoint extensions of symmetric operators takes special place in the operator theory. In many interesting problems of quantum physics (like the interaction of photons with electrons) the operators of the form of sum of tensor products occur. Up to this moment, the method of getting all self-adjoint extensions of such an operator has not been described. In particular, we consider a closed densely defined symmetric operator $S = A \otimes I_T + I_A \otimes T$, where A is a closed densely defined symmetric operator on the separable Hilbert space $\mathfrak{H}_A$ and T is a bounded self-adjoint operator acting on the separable infinite dimensional Hilbert space $\mathfrak{H}_T$. Notice that the deficiency indices of S are infinite even if A has finite deficiency indices. Our aim is to describe all self-adjoint extensions of S using the boundary triplet approach. More precisely, assuming that $\Pi_A = \{\mathcal{H}_A, \Gamma_0^A, \Gamma_1^A\}$ is a boundary triplet for A^* we construct a boundary triplet $\Pi_S = \{\mathcal{H}_S, \Gamma_0^S, \Gamma_1^S\}$ for S^*. In addition, using the γ-field $\gamma_A(\cdot)$ and the Weyl function $M_A(\cdot)$ of the boundary triplet Π_A we express the γ-field $\gamma_S(\cdot)$ and Weyl function $M_S(\cdot)$ of Π_S.
The talk is based on joint work with Hagen Neidhardt and Igor Popov.

[1] M.Malamud, H.Neidhardt: Sturm-liouville boundary value problems with operator potentials and unitary equivalence, *J. Differ. Equations*, 252(11):5875–5922, 2012.

[2] A.A. Boitsev, H.Neidhardt, I. Yu. Popov: Weyl function for sum of operators tensor products, *Nanosystems: physics, chemistry, mathematics*, 4(6):747–759,20138

Trajectory tracking for the heat equation with colocated boundary control and observation

Tilman Selig

On a bounded domain $\Omega \subset \mathbb{R}^n$ with smooth boundary $\partial\Omega$ and output normal ν we consider the following heat equation

$$\begin{aligned} \frac{\partial x}{\partial t}(\xi,t) &= \Delta_\xi x(\xi,t), && (\xi,t) \in \Omega \times \mathbb{R}_{\geq 0}, \\ u(t) &= \nu^\top(\xi)\nabla_\xi x(\xi,t), && (\xi,t) \in \partial\Omega \times \mathbb{R}_{\geq 0}, \\ y(t) &= \int_{\partial\Omega} x(\xi,t)\mathrm{d}\sigma_\xi, && (\xi,t) \in \partial\Omega \times \mathbb{R}_{\geq 0}, \\ x(\xi,0) &= x_0(\xi), && \xi \in \Omega. \end{aligned}$$

In [1] this system of equations was shown to constitute a regular well-posed linear system with real-valued input u and output y. We prove that, by application of the so-called funnel controller which is a special nonlinear output-feedback introduced in [2], it is possible to stabilize this system and have the signal y follow a prescribed reference trajectory $y_{\mathrm{ref}} \in W^{1,\infty}(\mathbb{R}_{\geq 0})$ in the following sense: The norm of the state variable x satisfies $\sup_{t>0} \|x(\cdot,t)\|_{L^2(\Omega)} < \infty$, and $|y(t) - y_{\mathrm{ref}}(t)| \leq \frac{1}{\varphi(t)}$ for all $t > 0$, where $\varphi \in W^{1,\infty}(\mathbb{R}_{\geq 0})$ can be chosen almost arbitrarily.
The talk is based on joint work with Prof. Dr. Timo Reis.

[1] C. I. Byrnes, D. S. Gilliam, V. I. Shubov, and G. Weiss: *Regular linear systems governed by a boundary controlled heat equation*, Journal of Dynamical and Control Systems, 8 (2002), 341–370.

[2] A. Ilchmann, E. P. Ryan, and C. J. Sangwin: *Tracking with prescribed transient behaviour*, ESAIM: Control, Optimisation and Calculus of Variations, 7 (2002), 471–493.

Operator Theory

Monday afternoon

Operator theory on the tetrablock

Tirthankar Bhattacharyya

In this talk, we discuss structure theory for a triple of commuting bounded operators which has the tetrablock as a spectral set.

Sectorial linear relations and associated multivalued forms

Gerald Wanjala

Let $\mathcal{H}$ be a Hilbert space with inner product $\langle\cdot,\cdot\rangle$ and let $\mathcal{H}^2$ be the Hilbert space $\mathcal{H}^2 = \mathcal{H} \oplus \mathcal{H}$. For a linear relation $\mathbf{S}$ on $\mathcal{H}^2$ consider its numerical range $\mathcal{W}(\mathbf{S}) = \{\langle u', u\rangle : (u, u') \in \mathbf{S},\ \|u\| = 1\}$. By analogy with linear operators ([1]), we say that $\mathbf{S}$ is *accretive* if $\mathcal{W}(\mathbf{S}) \subset \{\xi \in \mathbb{C} : \operatorname{Re} \xi \geq 0\}$ and that it is *sectorial* if it is accretive and $\mathcal{W}(\mathbf{S})$ is contained in the sector

$$|\arg\,(z-\gamma)| \leq \theta < \frac{\pi}{2},\ \gamma \in \mathbb{R}.$$

In this talk we discuss results related to Kato's representation theorems (see [1]) for sectorial linear relations and the associated multivalued forms.

[1] T. Kato: Perturbation theory for linear operators, *Springer-Verlag, Berlin-Heidelberg-New York*, (1980).

Conservative L-systems and the Livšic function

Sergey Belyi

We study the connections among: (i) the Livšic class of functions $s(z)$ that are the characteristic functions of a densely defined symmetric operators $\dot{A}$ with deficiency indices $(1,1)$; (ii) the characteristic functions $S(z)$ (the Möbius transform of $s(z)$) of a maximal dissipative extension T of $\dot{A}$ determined by the von Neumann parameter κ; (iii) the transfer functions $W_\Theta(z)$ of conservative L-systems Θ with the main operator T of the system. It is shown that under some natural hypothesis $S(z)$ and $W_\Theta(z)$ are reciprocal to each other. We establish that the impedance function of a conservative L-system coincides with the function from the Donoghue class if and only if the von Neumann parameter vanishes ($\kappa = 0$). Moreover, we introduce the generalized Donoghue class and provide the criteria for an impedance function to belong to this class.
The talk is based on joint work with K. A. Makarov and E. Tsekanovskiĭ (see also references below).

[1] Yu. Arlinskiĭ, S. Belyi, E. Tsekanovskiĭ: *Conservative Realizations of Herglotz-Nevanlinna functions*, Operator Theory: Advances and Applications, Vol. 217, Birkhäuser Verlag, 2011.

[2] K. A. Makarov and E. Tsekanovskiĭ: On the Weyl-Titchmarsh and Livšic functions, Proceedings of Symposia in Pure Mathematics, American Mathematical Society, 87 (2013), 291–313.

[3] K. A. Makarov and E. Tsekanovskiĭ: On the addition and multiplication theorems, arXiv:1310.8504 [math.SP], (to appear).

Stability of collocation methods for singular integral equations

Peter Junghanns

We give an overview on the development of the theory of collocation methods for some classes of singular integral equations. We focus on the application of C*-algebra techniques to prove the stability of such methods, where the integral equations are given on an interval. In particular, we deal with Cauchy singular integral equations with additional fixed singularities, for example of the form

$$a(x)u(x) + \frac{b(x)}{\pi \mathbf{i}} \int_{-1}^{1} \frac{u(y)\,dy}{y-x} + \sum_{k=1}^{m} \frac{\beta_k}{\pi \mathbf{i}} \int_{-1}^{1} \frac{(1+x)^{k-1} u(y)\,dy}{(y+x+2)^k} = f(x)\,.$$

Moreover, we apply our results to certain problems of two-dimensional elasticity theory, for example to the notched half plane problem and to the problem of a cut at the surface of a circular hole in an elastic plane.

On rational approximation on discrete sets

Vasiliy Prokhorov

In our talk we discuss questions related to the discrete Hankel operator and problems of rational approximation of functions given on a finite set E_N of points on the complex plane. We establish a connection between the lowest singular number of the discrete Hankel operator and the error of rational approximation (an analog of the AAK theorem). Moreover, we present results related to rational approximation of the Markov functions (Cauchy transforms of a positive Borel measure with compact support on the real line) on a finite set of points of the real line. We explore the degree of rational approximation of the Markov functions on discrete systems points (when a number of points N in the set E_N and degree of rational approximants n satisfy an asymptotic relation $N/n \to \theta > 2$). The degree of rational approximation can be described in terms of solutions of certain potential-theoretic problems, central among which is the constrained minimal energy problem.

The spectral problem for a Jacobi matrix related to the Hahn-Exton q-Bessel function

Pavel Stovicek

A family $\mathcal{T}^{(\nu)}$, $\nu \in \mathbb{R}$, of semiinfinite positive Jacobi matrices is introduced with matrix entries taken from the Hahn-Exton q-difference equation. The corresponding matrix operators defined on the linear hull of the canonical basis in $\ell^2(\mathbb{Z}_+)$ are essentially self-adjoint for $|\nu| \geq 1$ and have deficiency indices $(1,1)$ for $|\nu| < 1$.
A convenient description of all self-adjoint extensions is obtained and the spectral problem is analyzed in detail. The spectrum is discrete and the characteristic equation on eigenvalues is derived explicitly in all cases. Particularly, the Hahn-Exton q-Bessel function $J_\nu(z;q)$ serves as the characteristic function of the Friedrichs extension. As a direct application one can reproduce, in an alternative way, some basic results about the q-Bessel function due to Koelink and Swarttouw. The formal eigenvalue equation for the Jacobi matrix can be regarded as a three-term recurrence relation defining, as usual, a sequence of orthogonal polynomials. In case the Jacobi matrix operator is not essentially self-adjoint the orthogonality measure for the sequence of polynomials is indeterminate. The Nevanlinna parametrization of all possible orthogonality measures is derived explicitly.
These results extend and complete some recent results reported in [1], [2], where a number of examples of Jacobi operators has been constructed with discrete spectra and characteristic functions explicitly expressed in terms of various special functions.
The talk is based on joint work with Frantisek Stampach.

[1] F. Stampach, P. Stovicek: The characteristic function for Jacobi matrices with applications, *Linear Alg. Appl.*, 438 (2013), 4130-4155.

[2] F. Stampach, P. Stovicek: Special functions and spectrum of Jacobi matrices, *Linear Alg. Appl.*, (2014) in press, available online:
http://dx.doi.org/10.1016/j.laa.2013.06.024.

Additive mappings preserving certain local spectral quantities of operators

Mustapha Sarih

Let $\mathcal{L}(X)$ be the algebra of all bounded linear operators on a complex Banach space X. We describe additive maps from $\mathcal{L}(X)$ onto itself that preserve some local spectral quantities of operators. Extensions of this result to the case of different Banach spaces are also established.

Operator Theory

Tuesday afternoon second half

Regular and singular operators in rigged Hilbert spaces

Salvatore Di Bella

Spaces of linear maps acting on a *rigged Hilbert space* (RHS, for short)

$$\mathcal{D} \subset \mathcal{H} \subset \mathcal{D}^{\times}$$

have often been considered in the literature both from a pure mathematical point of view and for their applications to quantum theories (generalized eigenvalues, resonances of Schrödinger operators, quantum fields...). The spaces of test functions and the distributions over them constitute relevant examples of rigged Hilbert spaces and operators acting on them are a fundamental tool in several problems in Analysis (differential operators with singular coefficients, Fourier transforms) and also provide the basic background for the study of the problem of the multiplication of distributions by the duality method.

A notion of regularity and singularity for a special class of operators acting in a rigged Hilbert space $\mathcal{D} \subset \mathcal{H} \subset \mathcal{D}^{\times}$ is proposed. It is strictly related to the corresponding notion for sesquilinear form. A particular attention is devoted to those operators that are neither regular nor singular, pointing out that a part of them can be seen as a perturbation of a self-adjoint operator on $\mathcal{H}$. Some properties for such operators are derived and some examples are discussed.

The talk is based on the paper *Regular and singular operators in Rigged Hilbert spaces. (submitted).*

Operators defined by generalized Riesz bases in rigged Hilbert spaces

Giorgia Bellomonte

Let now $\mathcal{D}[t] \subset \mathcal{H} \subset \mathcal{D}^{\times}[t^{\times}]$ be a rigged Hilbert space and $\{\xi_n\}$ a basis for $\mathcal{D}[t]$ then, for every $f \in \mathcal{D}$, there exist coefficients $c_n = c_n(f)$ such that $\sum_{n=1}^{\infty} c_n(f)\xi_n$; let $c_n(f)$ be continuous functionals in $\mathcal{D}[t]$ and consider the sequence $\{\zeta_k\} \subset \mathcal{D}^{\times}$ which is biorthogonal to $\{\xi_n\}$.

The previous basis $\{\xi_n\}$ for $\mathcal{D}$ is a *generalized Riesz basis* if there exists a linear map $T : \mathcal{D} \to \mathcal{H}$ continuous from $\mathcal{D}[t]$ into $\mathcal{H}$, with continuous inverse, such that $\{T\xi_n\}$ is an orthonormal basis for the central Hilbert space $\mathcal{H}$.

Now we can consider the simplest operators that can be defined via a generalized Riesz basis $\{\xi_n\}$ and its dual basis $\{\zeta_n\}$. Let $\{\alpha_n\}$ is a sequence of complex numbers. We can formally define, for $f \in \mathcal{D}$,

$$A^{\boldsymbol{\alpha}} f = \sum_{n=1}^{\infty} \alpha_n \overline{\langle \zeta_n | f \rangle} \xi_n \quad (1), \qquad B^{\boldsymbol{\alpha}} f = \sum_{n=1}^{\infty} \alpha_n \langle f | \xi_n \rangle \zeta_n \quad (2)$$

$$R^{\boldsymbol{\alpha}} f = \sum_{n=1}^{\infty} \alpha_n \langle f | \xi_n \rangle \xi_n \quad (3), \qquad Q^{\boldsymbol{\alpha}} f = \sum_{n=1}^{\infty} \alpha_n \overline{\langle \zeta_n | f \rangle} \zeta_n \quad (4)$$

We study operators $(1) - (4)$ and determine some their properties, among which there are their dense domain definition, their closedness, conditions for their boundedness and the relations between them.

The talk is based on the joint paper with Camillo Trapani *Generalized Riesz bases in rigged Hilbert spaces. (submitted).*

Some operator techniques in wavelet theory

Fernando Gómez-Cubillo

Wavelets were introduced synthesizing ideas originated in engineering, physics and pure mathematics. Wavelet bases are built by application of translations and dilations to an appropriate function. A way to construct wavelets and obtain efficient algorithms is given by the concept of multiresolution analysis (MRA). Along the literature, the transfer functions of the filters associated with an MRA have been the keystone for constructing compactly supported orthonormal wavelets, as well as the study of its properties.
Almost all wavelet theory has been developed on the basis of Fourier transform, which diagonalizes the translation operator T. In this work explicit spectral representations of both, the dilation operator D and the translation operator T, are built on the basis of orthonormal bases with appropriate structure. This permit us to characterize orthonormal wavelets and MRA in terms of wandering and invariant subspaces and by means of rigid (operator-valued) functions defined on the functional spectral spaces of D and T.
The approach is useful for computational purposes. To illustrate this, we derive new methods for constructing compactly supported orthonormal wavelets.
The talk is based on joint work with Z. Suchanecki and S. Villullas.

[1] F. Gómez-Cubillo, Z. Suchanecki: Spectral models for orthonormal wavelets and multiresolution analysis of $L^2(\mathbb{R})$, *J. Fourier Anal. Appl.*, 17 (2011), 191–255.

[2] F. Gómez-Cubillo, Z. Suchanecki: Inner functions and local shape of orthonormal wavelets, *Appl. Comput. Harmon. Anal.*, 30 (2011), 273–287.

[3] F. Gómez-Cubillo, Z. Suchanecki, S. Villullas: Orthonormal MRA wavelets: spectral formulas and algorithms, *Int. J. Wavelets Multiresolut. Inf. Process.*, 10 (2012), 1250008 (19pp.).

[4] F. Gómez-Cubillo, Z. Suchanecki, S. Villullas: Spectral algorithms for orthonormal MRA wavelets, *preprint*.

Complex symmetric operators and their rank one perturbations

Ji Eun Lee

In this paper, we study properties of complex symmetric operator and its rank one perturbations $R = T + u \otimes v$. In particular, we prove that if T is a complex symmetric operator, then the spectral radius algebra $\mathcal{B}_T$ associated to T has a nontrivial invariant subspace under some conditions. Moreover, we consider decomposability of rank one perturbations of complex symmetric operator R. Finally, we find some conditions so that R satisfies a-Weyl's theorem.
The talk is based on joint work with Eungil Ko.
This research was supported by Basic Science Research Program through the National Research Foundation of Korea(NRF) funded by the Ministry of Education, Science and Technology (2012R1A1A3006841).

Operator Theory

Thursday afternoon second half

Some spectral properties of generalized derivation

Mohamed Amouch

Given Banach spaces $\mathcal{X}$ and $\mathcal{Y}$ and Banach space operators $A \in L(\mathcal{X})$ and $B \in L(\mathcal{Y})$, let $\rho\colon L(\mathcal{Y},\mathcal{X}) \to L(\mathcal{Y},\mathcal{X})$ denote the generalized derivation defined by A and B, i.e., $\rho(U) = AU - UB$ ($U \in L(\mathcal{Y},\mathcal{X})$). The main objective of this article is to study Weyl and Browder type theorems for $\rho \in L(L(\mathcal{Y},\mathcal{X}))$. To this end, however, first the isolated points of the spectrum and the Drazin spectrum of $\rho \in L(L(\mathcal{Y},\mathcal{X}))$ need to be characterized. In addition, it will be also proved that if A and B are polaroid (respectively isoloid), then ρ is polaroid (respectively isoloid).The talk is based on joint work with Enrico Boasso.

[1] E. Boasso and M. AMOUCH : Generalized Browder's and Weyl's Theorems for Generalized Derivations, *mediterranean journal of mathematics*, (2014).

Derivable maps and generalized derivations

Zhidong Pan

Let $\mathcal{A}$ be a unital algebra, $\mathcal{M}$ be an $\mathcal{A}$-bimodule, and $L(\mathcal{A},\mathcal{M})$ be the set of all linear maps from $\mathcal{A}$ to $\mathcal{M}$. A relation $\mathcal{R}_{\mathcal{A}}$ on $\mathcal{A}$ is a nonempty subset of $\mathcal{A} \times \mathcal{A}$. We say $\delta \in L(\mathcal{A},\mathcal{M})$ is derivable on $\mathcal{R}_{\mathcal{A}}$ if $\delta(AB) = \delta(A)B + A\delta(B)$ for all $(A,B) \in \mathcal{R}_{\mathcal{A}}$ and δ is called a derivation if it is derivable on $\mathcal{A} \times \mathcal{A}$. Note that Jordan derivations are precisely those linear maps that are derivable on $\mathcal{R}_{\mathcal{A}} = \{(A,B) \in \mathcal{A} \times \mathcal{A} : A = B\}$. For any $M \in \mathcal{M}$, a map $M_r \in L(\mathcal{A},\mathcal{M})$ is called a right multiplier if $M_r(A) = AM,\ \forall\ A \in \mathcal{A}$ and a map $M_l \in L(\mathcal{A},\mathcal{M})$ is called a left multiplier if $M_l(A) = MA,\ \forall\ A \in \mathcal{A}$. A generalized derivation is a sum of a derivation and a right (or left) multiplier. In this talk, we present a new relation, naturally related to generalized derivations, and show that for certain operator algebras $\mathcal{A}$ on a Hilbert space H, generalized derivations in $L(\mathcal{A}, B(H))$ can be characterized precisely as derivable maps on such a relation.

Cauchy theorem for a surface integral for extended classes of surfaces and functions in a finite-dimensional commutative algebra

Sergiy Plaksa

It is well known that in the case where a simply connected domain has a closed piece-smooth boundary, spatial analogues of the Cauchy integral theorem can be obtained with using the classical Gauss – Ostrogradskii formula, if a given function has specifically continuous partial derivatives of the first order up to the boundary (see, e.g., [1, p. 66] for the quaternion algebra and [2, p. 52] for Clifford algebras).
Generalizations of the Cauchy integral theorem have relations to weakening requirements to the boundary or the given function. Usually, such generalizations are based on generalized Gauss – Ostrogradskii – Green – Stokes formula (see, e.g., [3]) under the condition of continuity of partial derivatives of the given function, but for extended classes of surfaces of integration; see, e.g., [4], where regular surfaces are considered. In the papers [5, 6] the continuity of partial derivatives is changed by a differentiability of components of the given function taking values in the quaternion algebra.
We proved an analogue of the Cauchy integral theorem for extended classes of non piece-smooth three-dimensional surfaces of integration and for functions which take values in an arbitrary finite-dimensional commutative associative algebra and have differentiable components, but not continuously differentiable.
The talk is based on joint work with V. Shpakivskyi [7].

[1] V. V. Kravchenko, M. V. Shapiro: *Integral representations for spatial models of mathematical physics.* Pitman Research Notes in Mathematics, Addison Wesley Longman Inc., 1996.

[2] F. Brackx, R. Delanghe, F. Sommen: *Clifford analysis.* Pitman Research Notes in Mathematics, Boston, MA, 1982.

[3] J. Harrison, A. Norton: The Gauss – Green theorem for fractal boundaries, *Duke Math. J.*, **67** (1992), 575–588.

[4] R. Abreu Blaya, D. Peña Peña, J. Bory Reyes: Clifford Cauchy type integrals on Ahlfors–David regular surfaces in $\mathbb{R}^{m+1}$, *Adv. Appl. Cliff. Alg.*, **13**, 2 (2003), 133–156.

[5] A. Sudbery: Quaternionic analysis, *Math. Proc. Camb. Phil. Soc.*, **85** (1979), 199–225.

[6] O. F. Herus: On hyperholomorphic functions of the space variable, *Ukr. Math. J.*, **63**, 4 (2011), 530–537.

[7] S. A. Plaksa, V. S. Shpakivskyi: Cauchy theorem for a surface integral in commutative algebras, *Complex Variables and Elliptic Equations*, **59**, 1 (2014), 110–119.

Poletsky Stessin Hardy spaces in the plane

Nihat Gokhan Gogus

Recently weighted Hardy spaces were introduced by E. A. Poletsky and M. Stessin to generalize the notion of the classical Hardy spaces to more general hyperconvex domains and then to study the composition operators generated by the holomorphic mappings between such domains. In this talk we extend the classical result of Beurling which describes the invariant subspaces of the shift operator on these spaces in the plane.
The talk is based on joint work with Muhammed Ali Alan.

[1] E. A. Poletsky, M. I. Stessin: Hardy and Bergman spaces on hyperconvex domains and their composition operators, *Indiana Univ. Math. J.* 57 (2008), no. 5, 2153–2201.

[2] M. A. Alan, N. G. Göğüş: Poletsky-Stessin-Hardy spaces in the plane, *to appear in Complex Analysis and Operator Theory*, DOI:10.1007/s11785-013-0334-2.

Operator Theory

Friday afternoon

Spectra of random operators with absolutely continuous integrated density of states

Rafael del Rio

The structure of the spectrum of random operators is studied. It is shown that if the density of states measure of some subsets of the spectrum is zero, then these subsets are empty. In particular follows that absolute continuity of the IDS implies singular spectra of ergodic operators is either empty or of positive measure. Our results apply to Anderson and alloy type models, perturbed Landau Hamiltonians, almost periodic potentials and models which are not ergodic.

Conditional expectation operators in abstract L^2-type spaces

Wen-Chi Kuo

Various authors have considered generalizations of stochastic processes to vector lattices / Riesz spaces, with a variety of assumptions being made on the processes being considered. Most of this work has focused on martingale theory. The abstract properties of conditional expectation operators have also been explored in various settings. However, some of the more elementary processes such as Bernoulli processes and Poisson processes, which rely only on the concepts of a conditional expecation operator and independence, have received little attention. As these processes have little structure, their study relies heavily on properties of the underlying space, the representation of the conditional expectation operators and multiplication operations. Of particular importance is an understanding of the action of conditional expectations operators in the Riesz space vector analogue of L^2 and their averaging properties, which forms the focus of this talk.
The talk is based on joint work with Bruce A Watson and Jessica J. Vardy.

L^p-behavior of reproducing Bergman kernels

José Ángel Peláez

An explicit expression is known for the reproducing kernels of some classical Hilbert spaces of analytic functions, such as the Hardy space H^2 or a standard Bergman space A^2_α. However, this is not true for the reproducing kernels B^ω_ζ of a Bergman space A^2_ω, induced by a weight ω.
In spite of this obstacle, we shall describe the asymptotic L^p_v-behavior of B^ω_ζ (and their derivatives), where v, ω belong to any of both classes of radial weights considered in [2, Section 1.2]. We shall apply this result to study the action of the Bergman projection

$$P_\omega(f)(z) = \int_{\mathbb{D}} f(\zeta) B^\omega_\zeta(z)\, \omega(\zeta) dA(\zeta)$$

on L^p_v. In particular, we solve the two weight problem

$$\|P_\omega(f)\|_{L^p_v} \lesssim \|f\|_{L^p_v}, \quad f \in L^p_v, \quad p > 1,$$

for a large class of radial weights.
Finally, if we are in time, we shall present a characterization those positive measures μ for which the Toeplitz operator

$$T_\mu(f)(z) = \int_{\mathbb{D}} f(\zeta) B^\omega_\zeta(z)\, d\mu(\zeta).$$

belongs to the Schatten p-class $\mathcal{S}_p(A^2_\omega)$.
The talk is based on joint work with J. Rättyä.

[1] M. Dostanic, Unboundedness of the Bergman projections on L^p spaces with exponential weights, Proc. Edinb. Math. Soc. 47 (2004), 111–117.

[2] J. A. Peláez and J. Rättyä, Weighted Bergman spaces induced by rapidly increasing weights, Mem. Amer. Math. Soc. Vol. 227, n. 1066, (2014).

[3] J. A. Peláez and J. Rättyä, L^p-behavior of Bergman reproducing kernels, preprint (submitted).

Quaternionic harmonic Bergman spaces $\mathcal{A}(p, q, s)$

Luis M. Tovar S.

In this paper we develop the necessary tools to generalize the $F(p, q, s)$ functions spaces of analytic functions introduced by Zhao, to the case of quaternionic harmonic functions defined in the unit ball of $\mathbb{R}^4$.
The talk is based on joint work with Jorge Pérez H. and Lino F. Reséndis O.

Reconstruction of monogenic signals on the unit ball via Riemann-Hilbert approach

Min Ku

While it is well-known that to reconstruct analytic signals could be equal to solve a Riemann-Hilbert problem for Hardy spaces in the plane, there is not much known about the case of monogenic signals in three dimensions up to now. Our motivation is to reconstruct monogenic signals in terms of the study of Riemann-Hilbert boundary value problems for Hardy spaces in higher dimensional space. In this talk, we mainly focus on our recent work about the Riemann-Hilbert boundary value problems for poly-Hardy spaces on the unit ball of higher dimensional Euclidean space. As a special case, monogenic signals for Hardy space on the unit sphere will be reconstructed when the boundary data are given, which is the generalization of analytic signals for Hardy space on the unit circle of complex plane. We discuss the boundary behavior of functions in the poly-Hardy class, construct the Schwartz kernel function and the higher order Schwartz operator to study the Riemann-Hilbert boundary value problems for Hardy class and poly-Hardy class on the unit ball of higher dimensions, and obtain the expressions of solutions explicitly.

On L^p-projections and hyperstonean spaces

Banu Aytar Güntürk

It is a well-known fact that the Stonean space of the Boolean algebra of the L^p-projections of a classical Banach space $L^p(\mu)$, $1 \leq p < \infty$, $p \neq 2$, is hyperstonean and conversely, every hyperstonean space is obtained in this way. These projections are known to be pseudocharacteristic in general.
In this article we first show that the L^p-projections are actually characteristic projections, and then use them to give a second proof for the characterization of the surjective linear isometries of Bochner spaces.
The talk is based on joint work with Bahaettin Cengiz.

Operator Theory

Friday afternoon

Functional models and minimal contractive liftings

Rolf Gohm

Based on the analysis of functional models for contractive multi-analytic operators we establish a one-to-one correspondence between unitary equivalence classes of minimal contractive liftings of a row contraction and injective symbols of contractive multi-analytic operators. This allows an effective construction and classification of all such liftings with given defects. Popescu's theory of characteristic functions of completely non-coisometric row contractions is obtained as a special case satisfying a Szegö type condition. In another special case, single contractions and defects equal to 1, all scalar non-zero Schur functions on the unit disk appear in the classification. It is also shown that the process of constructing liftings iteratively reflects itself in a factorization of the corresponding symbols.
The talk is based on joint work with S. Dey and K.J. Haria.

[1] S.Dey, R.Gohm, K.J.Haria: Functional models and minimal contractive liftings, preprint arXiv:1402.4220

On the maximal function of a contraction operator

Ilie Valusescu

The maximal function of a contraction operator $T \in \mathcal{L}(\mathcal{H})$ arizes in the factorization process of an operator valued semispectral measure, i.e. it is the L^2-bounded analytic function attached to T and has the form $M_T(\lambda) = D_{T^*}(I - \lambda T^*)^{-1}$, where $\lambda \in \mathbb{D}$ and D_T is the defect operator of T. In the the particular $C_{.0}$ case, the Sz.-Nagy–Foias functional model reduces to the functional representation given by the maximal function $M_T(\lambda)$, i.e. $\mathbf{H} = M_T\mathcal{H} \subset H^2(\mathcal{D}_{T^*})$, where $(M_T h)(\lambda) = M_T(\lambda)h$. In this case M_T becomes an isometry, and the functional model for T^* is given by the restriction of the backward shift to $\mathbf{H}$, and can be expressed with the maximal function of T as $\mathbf{T}^* = \frac{1}{\lambda}[M_T(\lambda)h - M_T(0)h]$.
Analogously, the maximal function of T^* has the form $M_{T^*}(\lambda) = D_T(I - \lambda T)^{-1}$, and for the discrete linear system generated by the rotation operator $R_T = \begin{bmatrix} T & D_{T^*} \\ D_T & -T^* \end{bmatrix}$ the operators M_T and M_{T^*} become the controllability and the observability operators, respectively.
Some other properties of the maximal function are analyzed in various cases, too.

Strict and bistrict plus-operators

Victor Khatskevich

We consider strict and bistrict plus-operators between spaces with indefinite metrics, in particular Krein spaces (or J-spaces). We call a plus-operator T in a Krein space *strict* if $T = dA$, where $d > 0$ is a constant and A is a J-expansion, and we call T *bistrict* if both T and its conjugate adjoint T^* are strict plus-operators. It is well known that a plus-operator T defines an operator linear fractional relation. In particular, we consider special case of linear-fractional transformations. In the case of Hilbert spaces H_1 and H_2, each linear-fractional transformation of the closed unit ball $\mathcal{K}$ of the space $L(H_1, H_2)$ is of the form

$$F_T(K) = (T_{21} + T_{22}K)(T_{11} + T_{12}K)^{-1}$$

and is generated by the bistrict plus-operator T.
We consider applications of our results to the well-known Krein-Phillips problem of invariant subpaces of special type for sets of plus-operators acting in Krein spaces, and some other applications.

On some unitarily invariant norm inequalities

Shigeru Furuichi

We introduce some symmetric homogeneous means, and then show unitarily invariant norm inequalities for them. Our new inequalities give the tighter bounds of the logarithmic mean than the inequalities given by Hiai and Kosaki in 1999 [1,2]. Our main results are developments of our previous results given in [3]. Some properties such as monotonicity and norm continuities in parameter for our means are also discussed. I will give my talk based on the preprint [4].

[1] F.Hiai and H.Kosaki, Comparison of various means for operators, J. Func. Anal., Vol.163(1999), pp.300-323.

[2] F.Hiai and H.Kosaki, Means for matrices and comparison of their norms, Indiana Univ. Math. J., Vol.48 (1999), pp.899-936.

[3] S.Furuichi and K.Yanagi, Bounds of the logarithmic mean, Journal of Inequalities and Applications, Vol.2013(2013), No.535, pp.1-11. (http://arxiv.org/abs/1402.0613)

[4] S.Furuichi , Unitarily invariant norm inequalities for some means, submitted. (http://arxiv.org/abs/1402.2014)

Birkhoff-James orthogonality : Interplay between $\mathbb{X}$ and $\mathbb{B}(\mathbb{X})$

Kallol Paul

We obtain many important relations between Birkhoff-James orthogonality in a normed linear space $\mathbb{X}$ and the space of all bounded linear operators $\mathbb{B}(\mathbb{X})$. We show how the Birkhoff-James orthogonality relation in $\mathbb{X}$ can be induced from the one in $\mathbb{B}(\mathbb{X})$, like in [1] we proved that if a linear operator T on a finite dimensional real normed linear space $\mathbb{X}$ attains its norm only on $\pm D$, where D is a connected component of the unit sphere $\mathbb{S}_{\mathbb{X}}$ then for any $A \in \mathbb{B}(\mathbb{X})$, $T \perp_B A$ implies that there exists $x \in D$ such that $Tx \perp_B Ax$. In this talk we plan to discuss about the converse part of this result and other related topics.
The talk is based on joint work with Debmalya Sain.

(1) D. Sain and K.Paul: Operator norm attainment and inner product spaces, *Linear Algebra and its Applications*, 439 (**2013**), 2448-2452.

Strong orthogonality in the sense of Birkhoff-James in Normed linear spaces

Debmalya Sain

In [1] Paul et. al. introduced the notion of strong orthogonality in the sense of Birkhoff-James in a normed linear space. We study the properties of strongly orthonormal Hamel basis in the sense of Birkhoff-James in a finite dimensional real normed linear space that are analogous to the properties of orthonormal basis in an inner product space. We conjecture that a finite dimensional real smooth normed space of dimension (> 2) is an inner product space iff given any element on the unit sphere there exists a strongly orthonormal Hamel basis in the sense of Birkhoff-James containing that element.
The talk is based on joint work with Kallol Paul and Lokenath Debnath.

[1] K.Paul, D.Sain and K. Jha: On strong orthogonality and strictly convex normed linear spaces, *Journal of Inequalities and Applications* **2013**, 2013:242.

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