BMI paper

Analysis of poker strategies in heads-up poker

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Preface

One of the final parts of the master study Business Mathematics and Informatics (BMI) is the BMI paper. Its aim is to combine mathematics and informatics through a literature study that has a clear business focus while at the same time the student learns to write a thesis in a scientific way.

Combining the three perspectives of BMI in a paper is not easy. Some might think at first that a topic like poker has no relation to even one of the perspectives, let alone all three. In this paper I have tried to show that poker indeed is influenced by Business, Mathematics and Informatics. Because the 'poker industry' has grown enormously in the past few years, it has in fact become a business on its own. Mathematics plays a very important part, next to psychology, in the way people play and poker computers are being developed more and more to challenge poker professionals.

The reason why I wrote this paper on poker is that I enjoy playing the game very much. The combination of mathematics and psychology is fantastic. Reading and writing about poker has only made me love this game more.

First of all, I would like to thank Sandjai Bhulai for supervising me during this poker study. His advice and ideas were very useful. I would also like to thank my brother, Joost, for giving me my first poker lessons and his advice on poker strategies. Finally, I feel the urge to make an apology to the members of Disputum Stercus Tauri, with whom I play poker a lot. During this study I have learned much about poker and poker strategies. Therefore, I would like to apologize for winning your money in future games.

C.L. Alons
Amsterdam, November 2007

Summary

The introduction of the World Series of Poker and the media attention in the last decade has made poker one of the most popular card games in the world. Poker is a game of mathematics and psychology. The psychology plays a role in the analyses of opponents' strategies, bluffing, and hiding your own strategy. Mathematics is important in the calculation of poker odds and pot odds. Good calculations of poker and pot odds will definitely give a player profit in the long run.

The final phase in a tournament, when only two players are left, is called heads-up poker. Although heads-up is just a particular form of poker, it has some differences in hand values, but more important, in the strategy of the players. This paper contains a simulation study in heads-up poker for tournament and cash games, which are analyzed for four basic strategies and one self-developed strategy. The four basic strategies are the Rock (a tight and passive strategy), the Calling Station (a loose and passive strategy), the Maniac (a loose and aggressive strategy), and the Killer (a tight and aggressive strategy). The self-developed strategy "Korik" is an aggressive strategy that plays when the odds of its hand are above 1:1. So when the probability of winning is greater than 50%.

The strategy Korik is the best strategy in both the tournament and the cash game variant. The Killer scores best of the basic strategies. Korik and Killer are both strategy forms based on tight, aggressive play. This study shows that it is best to wait for a good hand and then bet a lot of money on that hand. In the long run those strategies will create the most profit.

In the program psychological qualities were not implemented. Therefore, in real life, good players will adapt their strategy when someone only bets on good hands. So changing the strategy from time to time will most definitely be the wisest thing to do in a real game.

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1. Introduction

Poker is perhaps the most popular and widely known card game. Unlike most casino card games, poker is a game in which players play against each other, not against the house. What sets poker (and those who play it well) apart from other casino games are the elements of skill and psychology, which are essential in good poker players. Skill in poker is an amalgamation of several elements. Skills such as gauging the strength of your hand and that of your opponents, building up a pot, knowing what cards to play, knowing when to raise or to call, and knowing when to fold and when to bluff, just to name a few. The element of psychology is equally important as that of mathematical skill. Each player brings his or her emotions to a game and being able to gauge the opponent's strengths and weaknesses are essential to holding the upper hand. But what must be kept in mind, a great poker player can win a game with an inferior hand, if they have the psychological skill to out-bluff their opponents.

In the past, research did not focus much on poker but more on chess and checkers. In 1997 Deep Blue, a supercomputer-based chess playing software system developed by I.B.M. researchers beat Gary Kasparov, the leading world chess champion at that time. The researchers of the University of Alberta won the world checkers championship in 1994, and earlier this year they reported that they had developed a program that cannot lose, and at best can be at tie with other checkers players. Poker is thought to be a more difficult challenge for software designers than games like chess and checkers[2]. Poker is more difficult because there is an uncertainty in the cards that are dealt on the table and of course bluffing is an unpredictable element in poker. However, rapid progress is being made in developing new algorithms with broad practical applications in areas such as negotiation and commerce. A computer beating the top poker players has not yet been developed, but may not be as far away as it used to be.

2. The problem

As mentioned in the introduction, poker has become more and more interesting for research. In this paper, research will be done for a particular variant of poker, namely Heads-Up Poker. In a tournament heads-up is the final phase. Only two players are left and it is a one-on-one confrontation.

In this paper we will use simulation to analyze which of the basic strategies is best in a heads-up poker game. After that, we will analyze whether or not a self-designed strategy can beat the basic strategies.

These five strategies will be analyzed in two different scenarios. The first scenario is the classic cash game. Here we assume players have an infinite amount of money. The simulation program will run 5,000 games and see which strategies win the most money. The second scenario is a tournament scenario. Here the players have a limited amount of money. The simulation program will simulate 100 tournaments. A tournament is over when one of the two players is out of stack. So, in this scenario every strategy will be analyzed by the number of tournaments it wins against the other strategies.

3 The game of Poker

3.1 Introduction

Poker is a game played with standard playing-cards. Poker is a so called vying game. A vying game is a game where the players can win in two cases:

- At the showdown the best hand wins all the stakes ('the pot')
- All players except for one have given up betting and dropped out of play. The last person wins the pot without a showdown.

So, in a vying game the best hand does not necessarily win. Therefore, bluffing is very important in poker.

A five-card vying game is one where the only valid combinations are those of five cards. In orthodox poker these are, from highest to lowest[1]:

Example	Rank of Hands from Highest to Lowest
Royal Flush, Ace high Hearts	**Royal Flush** The five highest ranking cards of a suit. A Royal Flush is essentially an Ace high Straight Flush.
Straight Flush, Five high	**Straight Flush** Any five cards of the same suit in consecutive order. The example is a five high Straight Flush. (Only the Ace can be high or low for Straights).
Four Kings	**Four of a Kind** Four cards of the same rank (for example, 4 Kings), with one dissimilar card.
Full House, Queens over	**Full House** A triple of one rank plus a pair of another. The example is Queens over Twos, which beats Jacks over Aces because the triple is taken first.

Twos	
 Flush, King high	**Flush** Any five non-consecutive cards of the same suit. (If they are consecutive, you have a Straight Flush).
 Straight, Ace high	**Straight** Any five consecutive cards of mixed suits. Ace can be high (next to a King) or low (next to a 2) but not both at the same time.
 Three Fives	**Three of a Kind** Three cards of the same rank (a triple), with two dissimilar cards. (If the other two made a pair, you would have a Full House).
 Two Pair, Sevens and Fours	**Two Pair** Any pair of one rank, plus any pair of another rank. If two hands have the same high pair, the second pair decides the winner. If both pairs tie, the remaining high cards decide.
 A pair of Tens	**Pair** Any two cards of the same rank. When two hands have the same pair, the remaining high cards decide.
 King high	**High Card** If no other hand rank is achieved, the highest card held wins. If two hands hold the same high card, then the remaining high cards decide.

Card Ranks
The Rank of Cards from highest (Ace) to lowest (Two) is:

Note: There is no ranking of Suits in Poker. For example, an Ace of Spades does not outrank an Ace of Clubs. They are of equal rank.

3.2 Birth and growth

Poker first appeared in the beginning of the 19th century around New Orleans. It was mostly played in gambling saloons. The first form of poker was played with a 20-card pack, which consisted of A, K, Q, J, and 10. The cards were evenly dealt amongst four players. The highest hand was four aces or four kings and an ace. These hands were

unbeatable, unlike the highest hand in classic poker, the "royal flush", which can be tied in another suit.

To make it possible for more than four players to join in a game the 52-card game made its first appearance. The 52-card game also made it possible to draw cards and introduce more than one betting round. This introduction was in the 1830s. The flush was introduced around this time as well. In this phase the hand ranking was as follows: one pair, two pairs, triplets, flush, full house, fours.

3.3 Coming of age

From the middle of the 19th century, poker experienced rapid changes and innovations as it became more widespread through the upheavals of the Civil War[1]. In this phase, the so-called Jack pots were introduced. This meant that when a player had a pair of Jacks or better cards he was obliged to open his cards and otherwise he was not allowed to open them at all. This rule was introduced to drive out wild players, who would bet on any cards they received. The rule was abandoned because it took out the idea of bluffing, and made poker more like a lottery.

In the 1860s the straight made its introduction. The 1864 edition gave the hands as: one pair, two pairs, straight sequence or rotation, triplets, flush, full house, fours. But when a straight was also a flush it outranked the full house. The fours were still the highest hand, even though it was mathematically more likely to 'hit' a four than a straight flush. The reason was that four aces or four kings and an ace could not be tied and a straight flush could be tied in a different suit.

It was not until 1892, when John Keller defended his view that the straight flush should be the highest rank in poker. His most important argument was a moral argument. He said that among gentlemen it was unethical and ungentlemanly to bet on four aces,

because the win was a certainty. A straight flush however, could be tied in a different suit which made it morally superior to betting on a certainty.

In the 20th century following Draw and Stud, a third major structural division of the poker game, represented today by Texas Hold'em, was first played. Here the introduction of one or more communal cards was a fact. The rise of modern tournament play dates from the World Series of Poker started in 1970.

3.4 Texas Hold'em

The most popular form of poker is Texas Hold'em. This is a poker game for two to ten players with fairly simple rules. The object is to make the best 5-card poker hand using any combination of the 2 cards dealt face down to you and the 5 common cards dealt face up on the table. Here is the flow of play given by www.everestpoker.com[3]:

- **The Dealer.** Cards are dealt from the position of the Dealer, which rotates clockwise around the table after each hand. The player designated as Dealer for a given hand is identified by a round "D" marker on the table. The software will automatically deal cards on behalf of the Dealer.
- **Blinds.** To start the game, the player to the left of the Dealer puts up the small blind (which is usually half the minimum bet) and the player to his left posts the big blind (equal to the minimum bet). This gets the pot started and encourages other players to place bets. They are called "blinds" because these bets are made before any card is seen.
- **The Deal.** Each player is dealt two cards, called Pocket Cards, which are visible only to the player holding them.
- **Betting.** After all players have received their two pocket cards, there is a Round of Betting starting with the player to the left of the player who posted the big blind. Depending on prior activity in the current hand of play, a player can fold,

check, call, bet, raise or re-raise within the bounds of the Game Limits and Table Stakes. A round of betting may circle the table several times if there are raises and re-raises. The round ends when all players have either called the last bet or folded.

- **The Flop.** Next comes the Flop, where 3 cards are dealt face-up in the common area of the table, followed by another round of betting.
- **The Turn.** Then comes the Turn, where a 4th common card is dealt face-up next to the Flop, and a third round of betting occurs.
- **The River.** The final common card, called the River, is placed face-up next to the Turn and is followed by the last round of betting.
- **The Showdown.** The Showdown occurs after betting concludes. The remaining players compare the best five-card poker hands they make using any combination of their 2 pocket cards and the 5 common cards. The player with the highest hand rank takes the pot. Occasionally, two or more players will have hands of equal rank or the best possible hand is comprised using all 5 common cards. In these cases, the players involved share the pot.
- **Next Hand.** After the pot is distributed, the Dealer button moves to the next player on the left (clockwise around the table), and the players to the left of the new Dealer post blinds so the deal for a new hand can start.

3.5 Betting Rules

There are different types of poker when it comes to betting. PartyPoker.com[6] gives the following betting options.

Limit Poker

In a game of limit poker, the amount you are allowed to bet is limited to a specific size. This limit is in the name of the poker game ($3/6, $20/40, etc.).

In the case of a '$1/2 limit' game, the minimum sizes of the bets would be:

- Pre-flop (before the flop): $1
- On the flop (when the three first community cards have been dealt): $1
- On the turn (when the fourth community card has been dealt): $2
- On the river (when the fifth and last community card has been dealt): $2

The smallest bet allowed is the big blind. Furthermore, all raises must be done in increments of the betting amounts.

For example, in a $1/2 limit game, the raises will be as follows, limited to four times the first bet that was made:

- Pre-flop: $1, $2, $3 up to $4
- On the flop: $1, $2, $3 up to $4
- On the turn: $2, $4, $6 up to $8
- On the river: $2, $4, $6 up to $8

No-Limit Poker

In no-limit poker there is no maximum to the amount a player can bet. The minimum, however, is the big blind. When a player wants to raise another player, he must raise with at least the bet of the other player.

When a player wants to call, but does not have enough chips to equal the amount that is bet, he is "all-in". This means that the player plays for all the chips he has left.

Pot-Limit Poker

Pot-limit is the most popular variant of the poker game in Europe. It is quite similar to no-limit poker but there are certain key differences.

In pot-limit poker, players are allowed to bet any amount from the size of the big blind to the size of the pot. The pot includes the sum of all bets and raises made during the current round.

3.6 Poker Etiquette

In the biggest tournaments and among the world's most successful poker players, the biggest winners almost always have two things in common - good sportsmanship and perfect poker etiquette.

Good manners at the table make the game more pleasant for everybody, help you get more hands per hour and in many cases can actually help add to your winnings.

In Las Vegas, there is a saying among old time poker pros: "Don't tap on the aquarium, it spooks the fish." More experienced players know better than to insult the lesser skilled players, as that is where the money comes from. They also know the negative outcome from alerting their less-skilled opponents to their own superior skills.

4. Heads-up

In a tournament heads-up is the final phase. Only two players are left and it is a one-on-one confrontation. While the game of Poker is the same with heads-up poker and 6-10 person play there are also some important differences that need to be taken into account. Especially the rank of the starting hand is different and the probability that another person has a stronger hand is smaller in a heads-up game.

4.1 Rank of Starting Hands for Heads-Up Poker[4]

The ranks of starting hands in heads-up poker are very different from 6-10 poker play. This, of course, is due to the number of possible good hands that your opponents have. When you play one opponent, his hand will have a smaller probability to be good than an opponent that has called when 8 opponents have folded. This opponent called with a reason, most likely, because he has a very strong hand.

As can be seen in the chart of Top 20 heads-up poker starting hands, pairs are very strong. Almost any pair is a favourite against a non-paired hand before the flop. A player is dealt a pocket pair every 17 deals, so the probability of both players holding a pair is approximately (17 × 17) = 289.

Chart of Top 20 Heads-Up Poker Starting Hands

1 – A-A
2 – K-K
3 – Q-Q
4 – J-J
5 – 10-10

6 – 9-9

7 – 8-8

8 – A-K (suited)

9 – 7-7

10 – A-Q (suited)

11 – A-J (suited)

12 – A-K (not suited)

13 – A-10 (suited)

14 – A-Q (not suited)

15 – A-J (not suited)

16 – K-Q (suited)

17 – 6-6

18 – A-9 (suited)

19 – A-10 (not suited)

20 – K-J (suited)

5. Mathematics in Texas Hold'em Poker

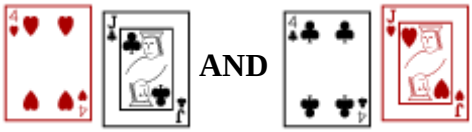
Poker is a game of psychology and mathematics. The best players in the world all have a strong handle on both of those aspects. The deck of cards and its possibilities form the mathematics that every poker player should be aware of and the other players and their cards. To play poker, intuition can play a big roll, but some awareness of the mathematics in poker is essential for a good player. Without that awareness, it is difficult to estimate the best course of action based on what incomplete information you have at any point in the poker hand up until showdown[7].

Learning how to properly count the outs and calculate poker odds is a fundamental requirement of Texas Hold'em. But first the number of possible starting hands will be discussed.

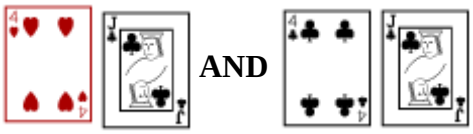
5.1 Number of Possible Starting Hands

In Texas Hold'em there are 2,652 possible starting hands (52 times 51) because the first two cards dealt can be anything out of the deck. But not all 2,652 are different hands, because the same two cards dealt in two different orders are still the same hand.

So, that gives 1,326 possible starting hands in Texas Hold'em poker. These 1,326 starting hands are not all different in their value, because there is no hierarchy of suits in poker.



The hands above are both equal in value as a starting hand. Viewed in this way, there are no more than 169 possible starting hands in Texas Hold'em poker[7].
The possibility of a hand being suited is very important to the Hold'em player so that there are 78 possible suited starting hands (13 times 12 divided by 2) and 78 possible non-suited starting hands.



The hands above are different in their value, because a suited hand gives the extra probability on a flush.

To sum it up, the total 169 possibilities break down as[7]:

- 13 possible pairs;
- 78 mismatched cards that are not suited;
- 78 mismatched cards that are suited.

5.2 The importance of odds in Poker

Poker odds are a very important concept in poker. It gives the player an idea on how good or bad his situation is. The following example gives an idea on how odds work[8].

Let us say you and a friend are flipping a quarter and he gives you 1:1 odds that the next flip will land on heads. You already know that it will land on heads 50% of the time, and

it will land on tails the rest of the time. In this case, he is giving you an even bet, because nobody has a statistical advantage.

But what if we said that the friend feels very lucky today and offers you 2:1 odds that the next coin flip will be heads. Of course you will take this bet, because the chances of heads or tails coming up are still 1:1, while he is paying you at the 2:1 rate. Your friend is hoping to ride his luck a little longer, but if he gambles with you long enough, he is bound to loose a lot of money with these kinds of odds.

The above example is a simplified version of what goes on in Texas Hold'em all the time. This is summed up in this short principle[8]:

In poker, there are two types of players. The first group consists of players who take bad odds in hopes of getting lucky. The second group consists of players who cash in on the good odds that are left by the first group.

5.3 Hand Odds and Poker Odds

Next to poker odds there are the hand odds. Hand odds are the chances of making a hand in Texas Hold'em poker.

X to 1 odds $\rightarrow$ You expect to hit your hand 1 out of $(X + 1)$ times.

X to 1 odds: $\frac{1}{X+1}$ chance to hit your hand.

Example: 3 to 1 odds $\rightarrow \frac{1}{4} = 25\%$ chance to hit your hand

To give an example: if a player holds a pair of 2's and the flop does not contain a 2, the odds of hitting a 2 on the turn is 22:1 and the odds of hitting a 2 on the turn or the river is 12:1. How to calculate these odds is described in Chapter 5.4 and 5.5.

In Table 1 for some hands the odds are shown.

Outs	One Card %	Two Card %	One Card Odds	Two Card Odds	Draw Type
1	2%	4%	46	23	Backdoor Straight or Flush (Requires two cards)
2	4%	8%	22	12	Pocket Pair to Set
3	7%	13%	14	7	One Overcard
4	9%	17%	10	5	Inside Straight / Two Pair to Full House
5	11%	20%	8	4	One Pair to Two Pair or Set
6	13%	24%	6.7	3.2	No Pair to Pair / Two Overcards
7	15%	28%	5.6	2.6	Set to Full House or Quads
8	17%	32%	4.7	2.2	Open Straight
9	19%	35%	4.1	1.9	Flush
10	22%	38%	3.6	1.6	Inside Straight & Two Overcards
11	24%	42%	3.2	1.4	Open Straight & One Overcard
12	26%	45%	2.8	1.2	Flush & Inside Straight / Flush & One Overcard
13	28%	48%	2.5	1.1	
14	30%	51%	2.3	0.95	
15	33%	54%	2.1	0.85	Flush & Open Straight / Flush & Two Overcards
16	34%	57%	1.9	0.75	
17	37%	60%	1.7	0.66	

Table 1: Card odds for some draw types[8]

To calculate hand odds, the number of *outs* is very important. An out is defined as a card in the deck that helps you make your hand[8]. If a player holds two spades in his hand and there are two spades on the flop, there are 9 more spades in the deck (since there are 13 cards of each suit). This means that the player has 9 outs to hit his flush. This however does not mean that this players' hand is the winning hand.

It is, of course, possible that one of the other players has a spade as well. Normally, a player does not know what his opponents hold, so he can only calculate odds with the knowledge that is available to him.

In Table 2 the number of outs for some hand draws can be seen.

Draw	Hand	Flop	Specific Outs	# Outs
Pocket Pair to Set	[4♠ 4♥]	[6♣ 7♦ T♠]	4♦, 4♣	2
One Overcard	[A♠ 4♥]	[6♥ 2♦ J♣]	A♦, A♥, A♣	3
Inside Straight	[6♣ 7♦]	[5♠ 9♥ A♦]	8♣, 8♦, 8♥, 8♠	4
Two Pair to Full House	[A♦ J♥]	[5♠ A♠ J♦]	A♥, A♣, J♠, J♣	4
One Pair to Two Pair or Set	[J♣ Q♦]	[J♦ 3♣ 4♠]	J♥, J♠, Q♠, Q♥, Q♣	5
No Pair to Pair	[3♦ 6♣]	[8♥ J♦ A♣]	3♣, 3♠, 3♥, 6♥, 6♠, 6♦	6
Two Overcards to Over Pair	[A♣ K♦]	[3♦ 2♥ 8♥]	A♥, A♠, A♦, K♥, K♣, K♠	6
Set to Full House or Quads	[5♥ 5♦]	[5♣ Q♥ 2♠]	5♠ Q♠, Q♦, Q♣, 2♥, 2♦, 2♣	7
Open Straight	[9♥ T♣]	[3♣ 8♦ J♥]	Any 7, Any Q	8
Flush	[A♥ K♥]	[3♥ 5♠ 7♥]	Any heart (2♥ to Q♥)	9
Inside Straight & Two Overcards	[A♥ K♣]	[Q♠ J♣ 6♦]	Any Ten, A♠, A♦ A♣, K♠, K♥, K♦	10
Flush & Inside Straight	[K♣ J♣]	[A♣ 2♣ T♥]	Any Q, Any heart	12
Flush and Open Straight	[J♥ T♥]	[9♣ Q♥ 3♥]	Any heart;, 8♦, 8♠, 8♣, K♦, K♠, K♣	15

Table 2: Examples of draws and specific outs[8].

Keyword Definitions[8]

- **Backdoor:** A straight or flush draw where you need two cards to help your hand out.
  You have [A K]. Flop shows [T 2 5]. You need both a [J] and [Q] for a straight.
- **Overcard Draw:** When you have a card above the flop.
  You have [A 3]. Flop shows [K 5 2]. You need a [A] overcard to make top pair. 3 total outs.
- **Inside Straight Draw (aka 'Gutshot'):** When you have one way to complete a straight.
  You have [J T]. Flop shows [A K 5]. You need a [Q] to complete your straight. 4 total outs.
- **Open Straight Draw:** When you have two ways to complete a straight.
  You have [5 6]. Flop shows [7 8 A]. You need a [4] or [9] to complete your straight. 8 total outs.
- **Flush Draw:** Having two cards to a suit with two suits already on the flop.
  You have [A♥ K♥]. Flop shows [7♥ 8♥ J♣]. You need any heart to make a flush. 9 total outs.

A very common mistake players make is that they overcount their outs. The following example shows how easy it is to overcount the outs.

Example: You hold [J♦ T♦] and the board shows [8♦ Q♦ K♠]. A Nine or Ace gives you a straight (8 outs), while any diamond gives you the flush (9 outs). However, there is an [A♦] and a [9♦], so you do not want to count these twice toward your straight draw and flush draw. The true number of outs is actually 15 (8 outs + 9 outs - 2 outs) instead of 17 (8 outs + 9 outs).

One of the most difficult things in poker odds is to keep your opponent's options and outs in mind. In some cases, an out can turn out to be a better out for another player. An example of this is shown in below.

Example[8]: You hold [J♠ 8♣]o (off-suit, or not of the same suit) and the flop comes [9♠ T♥ J♣] rainbow (all of different suits). To make a straight, you need a [Q] or [7] to drop, giving you 4 outs each or a total of 8 outs. But, you have to look at what will happen if a [Q♥] drops, because the board will then show [9♠ T♥ J♣ Q♥]. This means that anyone holding a [K] will have made a King-high straight, while you hold the second-best Queen-high straight.

So, the only card that can really help you is the [7], which gives you 4 outs, or the equivalent of a gutshot draw. While it is true that someone might not be holding the [K] (especially in a short or heads-up game), in a big game, it is a very scary position to be in.

5.4 How to calculate hand odds (the longer way):

In the last paragraph we saw how to calculate the number of outs. Now we need a way to calculate the percentage of time the hand is hit on the river. For a single card the probability can be calculated easily: $\frac{\text{Total Outs}}{\text{Remaining Cards}}$. For two cards however, like from the Flop to the River, it is a bit trickier. This can be calculated as shown below[8]:

$$\text{Flop to River: } 1-\left[\left(\frac{47-\text{Outs}}{47}\right)\times\left(\frac{46-\text{Outs}}{46}\right)\right]$$

$$\text{Turn to River: } 1-\left(\frac{46-\text{Outs}}{46}\right)$$

The number 47 represents the remaining cards left in the deck after the flop (52 total cards, minus 2 in our hand and 3 on the flop = 47 remaining cards). As mentioned earlier, there are not actually 47 cards left in the deck, because other players also received cards, but each player can only calculate odds based on the information he has. The next example shows the probability that a two-overcard draw, hits a top-pair. Each overcard has 3 outs, giving a total of 6 outs for a top-pair draw:

Two Overcard Draw

$$= 1 - \left[\left(\frac{47-6}{47} \right) \times \left(\frac{46-6}{46} \right) \right]$$

$$= 1 - \left[\frac{41}{47} \times \frac{40}{46} \right] = 1 - [0.87 \times 0.87]$$

$$= 1 - .76$$

$$= .24 = 24\% \text{ chance to draw overcards from flop to river}$$

To calculate the odds from the probability is easy:

$$\text{Odds} = \left(\frac{1}{\text{Percentage}} \right) - 1$$

Thus, to change the 24% draw into odds that we can use, the calculation is as follows:

Odds

$$= \left(\frac{1}{24\% \text{ Overcard Draw}} \right) - 1$$

$$= \left(\frac{1}{0.24} \right) - 1$$

5.5 How to calculate hand odds (the shorter way):

The calculations shown in Paragraph 5.4 give exact probability and odds. This however is not always easy to do while playing. Therefore we present a shorter way that also gives a quite satisfactory result.

This calculation goes as follows. When a player wants to know the probability that he hits his hand from the flop to the river, he has to multiply the number of outs by 4 and divide by 100. When a player wants to know the probability that he hits his hand from the turn to the river, he has to multiply the number of outs by 2 and divide by 100. This method gives an approximation of the actual probability. This can be seen in the table below.

Sample Outs and Percentages from Above Chart

Outs	One Card %	Two Card %	One Card Odds	Two Card Odds	Draw Type

6• 13%• 24%• 6.7• 3.2• No Pair to Pair / Two Overcards• • 7• 15%• 28%• 5.6• 2.6• Set to Full House or Quads• •

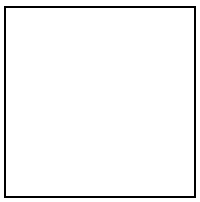
24% 6.7

The odds are still

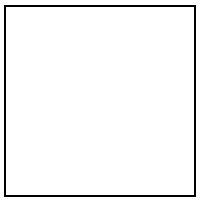
13%

7

• calculated using the already-known formula:



Or the equivalent:



The next example shows how easy it is to calculate the odds[8]:

You hold: A♣ J♠
Flop is: 5♣ T♦ K♦

Total Outs: 4 Queens (Inside Straight) + 3 Aces (Overcard) - Q♦ or A♦ = 5 Outs

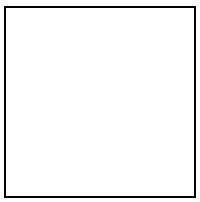
Percentage for Draw = 5 Outs × 4 = 20%

Odds = (100 / 20) – 1
= 5 – 1
= 4:1

So, this means that a player can expect to make his hand 1 out of every 5 times.

5.6 Pot Odds and Poker Odds

In the paragraphs before, poker odds in terms of hand odds were discussed. In this paragraph the reason why it is important to calculate poker odds is described. First, a new type of odds is introduced, the pot odds. Pot odds are simply the ratio of the amount of money in the pot to how much money it costs to call[8]. So for instance, if there is $100 in the pot and it takes $10 to call, the pot odds are 100:10, or 10:1.



The usefulness of hand and pot odds becomes clear when you compare the two. As seen earlier, in a flush draw, the *hand odds* for making a flush are 1.9 to 1. When the *pot odds* are higher than 1:1.9 it is best to call, otherwise the player should fold. The next example will illustrate this.

Example[8]:
If there is $15 in the pot plus a $5 bet from an opponent, then you are getting 20:5 or 4:1 pot odds. This means that, in order to break even, you must win 1 out of every 5 times. However, with your flush draw, your odds of winning are 1 out of every 3 times! You should quickly realize that not only are you breaking even, but you are making a nice profit on this in the long run. Let us calculate the profit margin on this by theoretically playing this hand 100 times from the flop, which is then checked to the river.
Net Cost to Play = 100 hands × $5 to call = -$500
Pot Value = $15 + $5 bet + $5 call

Odds to Win = 1.9:1 or 35% (From the flop)
Total Hands Won = 100 × Odds to Win (35%) = 35 wins

Net Profit = Net Cost to Play + (Total Times Won × Pot Value)
= -$500 + (35 × $25)
= -$500 + $875
= $375 Profit
= $3.75 Profit/Hand

The example above shows that when the pot odds are greater than the poker odds a player should always call. In the long run he will definitely be making money.
Many players find this hard to do, because in some cases the hand might not look good enough to call. For instance, a gutshot straight draw has 5 to 1 hand odds, but when the pot odds are higher calling is definitely worth it in the long run. On the other hand, when a player has a flush draw, but an opponent raises so much that the pot odds become 1:1 it is wiser to fold, because in the long run this is not profitable.

The ability to memorize or calculate the hand odds and pot odds lead players to make many of the right decisions. The most fundamental principle of playing drawing hands is that the pot odds must be greater than the hand odds. There are of course some things to take into account. These are discussed in the next paragraph.

5.7 Poker Odds from the Flop to Turn and Turn to River

An important note that has to be made is that many players who understand Texas Hold'em odds tend to forget that much of the theoretical odds calculations from the flop to the river assume there is no betting on the turn[8]. So when the hand odds are 2:1, calling 2:1 pot odds is only justified when both the turn and the river can be seen without betting. Since, a player does not know in advance whether his opponents are going to bet or not this strategy is wrong. The right strategy is to calculate the hand odds card by card. The next two examples illustrate the wrong and the right strategy.

Example of Incorrect Pot Odds Math[8]
You Hold: Flush Draw
Flop: $10 Pot + $10 Bet
You Call: $10 (getting 2 to 1 odds)

Turn: $30 Pot + $10 Bet
You Call: $10 (getting 4 to 1 odds)

Long-Term Results Over 100 Hands
Cost to Play = 100 Hands × ($10 Flop Call + $10 Turn Call) = $2,000
Total Won = 100 Hands × 35% Chance to Win × $50 Pot = $1,750

Total Net = $1,750 (Won) - $2,000 (Cost)
= -$250 Profit
= -$2.5/Hand

Example of Correct Pot Odds Math[8]
You Hold: Flush Draw
Flop: $30 Pot + $10 Bet

You Call: $10 (getting 4 to 1 odds)

Turn: $50 Pot + $16 Bet
You Call: $16 (getting about 4 to 1 odds)

Long-Term Results Over 100 Hands
Cost to Play = 100 Hands × ($10 Flop Call + $16 Turn Call) = $2,600
Total Won = 100 Hands × 35% Chance to Win × $82 Pot = $2,870

Total Net = $2,870 (Won) - $2,600 (Cost)
= $270 Profit
= $2.7/Hand

As can be seen from these examples, calling a flush draw with 2 to 1 pot odds on the flop can lead to a long term loss, if there is additional betting past the flop. There is, however, a concept called Implied Value that is able to help some draws to remain profitable even with seemingly 'bad' odds.

Implied Value

Implied Value is a concept that takes future betting into account. Implied value is most often used to anticipate the opponent calling on the river. The general idea is that a player can call on the turn, when his odds may give reason to fold, in the case that he knows that his opponent will call when the player bets on the river card. For example, when a player has a flush draw and is offered 3 to 1 pot odds on the turn. Normally, he needs 4 to 1 pot odds to make this a profitable call. But the player may likely anticipate his opponent calling him on the river if he does hit his flush. This means that even though the player only gets 3 to 1 pot odds, by anticipating the opponent calling a bet on the river, he is able to make this call on the turn.

6. Basic Strategies

All poker players play according to certain strategies. In this chapter the basic strategies[5] are discussed.

6.1 Tight Play vs Loose Play

The first thing you can see about a player's strategy is how many hands he plays. A player is considered a "loose" player when he plays a lot of hands. When a player only plays good hands and therefore folds many hands, he is considered a "tight" player.

6.2 Aggressive Play vs Passive Play

The second way to define a player's strategy is to look at how often a player raises or bets. A player that raises or bets on many hands is considered an "aggressive" player and a player that checks or calls a lot is considered a "passive" player.

Most experts believe that an aggressive strategy is superior to a passive strategy, because when a player bets or raises, other players might fold. This increases the number of hands that an aggressive player wins. The pitfall however is that when an aggressive player bets or raises on a weak hand and gets called by a strong hand, he is bound to loose a lot of money.

6.3 Rocks - Killers - Maniacs - Calling Stations

You can use these distinctions to categorize your opponents' play into one of four categories:

- **Rocks:** These are tight and passive. They do not play many hands, but when they do play a hand, they usually check or call. “Rocks” often play timid poker, and fold any time they do not have the best possible hand.
- **Maniacs:** “Maniacs” are loose and aggressive. This type of player is the polar opposite of a rock. Any starting hand will do, and any starting hand warrants a raise. In some games, maniacs can play quite profitably by stealing blinds and antes.
- **Calling Stations:** The most profitable type of player to play with is a “calling station”. Loose and passive, they play lots of hands but do not raise with them. They usually call bets in hopes that their hand will improve and win.
- **Killers:** This type of player is tight and aggressive. This player does not play many hands, but when he does he raises or bets trying to win a big pot with his good hand.

7. The program

To analyze different strategies in heads-up poker a simulation program has been written in JAVA. All different strategies play against each other in two different scenarios of poker. In the first scenario players have an infinite amount of money. After 5,000 games the program checks which player has won the most money. In the second scenario all players have a fixed starting stack of money and play 100 tournaments against each other. Each of the 100 tournaments will be played until one of the two players is out of stack. During a tournament the Small and Big Blind will double after every ten games.

A game is simulated as follows. The program first shuffles the cards. This is done by generating 52 random numbers between 0 and 1 and link these number each to one of the 52 cards. After that all cards are ranked by their random number and the cards are shuffled. The two cards are dealt to both players and the betting round begins. Each player bets according to his strategy. If both players call or raise the five community cards are dealt. After that the winner is determined and receives cash from the other player. To simplify the simulation program and leave out the "human component" the betting rounds before the showdown have been cancelled.

7.1 Used Strategies

In the scenario with finite money, each player receives 250 euros.

Rock

The rock calls if is his hand is one of the top-20 starting hands, otherwise he folds.

Maniac

The maniac raises every hand he gets to 100 euros.

Calling Station

The calling station calls every hand he gets.

Killer

The killer raises to 100 euros if his hand is one of the top-20 starting hands, otherwise he folds.

The self-developed strategy "Korik"

Korik raises to 100 euros if his starting hand has more than 50% probability to win (see Appendix A), otherwise he folds.

7.2 Scenario 1: Infinite money

In cash games, players can re-buy coins whenever they want. So the amount of money they can win or loose is pretty much unlimited. In this scenario players have an infinite amount of money. After 5,000 games the program checks which player has won the most money. The Small and big blind are fixed at 1 and 2 euros, respectively.

7.3 Scenario 2: Finite money

In tournaments, players do not have unlimited money. Every player starts with a fixed stack of coins and the winner is the player that has all the coins in play. In this scenario all players have a fixed starting stack of money, which is 250 euros. They play 100 tournaments against each other. Each of the 100 tournaments will be played until one of the two players is out of stack. During a tournament the small and big blind start at 1 and 2 euros and are doubled after every ten games. If a player wants to raise to 100 euros, but does not have enough money, he goes "all-in". This means that he plays for every coin he has. If the player wins, he doubles up his stack. If the player looses, he looses that tournament and another tournament is started until all 100 tournaments have been played.

8. Results

8.1 Scenario 1: Infinite money

After simulating each basic strategy against every other strategy in 5,000 games, the following results were found.

	Rock	Calling Station	Maniac	Killer	money	#players beaten
Rock	€0	-€1,829	€12,450	-€831	**€9,790**	**1**
Calling Station	€1,829	€0	-€500	€9,828	**€11,157**	**2**
Maniac• -€12,450 -€12,450	€500	€0	-€6,552	**-€18,502**	**1**	**Killer**

• **Killer**• €831• -€9,828• €6,552• €0• **€2,445**• **2**• • Table 3 Financial results of basic strategies after 5,000 games against each other

Table 3: Financial results of basic strategies after 5,000 games against each other

We see that the Calling Station and Killer both defeat two other strategies. The Calling Station, however, has won the most money. It is surprising to see that although the Killer has beaten two other strategies, his revenues are negative. The Rock on the other hand is only victorious against the Maniac but wins so much that his revenues are very high.

In the table below the strategy Korik is also included.

	Rock	Calling Station	Maniac	Killer	Korik
Rock• €0• -€1,829• €12,450• -€831 €0	-€1,829	€12,450	-€831	€703	**€10,493**

Table 4: Financial results of basic strategies and Korik after 5,000 games against each other

We see that Korik looses some money against the Rock and quite a lot against the Killer, but due to enormous profits against especially the Maniac and also the Calling Station, Korik is able to make the most profit of all strategies. The killer has beaten most of the players, namely three.

A graph of the stack heights of Calling Station vs Killer can be seen below.

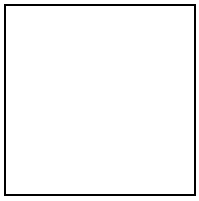


Figure 1: Stack heights of Calling Station vs Killer during 5,000 games.

What can be seen is that the Calling Station has a profit around 2.500 euros in the first 3,200 games and after that the profit rises fast to 10,000 euros at 5,000 games.

A graph of the stack heights of Calling Station vs Korik can be seen below.

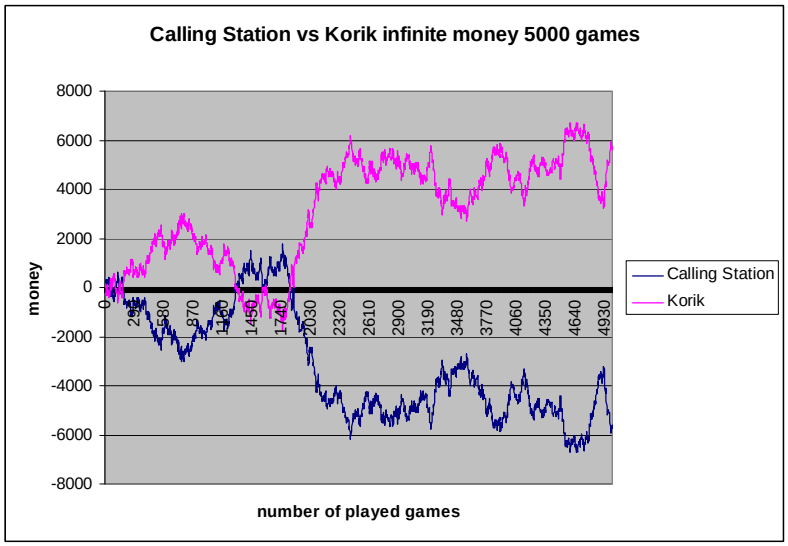


Figure 2: Stack heights of Calling Station vs Korik during 5,000 games.

What can be seen in this graph is that in the beginning the stack of both players are very unstable, but after about 2,000 games the profit of Korik rises to 6,000 euros at 5,000 games.

A graph of the stack heights of Korik vs Killer can be seen below.

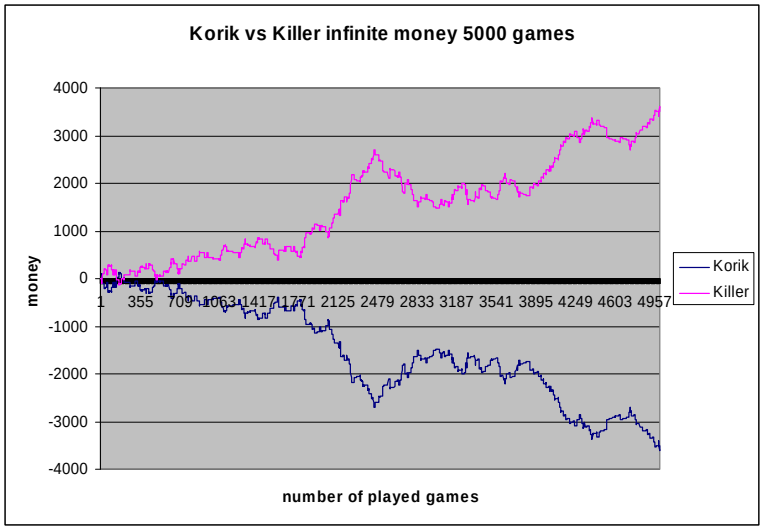


Figure 3: Stack heights of Korik vs Killer during 5,000 games.

In this graph it can be seen that the killer just wins more games than Korik. The total amount is around 3,500 euros after 5,000 games.

In the graph below the stack heights of Maniac vs Korik can be seen.

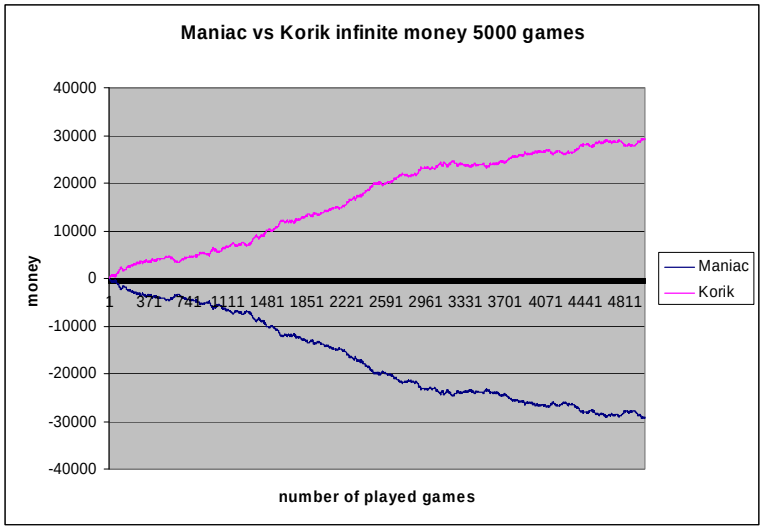


Figure 4: Stack heights of Maniac vs Korik during 5,000 games.

Korik plays only the hands which have more than 50% probability of beating the opponent. The maniac loses too many hands due to refusing to fold weak hands. The profit of Korik is around 30,000 euros after 5,000 games.

In the graph below the stack heights of Korik vs Rock can be seen.

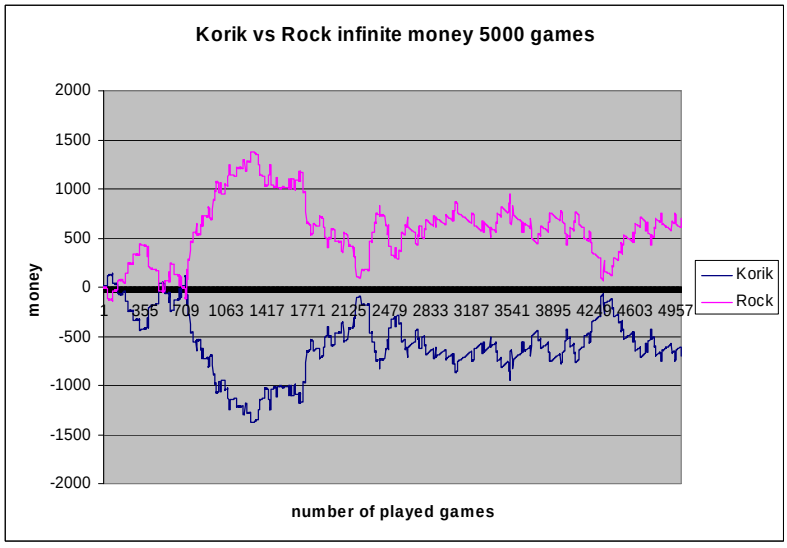


Figure 5: Stack heights of Korik vs Rock during 5,000 games.

These strategies are almost equal in profit, with a slight advantage for the Rock. After 5,000 games the rock has a profit around 700 euros.

Below, the graph of the stack heights of Maniac vs Calling Station can be seen.

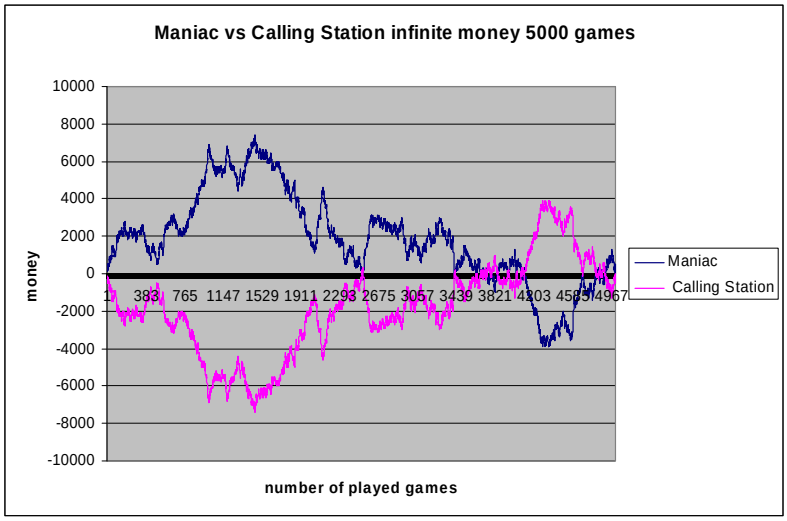


Figure 6: Stack heights of Maniac vs Calling Station during 5,000 games.

These strategies are very similar, especially when they play against each other. This is because they never fold. Therefore, it is not a surprise that the profit of the winner, in this case the Maniac, is near zero (500 euros).

In the graph below the stack heights of Maniac vs Killer can be seen.

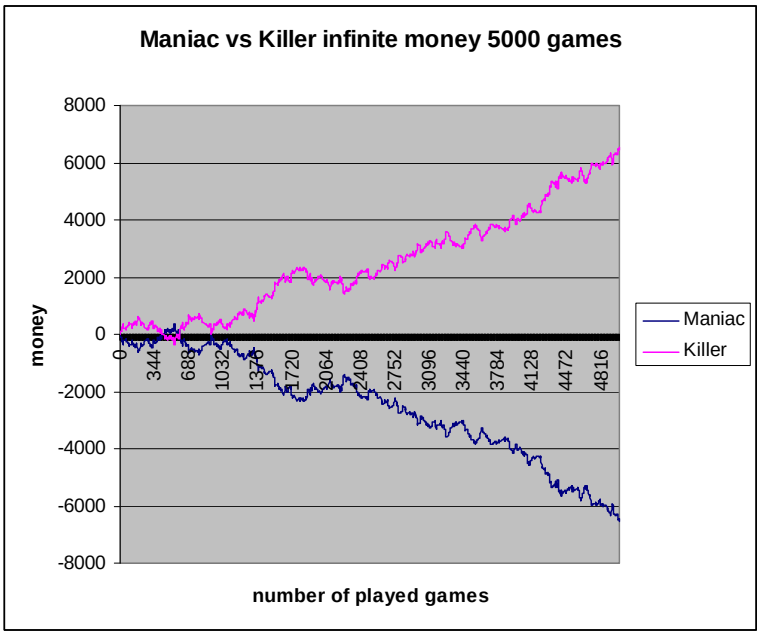


Figure 7: Stack heights of Maniac vs Killer during 5,000 games.

These strategies are both very aggressive, but the Killer only plays with good cards. Therefore, the killer makes a profit over 6,000 euros in 5,000 games.

A graph of the stack heights of Rock vs Killer can be seen below.

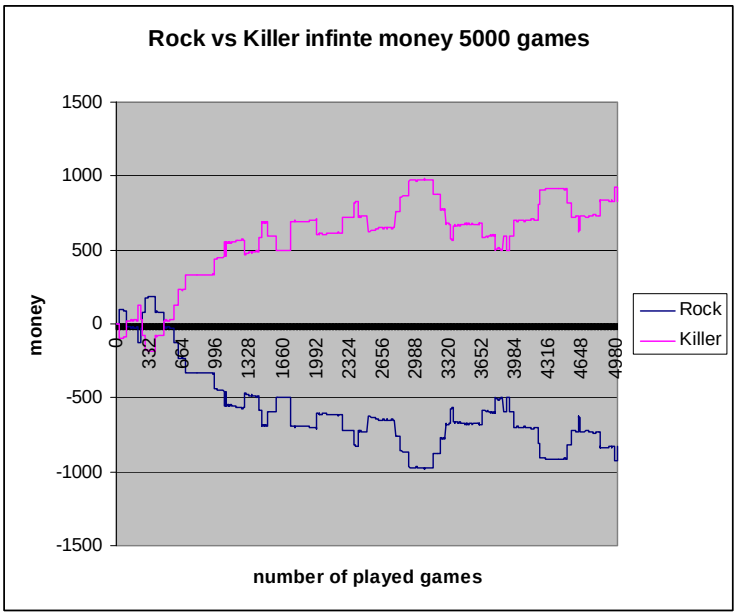


Figure 8: Stack heights of Rock vs Killer during 5,000 games.

When these strategies play against each other, they only bet when they both have a good hand. Therefore, it is not a surprise that the profit of the winner, in this case the Killer, is not high. What is interesting is that the graph shows large steps instead of small ones, like the other graphs. This occurs because of the aggressive behaviour of the killer and the tight play of both players. When the players both participate, the bet is 100 euros. So the steps in the graph are mostly of 100 up or down.

The graph of Rock vs Maniac can be seen below.

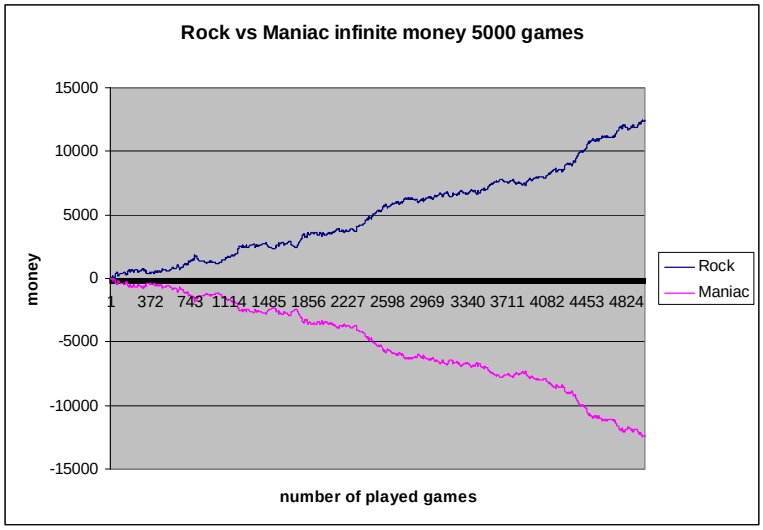


Figure 9: Stack heights of Rock vs Maniac during 5,000 games.

The Rock plays only the top-20 hands. The maniac loses too many hands and too much per hand due to refusing to fold weak hands. The profit of the Rock is around 12,000 euros after 5,000 games.

A graph of the stack heights of Rock vs Calling Station can be seen below.

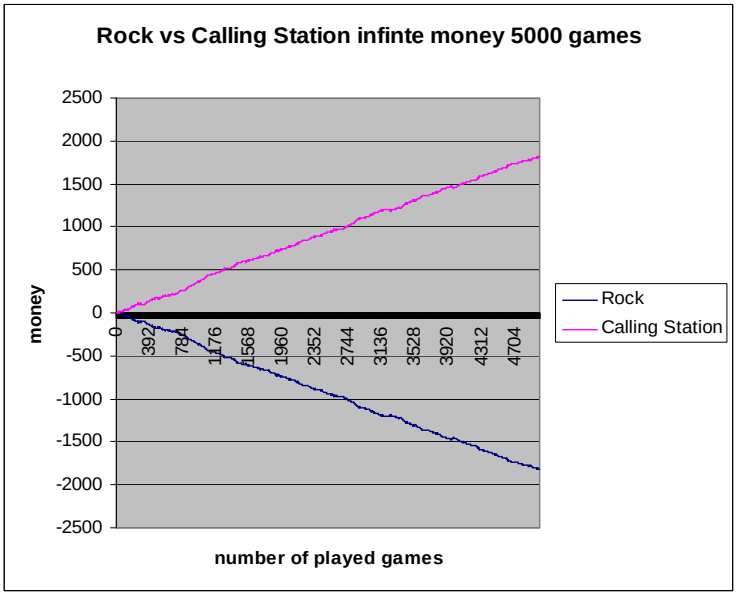


Figure 10: Stack heights of Rock vs Calling Station during 5,000 games.

The Rock plays only the top-20 hands, but does not win much money per hand due to the fact that the Rock only calls instead of raising good hands. The Calling Station picks up money for every hand the Rock folds. The profit of the Calling station therefore is around 1,800 euros after 5,000 games.

8.2 Scenario 2: Finite money

Now we have seen how the strategies perform in the long run with infinite money. However, in tournaments, players do not have unlimited money. Every player starts with a fixed stack of coins and the winner is the player that has all the coins in play.

The results after simulating every strategy against every other strategy during 100 tournaments are shown below in Table 5.

	Rock	Calling Station	Maniac	Killer	Korik	Tournaments won	#players beaten
Rock● -	25	24	0	6	**55**	**0**	5: Number of tournaments won after 100 tournaments against every other strategy. **Calling Station**
-				76	46	-	14	34

Table 5: Number of tournaments won after 100 tournaments against every other strategy.

We see here that strategy Korik wins the most tournaments and is also victorious against every other strategy. Strategy Rock looses against every strategy and has the least tournament wins.

9. Conclusion

Poker has grown throughout the years to become the most popular card game in the world. Especially the introduction of the World Series of Poker and the media attention in the last decade has made poker so well known.

Poker is a game in which mathematics and psychology are two very important aspects. In Chapter 5 we described how important it is to calculate poker odds (or hand odds) and pot odds. The poker odds are defined as the ratio of the number of games a player makes his hand and does not make his hand. Pot odds are defined as the ratio of the amount of money in the pot to how much money it costs to call. In conclusion, when the pot odds are higher than the poker odds calling will give a player profit in the long run.

The final phase in a tournament is heads-up poker. Although heads-up is just a particular form of poker, it has some differences in hand values, but more important, in the strategy of the players. In Chapter 6 the basic strategies were described and in Chapters 7 and 8 a simulation study in heads-up poker for tournament and cash games analyzed four basic strategies and one self-developed strategy. The conclusions of that study are the following. Strategy Rock is a very poor strategy. In the tournament scenario it looses against every other strategy and in the cash game scenario, Rock can make money against the Maniac due to winning large pots with good hands. Against other strategies, the Rock simply does not win enough money with the hands that he plays to compensate for the blinds he has to pay when he folds. So the combination of tight and passive play is not good.

On the other hand loose and aggressive is not that good as well. The Maniac looses against all strategies in the cash game except the Calling Station. In the tournament variant only the Rock can be defeated. Due to the combination of aggressive and loose play, the Maniac looses very much money each game against the tighter players. So this

strategy is very dangerous to play. Players that use this strategy are bound to loose a lot of money.

The Calling Station scores pretty good. This strategy picks up many blinds against the tighter players, but a strategy which has aggressive and tight play can win major pots against the Calling Station, because the Calling Station does not fold weak hands. In real play when players start to notice that one player uses this strategy, they will take advantage of it.

In the scenario with infinite money we saw that the Killer beats all strategies except the Calling Station. This is because the Killer plays too tight against the Calling Station and therefore looses many blinds to him. In the tournament the killer only is beaten by Korik. The strength of the Killer is that he only plays very good hands and bets a lot of money on them. Against loose players this is a very good strategy. However, folding too many hands is in the long run in cash games not profitable against strategies like the Calling Station.

Strategy Korik is able to make the most profit of all strategies in the cash game and the tournament. We saw that Korik looses some money against the Rock and quite a lot against the Killer, but due to enormous profits against especially the Maniac and also the Calling Station those losses are well compensated. The strength of Korik is that he plays by his odds. In a one-on-one game the pot odds are always 1:1, so if the hand odds are higher, in the long run this strategy will win money. However, against a very tight player Korik must be careful. A tight player only plays very good hands, so the hand odds may look better than 1:1, but if a tight player calls are raises those may be worse than 1:1.

Of course the strategy Korik is probably not the best strategy. Better strategies are strategies, which adapt to the opponents' strategy by analyzing it. When a player knows

the strategy of his opponent he has a major advantage. Neural networks and data mining can play an important role in such developments and probably will in the near future.

Almost all strategies have their strengths against certain types of strategies. A good poker player masters all these strategies and is able to detect his opponent's strategy. Once you know your opponent's strategy switching to different strategies will be very profitable. But always be careful, the other player may pull the same trick on you!

Appendix

In the next table the probability that a given hand will end up being the best hand are shown.

o = offsuited, not of the same suit.
s = suited, of the same suit.

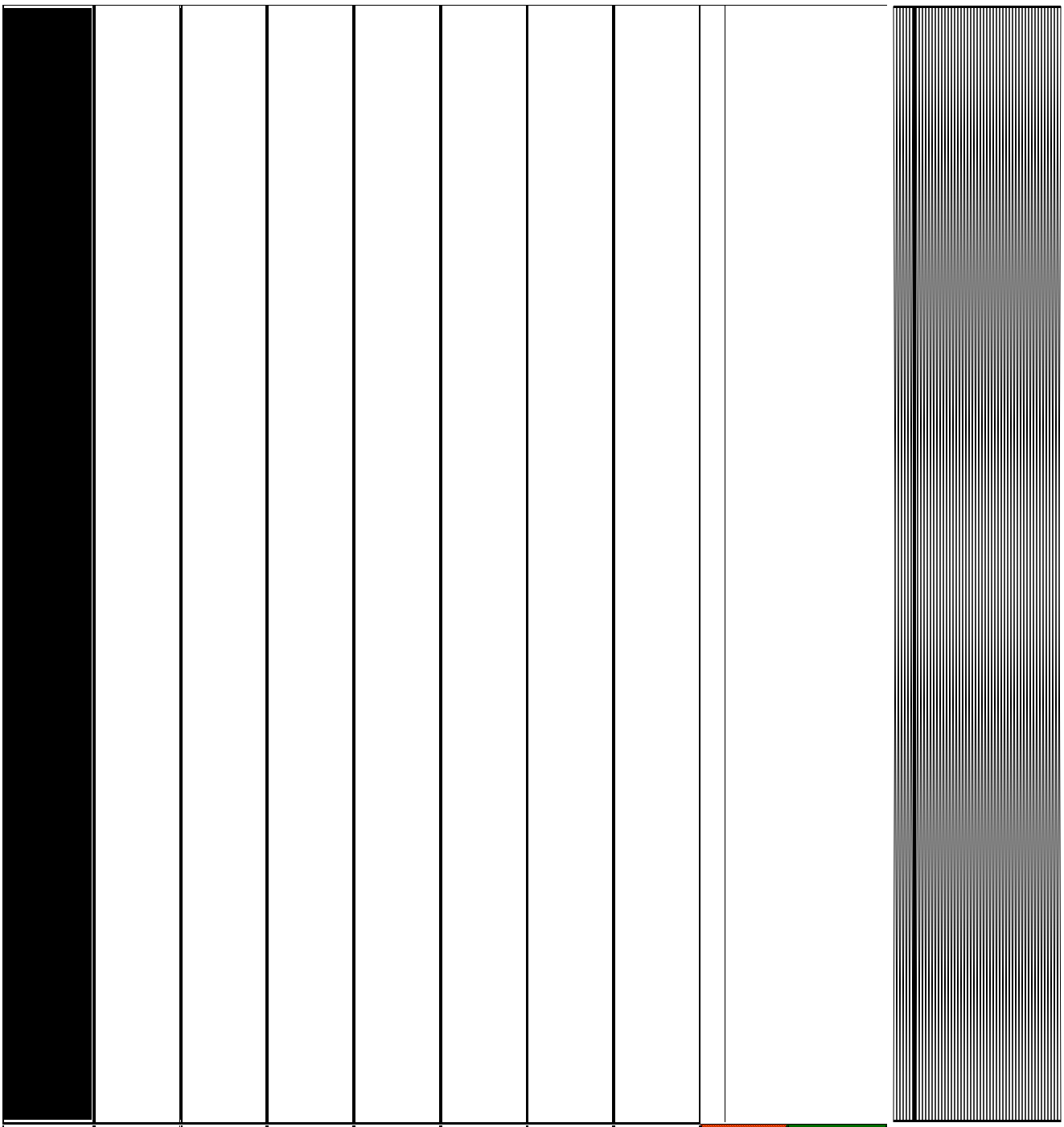
Card										
		85.3% 73.4% 63.9% 55.9% 49.2% 43.6% 38.6%								

		8% ● 34.7% ● 31.1% ● ● AKs ● 67.0% ● 50.7% ● 41.4% ● 35.4% ● 31.1% ● 27.7% ● 25								

		0% ● 22.7% ● 20.7% ●). 4% ● AKo ● 65.4% ● 48.2% ● 38.6% ● 32.4% ● 27.9% ● 24.4%								

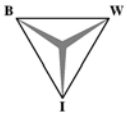
		% ● 2 1 . 6 % ● 1 9 . 2 % ● 1 7 . 2 % ● ● A Q s ● 6 6 . 1 % ● 4 9 . 4 % ● 3 9 . 9 % ● 3 3 . 7 % ● 2 9 . 4 % ● 2 6 .							

		0% • 23.3% • 21.1% • 19.3% • • AQo • 64.5% • 46.8% • 36.9% • 30.4% • 25.9% • 22								



ATo● 62.9 % 62.9%	44.4%	34.1%	27.6%	23.1%	19.8%	17.2%	15.1%	13.4%	● A9s● 63. 0% A9s
A8s● 62.1 % 62.1%	43.7%	33.6%	27.4%	23.3%	20.3%	18.0%	16.2%	14.8%	● A8o● 60. 1% A8o
A7o	59.1%	39.4%	28.9%	22.6%	18.4%	15.4%	13.2%	11.4%	10.1%

A6s	60.0%	41.3%	31.4%	25.6%	21.7%	19.0%	16.9%	15.3%	14.0%
A6o	57.8%	38.0%	27.6%	21.5%	17.5%	14.7%	12.6%	10.9%	9.6%
A5s	59.9%	41.4%	31.8%	26.0%	22.2%	19.6%	17.5%	15.9%	14.5%
A5o	57.7%	38.2%	27.9%	22.0%	18.0%	15.2%	13.1%	11.5%	10.1%
A4s	58.9%	40.4%	30.9%	25.3%	21.6%	19.0%	17.0%	15.5%	14.2%
A4o	56.4%	36.9%	26.9%	21.1%	17.3%	14.7%	12.6%	11.0%	9.8%
A3s	58.0%	39.4%	30.0%	24.6%	21.0%	18.5%	16.6%	15.1%	13.9%
A3o	55.6%	35.9%	26.1%	20.4%	16.7%	14.2%	12.2%	10.7%	9.5%
A2s	57.0%	38.5%	29.2%	23.9%	20.4%	18.0%	16.1%	14.6%	13.4%
A2o	54.6%	35.0%	25.2%	19.6%	16.1%	13.6%	11.7%	10.2%	9.1%
Cards	2 plyrs	3 plyrs	4 plyrs	5 plyrs	6 plyrs	7 plyrs	8 plyrs	9 plyrs	10 plyrs
KK	82.4%	68.9%	58.2%	49.8%	43.0%	37.5%	32.9%	29.2%	26.1%
KQs	63.4%	47.1%	38.2%	32.5%	28.3%	25.1%	22.5%	20.4%	18.6%
KQo	61.4%	44.4%	35.2%	29.3%	25.1%	21.8%	19.1%	16.9%	15.1%
KJs	62.6%	45.9%	36.8%	31.1%	26.9%	23.8%	21.3%	19.3%	17.6%
KJo	60.6%	43.1%	33.6%	27.6%	23.5%	20.2%	17.7%	15.6%	13.9%
KTs	61.9%	44.9%	35.7%	29.9%	25.8%	22.8%	20.4%	18.5%	16.9%
KTo	59.9%	42.0%	32.5%	26.5%	22.3%	19.2%	16.7%	14.7%	13.1%
K9s	60.0%	42.4%	32.9%	27.2%	23.2%	20.3%	18.1%	16.3%	14.8%
K9o	58.0%	39.5%	29.6%	23.6%	19.5%	16.5%	14.1%	12.3%	10.8%
K8s	58.5%	40.2%	30.8%	25.1%	21.3%	18.6%	16.5%	14.8%	13.5%
K8o	56.3%	37.2%	27.3%	21.4%	17.4%	14.6%	12.5%	10.8%	9.4%
K7s	57.8%	39.4%	30.1%	24.5%	20.8%	18.1%	16.0%	14.5%	13.2%
K7o	55.4%	36.1%	26.3%	20.5%	16.7%	13.9%	11.8%	10.2%	9.0%
K6s	56.8%	38.4%	29.1%	23.7%	20.1%	17.5%	15.6%	14.0%	12.8%
K6o	54.3%	35.0%	25.3%	19.7%	16.0%	13.3%	11.3%	9.8%	8.6%
K5s	55.8%	37.4%	28.2%	23.0%	19.5%	17.0%	15.2%	13.7%	12.5%
K5o	53.3%	34.0%	24.5%	19.0%	15.4%	12.9%	11.0%	9.5%	8.3%
K4s	54.7%	36.4%	27.4%	22.3%	19.0%	16.6%	14.8%	13.4%	12.3%
K4o	52.1%	32.8%	23.4%	18.1%	14.7%	12.3%	10.5%	9.1%	8.0%
K3s	53.8%	35.5%	26.7%	21.7%	18.4%	16.2%	14.5%	13.1%	12.1%
K3o	51.2%	31.9%	22.7%	17.6%	14.2%	11.9%	10.2%	8.9%	7.8%
K2s	52.9%	34.6%	26.0%	21.2%	18.1%	15.9%	14.3%	13.0%	11.9%
K2o	50.2%	30.9%	21.8%	16.9%	13.7%	11.5%	9.8%	8.6%	7.6%
Cards	2 plyrs	3 plyrs	4 plyrs	5 plyrs	6 plyrs	7 plyrs	8 plyrs	9 plyrs	10 plyrs
QQ	79.9%	64.9%	53.5%	44.7%	37.9%	32.5%	28.3%	24.9%	22.2%
QJs	60.3%	44.1%	35.6%	30.1%	26.1%	23.0%	20.7%	18.7%	17.1%
QJo	58.2%	41.4%	32.6%	26.9%	22.9%	19.8%	17.3%	15.3%	13.7%
QTs	59.5%	43.1%	34.6%	29.1%	25.2%	22.3%	19.9%	18.1%	16.6%
QTo	57.4%	40.2%	31.3%	25.7%	21.6%	18.6%	16.3%	14.4%	12.9%
Q9s	57.9%	40.7%	31.9%	26.4%	22.5%	19.7%	17.6%	15.9%	14.5%
Q9o	55.5%	37.6%	28.5%	22.9%	19.0%	16.1%	13.8%	12.1%	10.7%
Q8s	56.2%	38.6%	29.7%	24.4%	20.7%	18.0%	16.0%	14.4%	13.2%
Q8o	53.8%	35.4%	26.2%	20.6%	16.9%	14.1%	12.1%	10.5%	9.2%

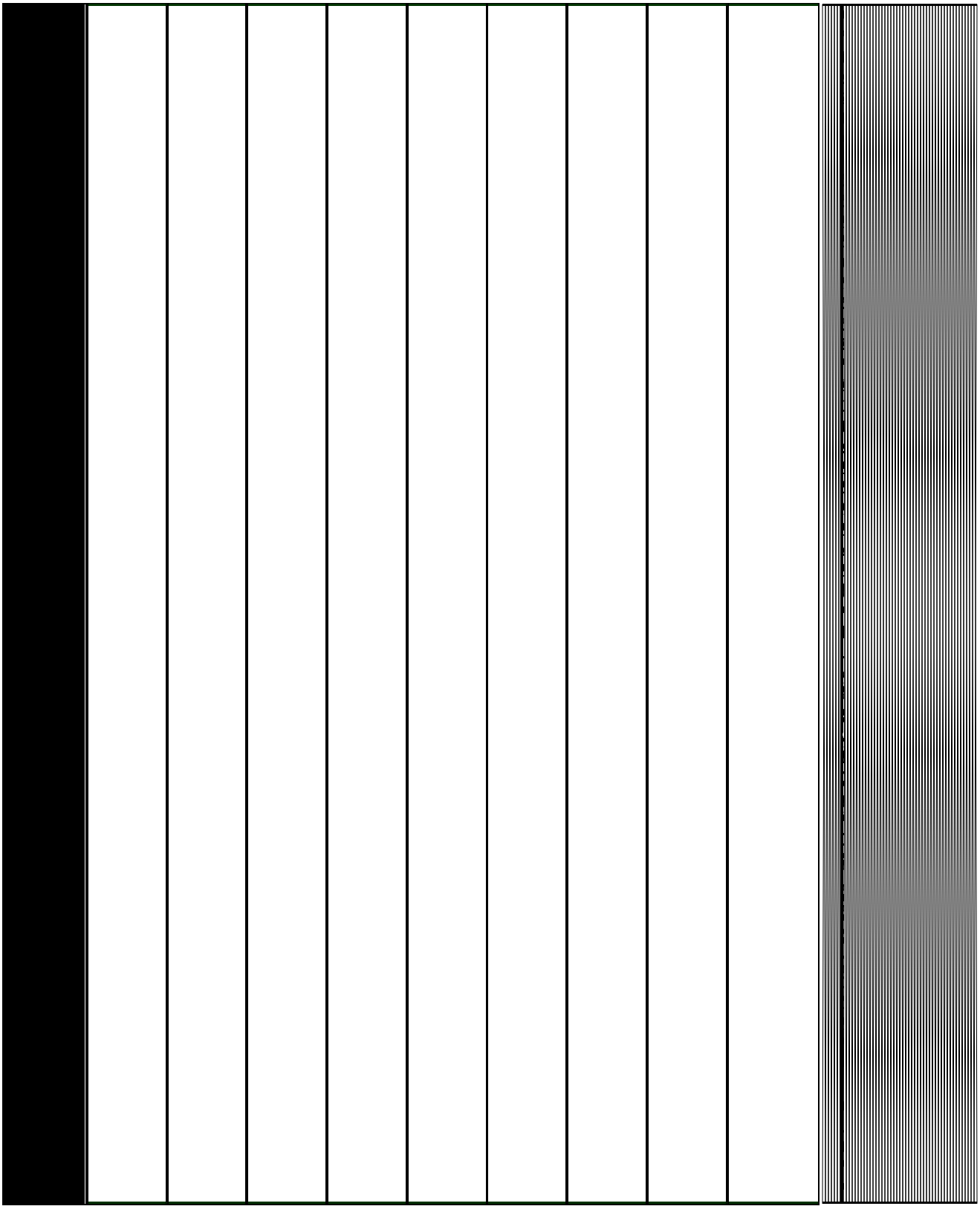


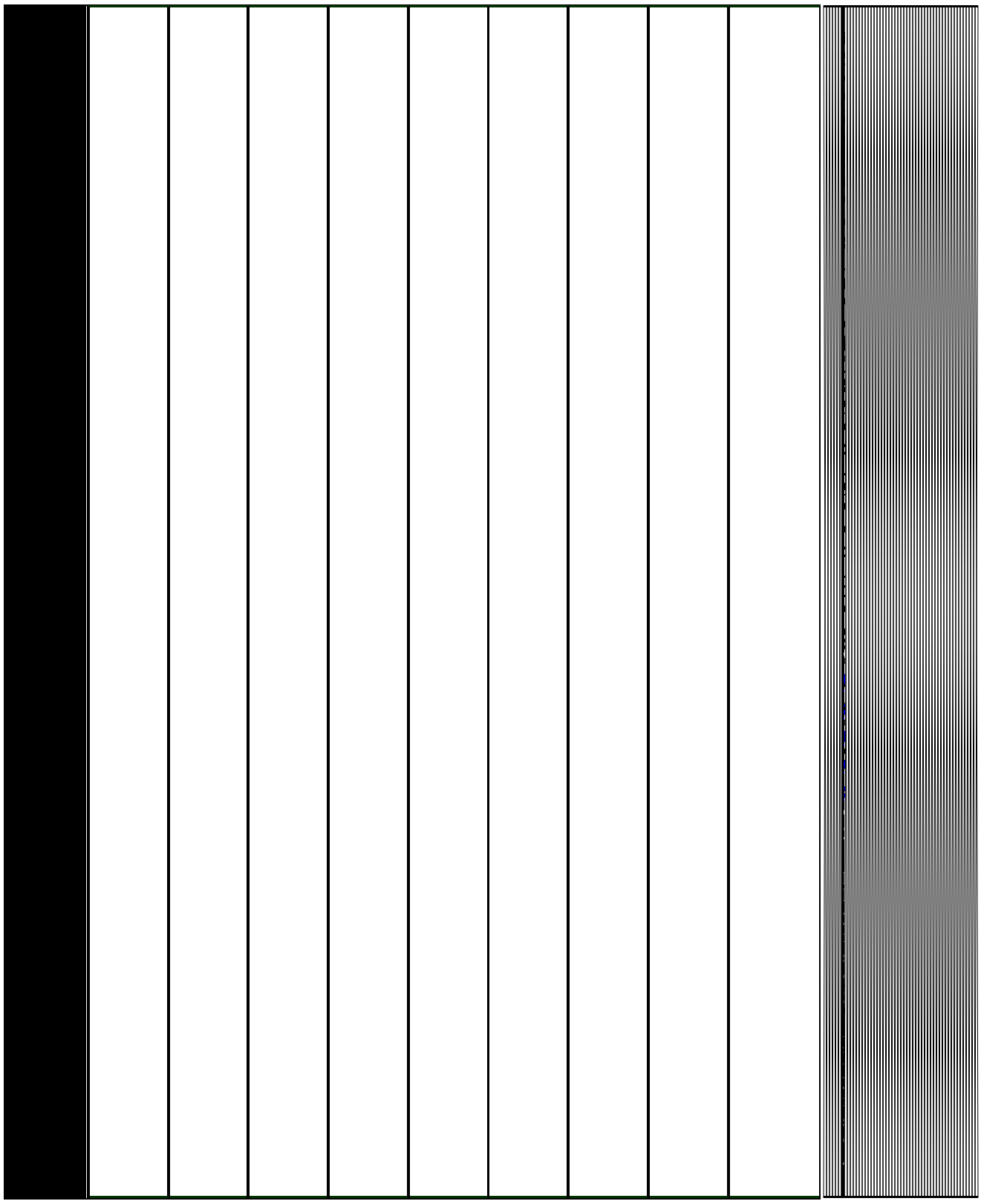
Q7s	54.5%	36.7%	27.9%	22.7%	19.2%	16.7%	14.8%	13.3%	12.1%
Q7o	51.9%	33.2%	24.0%	18.6%	15.1%	12.5%	10.6%	9.2%	8.0%
Q6s	53.8%	35.8%	27.1%	21.9%	18.5%	16.1%	14.3%	12.9%	11.7%
Q6o	51.1%	32.3%	23.2%	17.9%	14.4%	12.0%	10.1%	8.8%	7.6%
Q5s	52.9%	34.9%	26.3%	21.4%	18.1%	15.8%	14.1%	12.7%	11.6%
Q5o	50.2%	31.3%	22.3%	17.3%	13.9%	11.6%	9.8%	8.5%	7.4%
Q4s	51.7%	33.9%	25.5%	20.7%	17.6%	15.4%	13.7%	12.4%	11.3%
Q4o	49.0%	30.2%	21.4%	16.4%	13.3%	11.0%	9.4%	8.1%	7.1%
Q3s	50.7%	33.0%	24.7%	20.1%	17.0%	14.9%	13.3%	12.1%	11.1%
Q3o	47.9%	29.2%	20.7%	15.9%	12.8%	10.7%	9.1%	7.9%	6.9%
Q2s	49.9%	32.2%	24.0%	19.5%	16.6%	14.6%	13.1%	11.9%	10.9%
Q2o	47.0%	28.4%	19.9%	15.3%	12.3%	10.3%	8.8%	7.7%	6.8%

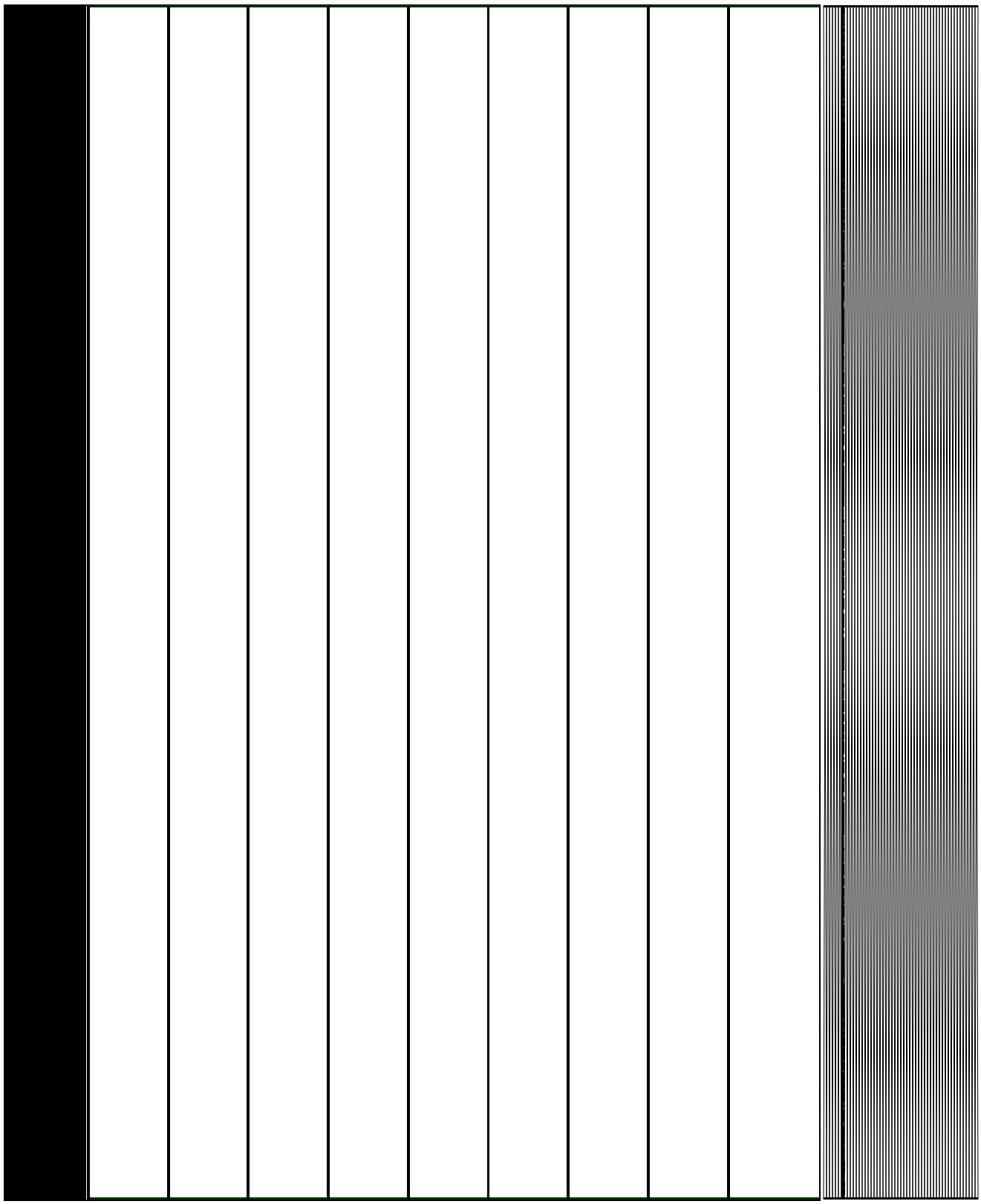
Cards	2 plyrs	3 plyrs	4 plyrs	5 plyrs	6 plyrs	7 plyrs	8 plyrs	9 plyrs	10 plyrs
JJ	77.5%	61.2%	49.2%	40.3%	33.6%	28.5%	24.6%	21.6%	19.3%
JTs	57.5%	41.9%	33.8%	28.5%	24.7%	21.9%	19.7%	17.9%	16.5%
JTo	55.4%	39.0%	30.7%	25.3%	21.5%	18.6%	16.3%	14.5%	13.1%
J9s	55.8%	39.6%	31.3%	26.1%	22.4%	19.7%	17.6%	15.9%	14.6%
J9o	53.4%	36.5%	27.9%	22.5%	18.7%	15.9%	13.8%	12.1%	10.8%
J8s	54.2%	37.5%	29.1%	24.0%	20.5%	17.9%	15.9%	14.4%	13.2%
J8o	51.7%	34.2%	25.6%	20.4%	16.8%	14.1%	12.2%	10.7%	9.5%
J7s	52.4%	35.4%	27.1%	22.2%	18.9%	16.4%	14.6%	13.2%	12.0%
J7o	49.9%	32.1%	23.5%	18.3%	14.9%	12.4%	10.6%	9.2%	8.1%
J6s	50.8%	33.6%	25.4%	20.6%	17.4%	15.2%	13.5%	12.1%	11.1%
J6o	47.9%	29.8%	21.4%	16.5%	13.2%	11.0%	9.3%	8.0%	7.0%
J5s	50.0%	32.8%	24.7%	20.0%	17.0%	14.7%	13.1%	11.8%	10.8%
J5o	47.1%	29.1%	20.7%	15.9%	12.8%	10.6%	8.9%	7.7%	6.7%
J4s	49.0%	31.8%	24.0%	19.4%	16.4%	14.3%	12.8%	11.5%	10.6%
J4o	46.1%	28.1%	19.9%	15.3%	12.3%	10.2%	8.6%	7.5%	6.5%
J3s	47.9%	30.9%	23.2%	18.8%	16.0%	14.0%	12.5%	11.3%	10.4%
J3o	45.0%	27.1%	19.1%	14.6%	11.7%	9.8%	8.3%	7.2%	6.3%
J2s	47.1%	30.1%	22.6%	18.3%	15.6%	13.7%	12.2%	11.1%	10.2%
J2o	44.0%	26.2%	18.4%	14.1%	11.3%	9.4%	8.0%	7.0%	6.2%

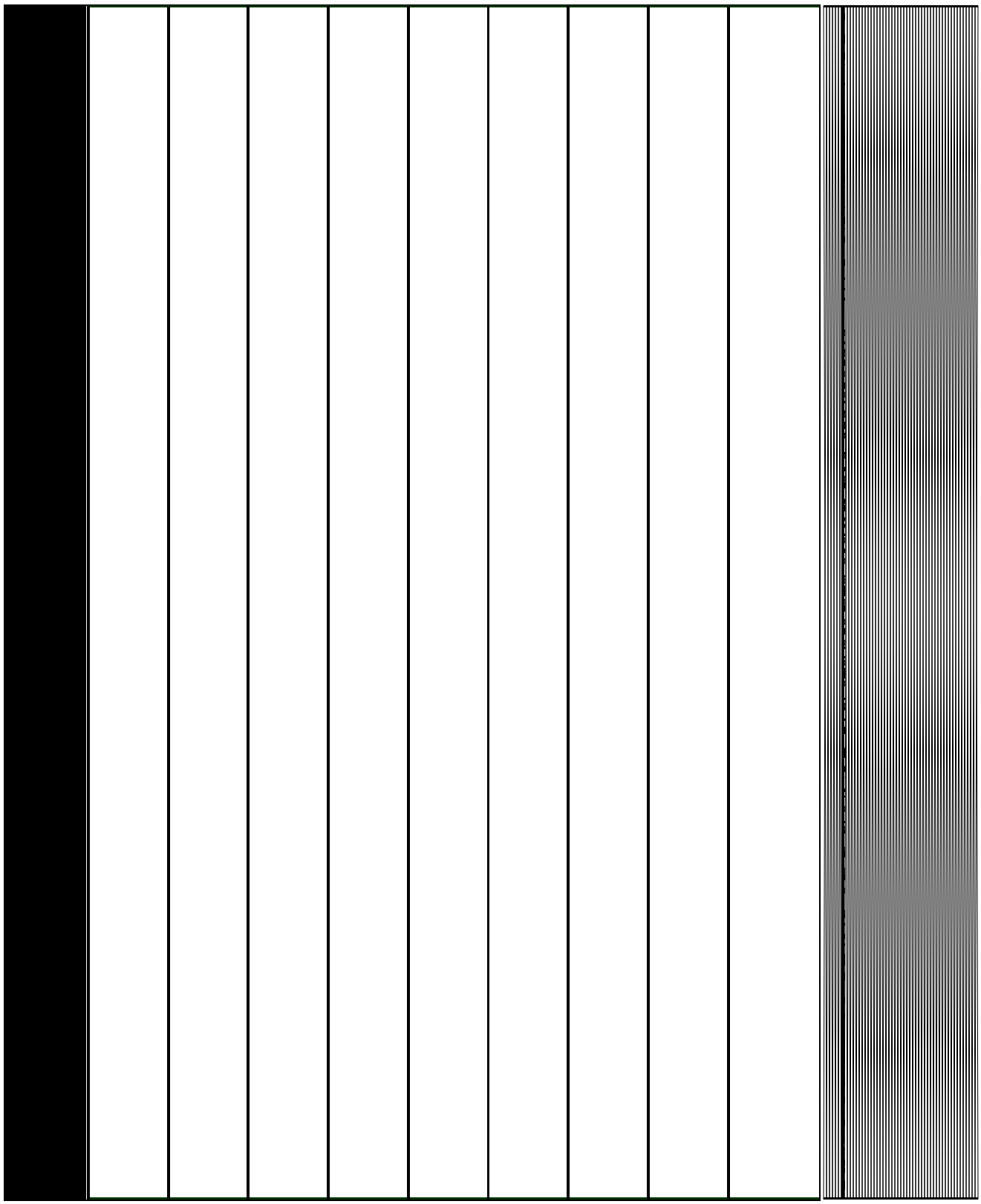
Cards	2 plyrs	3 plyrs	4 plyrs	5 plyrs	6 plyrs	7 plyrs	8 plyrs	9 plyrs	10 plyrs
TT	75.1%	57.7%	45.2%	36.4%	30.0%	25.3%	21.8%	19.2%	17.2%
T9s	54.3%	38.9%	31.0%	26.0%	22.5%	19.8%	17.8%	16.2%	14.9%
T9o	51.7%	35.7%	27.7%	22.5%	18.9%	16.2%	14.1%	12.6%	11.3%
T8s	52.6%	36.9%	29.0%	24.0%	20.6%	18.1%	16.2%	14.8%	13.6%
T8o	50.0%	33.6%	25.4%	20.4%	16.9%	14.4%	12.5%	11.0%	9.9%
T7s	51.0%	34.9%	27.0%	22.2%	19.0%	16.6%	14.8%	13.5%	12.4%
T7o	48.2%	31.4%	23.4%	18.4%	15.1%	12.8%	11.0%	9.7%	8.6%
T6s	49.2%	32.8%	25.1%	20.5%	17.4%	15.2%	13.6%	12.3%	11.2%
T6o	46.3%	29.2%	21.2%	16.5%	13.4%	11.2%	9.5%	8.3%	7.3%
T5s	47.2%	30.8%	23.3%	18.9%	16.0%	13.9%	12.4%	11.2%	10.2%
T5o	44.2%	27.1%	19.3%	14.8%	11.9%	9.9%	8.4%	7.2%	6.4%

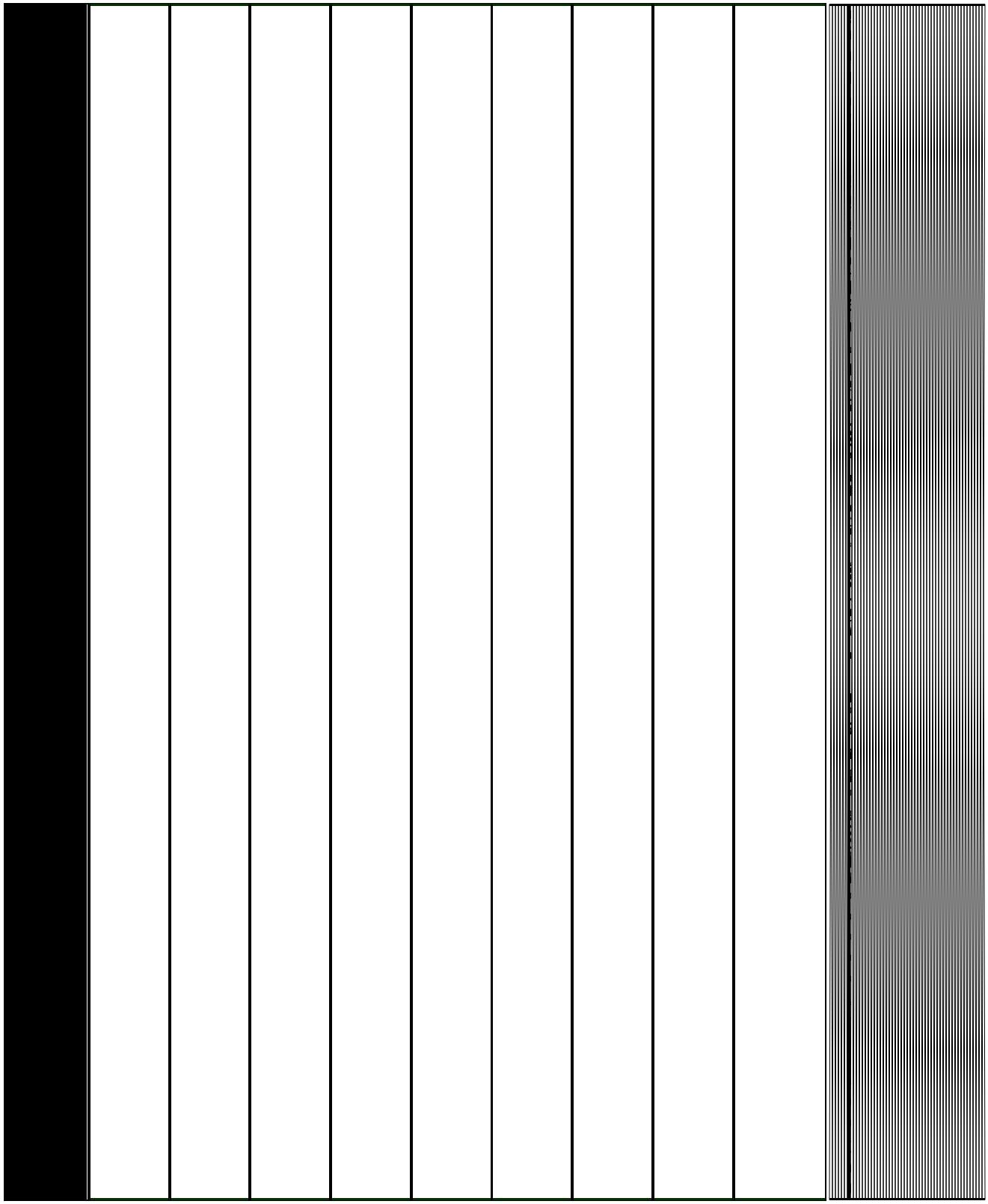
T4s	46.4%	30.1%	22.7%	18.4%	15.6%	13.6%	12.1%	11.0%	10.0%
T4o	43.4%	26.4%	18.7%	14.3%	11.5%	9.5%	8.1%	7.0%	6.2%
T3s	45.5%	29.3%	22.0%	17.8%	15.1%	13.2%	11.8%	10.7%	9.8%
T3o	42.4%	25.5%	18.0%	13.7%	11.0%	9.1%	7.8%	6.8%	6.0%
T2s	44.7%	28.5%	21.4%	17.4%	14.8%	13.0%	11.6%	10.5%	9.7%
T2o	41.5%	24.7%	17.3%	13.2%	10.6%	8.8%	7.5%	6.6%	5.8%
Cards	2 plyrs	3 plyrs	4 plyrs	5 plyrs	6 plyrs	7 plyrs	8 plyrs	9 plyrs	10 plyrs
99	72.1%	53.5%	41.1%	32.6%	26.6%	22.4%	19.4%	17.2%	15.6%
98s	51.1%	36.0%	28.5%	23.6%	20.2%	17.8%	15.9%	14.5%	13.4%
98o	48.4%	32.9%	25.1%	20.1%	16.6%	14.2%	12.3%	10.9%	9.9%
97s	49.5%	34.2%	26.8%	22.1%	18.9%	16.6%	14.9%	13.6%	12.5%
97o	46.7%	30.9%	23.1%	18.4%	15.1%	12.8%	11.1%	9.8%	8.8%

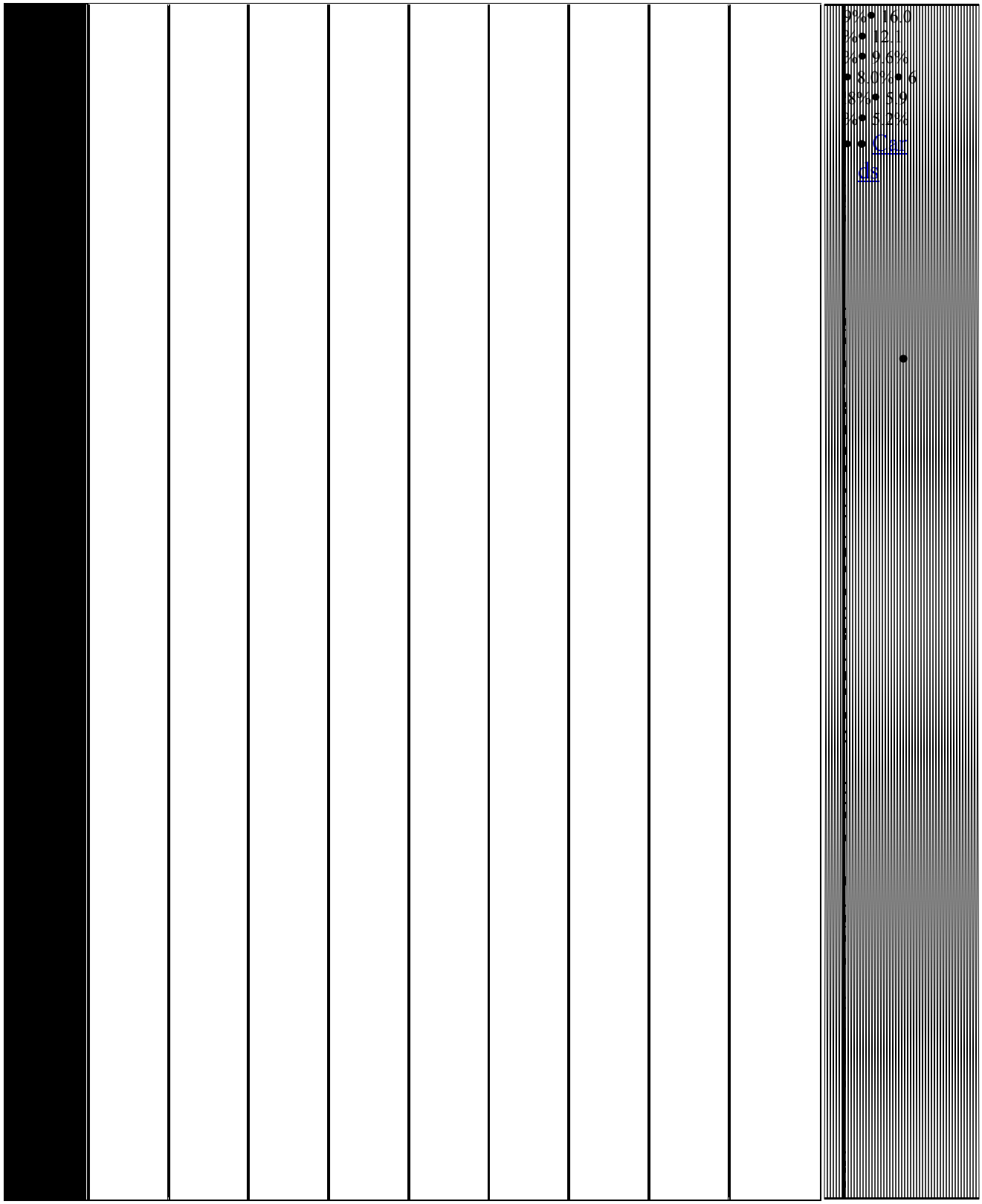


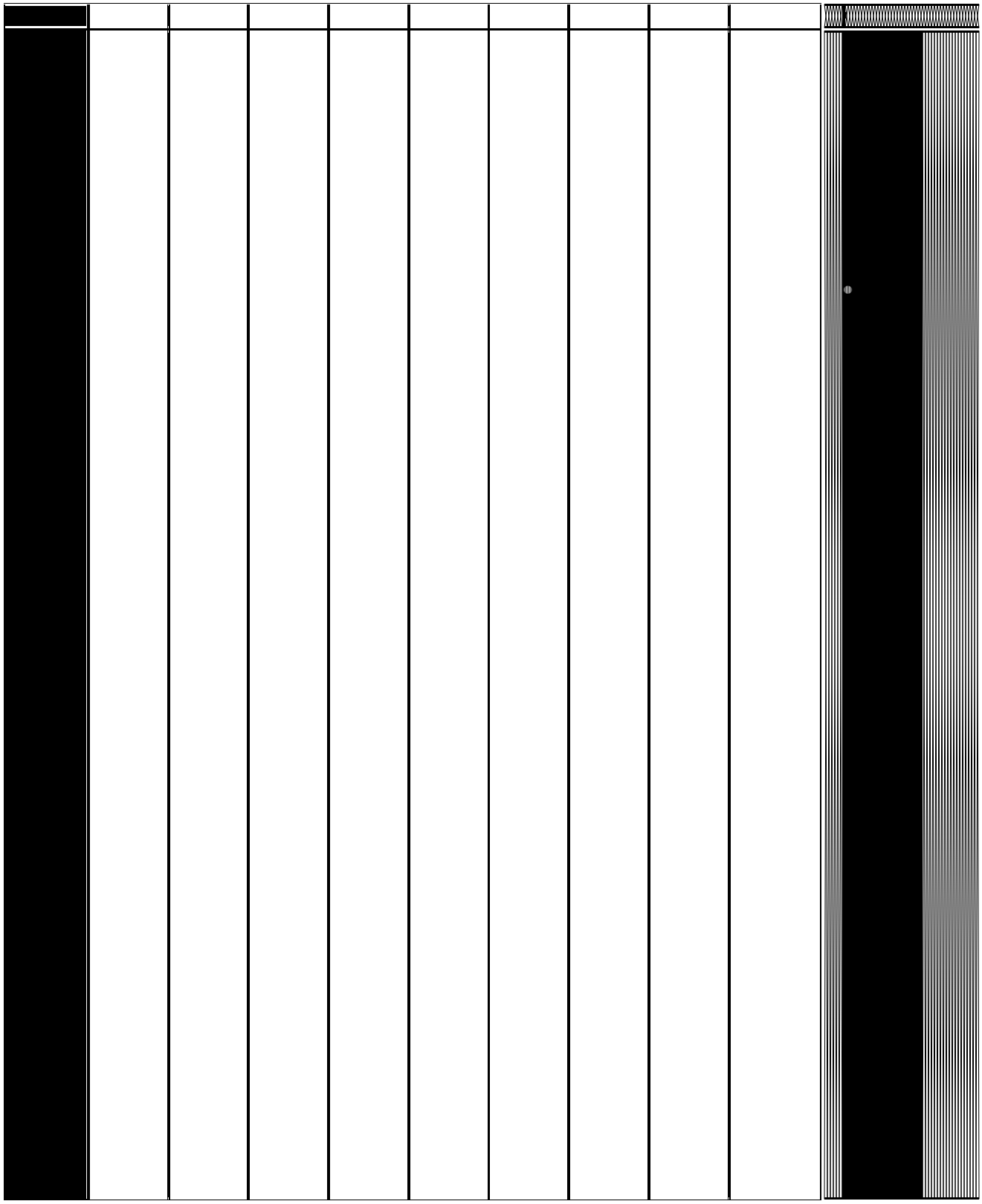


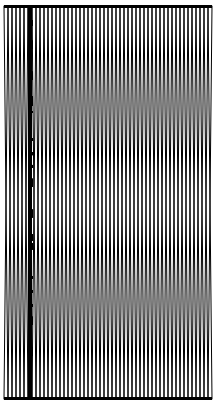







9%
16.0
3%
12.1
3%
9.6%
8.0%
6
8%
5.9
3%
5.2%
Cards





85s	44.8%	30.2%	23.2%	19.1%	16.3%	14.3%	12.9%	11.8%	10.9%
85o	41.7%	26.5%	19.4%	15.2%	12.4%	10.5%	9.1%	8.1%	7.3%
84s	42.7%	28.1%	21.4%	17.4%	14.8%	13.0%	11.7%	10.6%	9.8%
84o	39.6%	24.4%	17.5%	13.4%	10.8%	9.0%	7.8%	6.8%	6.1%
83s	40.8%	26.3%	19.8%	16.0%	13.6%	11.9%	10.7%	9.7%	8.9%
83o	37.5%	22.4%	15.7%	11.9%	9.5%	7.9%	6.7%	5.8%	5.1%
82s	40.3%	25.8%	19.4%	15.7%	13.3%	11.7%	10.5%	9.6%	8.8%
82o	36.8%	21.7%	15.1%	11.4%	9.1%	7.5%	6.4%	5.6%	4.9%
Cards	2 plyrs	3 plyrs	4 plyrs	5 plyrs	6 plyrs	7 plyrs	8 plyrs	9 plyrs	10 plyrs
77	66.2%	46.4%	34.4%	26.8%	21.9%	18.6%	16.4%	14.8%	13.7%
76s	45.7%	32.0%	25.1%	20.8%	18.0%	15.9%	14.4%	13.2%	12.3%
76o	42.7%	28.5%	21.5%	17.1%	14.2%	12.2%	10.8%	9.6%	8.8%
75s	43.8%	30.1%	23.4%	19.4%	16.7%	14.8%	13.4%	12.3%	11.4%
75o	40.8%	26.5%	19.7%	15.5%	12.8%	11.0%	9.7%	8.7%	7.9%
74s	41.8%	28.2%	21.7%	17.9%	15.3%	13.5%	12.2%	11.2%	10.4%
74o	38.6%	24.5%	17.9%	13.9%	11.4%	9.7%	8.5%	7.6%	6.8%
73s	40.0%	26.3%	20.0%	16.4%	14.0%	12.3%	11.1%	10.1%	9.3%
73o	36.6%	22.4%	16.0%	12.3%	9.9%	8.4%	7.2%	6.4%	5.7%
72s	38.1%	24.5%	18.4%	15.0%	12.8%	11.2%	10.1%	9.2%	8.5%
72o	34.6%	20.4%	14.2%	10.7%	8.6%	7.2%	6.1%	5.4%	4.8%
Cards	2 plyrs	3 plyrs	4 plyrs	5 plyrs	6 plyrs	7 plyrs	8 plyrs	9 plyrs	10 plyrs
66	63.3%	43.2%	31.5%	24.5%	20.1%	17.3%	15.4%	14.0%	13.1%
65s	43.2%	30.2%	23.7%	19.7%	17.0%	15.2%	13.8%	12.7%	11.9%
65o	40.1%	26.7%	20.0%	15.9%	13.3%	11.5%	10.2%	9.2%	8.5%
64s	41.4%	28.5%	22.1%	18.4%	15.9%	14.2%	12.9%	11.9%	11.1%
64o	38.0%	24.7%	18.2%	14.4%	12.0%	10.3%	9.2%	8.3%	7.6%
63s	39.4%	26.5%	20.4%	16.8%	14.5%	12.9%	11.7%	10.8%	10.0%
63o	35.9%	22.7%	16.4%	12.8%	10.6%	9.1%	8.0%	7.2%	6.5%
62s	37.5%	24.8%	18.8%	15.4%	13.3%	11.8%	10.7%	9.8%	9.1%
62o	34.0%	20.7%	14.6%	11.2%	9.1%	7.8%	6.8%	6.0%	5.4%
Cards	2 plyrs	3 plyrs	4 plyrs	5 plyrs	6 plyrs	7 plyrs	8 plyrs	9 plyrs	10 plyrs
55	60.3%	40.1%	28.8%	22.4%	18.5%	16.0%	14.4%	13.2%	12.3%
54s	41.1%	28.8%	22.6%	18.9%	16.5%	14.8%	13.5%	12.5%	11.7%

54o	37.9%	25.2%	18.8%	15.0%	12.6%	11.0%	9.8%	8.9%	8.2%
53s	39.3%	27.1%	21.1%	17.5%	15.2%	13.7%	12.5%	11.6%	10.8%
53o	35.8%	23.3%	17.1%	13.6%	11.4%	9.9%	8.8%	8.0%	7.3%
52s	37.5%	25.3%	19.5%	16.1%	14.0%	12.5%	11.4%	10.6%	9.8%
52o	33.9%	21.3%	15.3%	12.0%	10.0%	8.6%	7.6%	6.8%	6.2%
Cards	2 plyrs	3 plyrs	4 plyrs	5 plyrs	6 plyrs	7 plyrs	8 plyrs	9 plyrs	10 plyrs
44	57.0%	36.8%	26.3%	20.6%	17.3%	15.2%	13.9%	12.9%	12.1%
43s	38.0%	26.2%	20.3%	16.9%	14.7%	13.1%	12.0%	11.1%	10.3%
43o	34.4%	22.3%	16.3%	12.8%	10.7%	9.3%	8.3%	7.5%	6.8%
42s	36.3%	24.6%	18.8%	15.7%	13.7%	12.3%	11.2%	10.4%	9.6%
42o	32.5%	20.5%	14.7%	11.5%	9.5%	8.3%	7.3%	6.6%	6.0%
Cards	2 plyrs	3 plyrs	4 plyrs	5 plyrs	6 plyrs	7 plyrs	8 plyrs	9 plyrs	10 plyrs
33	53.7%	33.5%	23.9%	19.0%	16.2%	14.6%	13.5%	12.6%	12.0%
32s	35.1%	23.6%	18.0%	14.9%	13.0%	11.7%	10.7%	9.9%	9.2%
32o	31.2%	19.5%	13.9%	10.8%	8.9%	7.7%	6.8%	6.1%	5.6%
Cards	2 plyrs	3 plyrs	4 plyrs	5 plyrs	6 plyrs	7 plyrs	8 plyrs	9 plyrs	10 plyrs
22	50.3%	30.7%	22.0%	17.8%	15.5%	14.2%	13.3%	12.5%	12.0%

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