Concurrent Access to Mobile Networks

Processor Sharing Servers in Parallel

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Master's Thesis

vrije Universiteit amsterdam

CWI

The whole is more than the sum of its parts

Aristotle

May 2008

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Preface

This Master's thesis is part of my internship at the *Probability and Stochastic Networks* or *PNA2* department of the *Center for Mathematics and Computer Science (CWI)* in Amsterdam. PNA is an abbreviation for *Probability, Networks and Algorithms*, which is the name of the scientific cluster that comprises this department. The internship is part of my Mathematics study at the *VU University* in Amsterdam, where VU stands for its Dutch name "Vrije Universiteit." It is also the final assignment for obtaining my Master of Science (M.Sc.) degree. My supervisors were *prof. dr. Rob van der Mei* at the CWI and *dr. Sandjai Bhulai* at the VU. *Ir. Gerard Hoekstra* was my subsupervisor at the CWI.

Prof. dr. Rob van der Mei is heading the PNA2 department at the CWI and a professor in performance analysis and communication networks at the VU. He has sufficiently supported me during my internship with a lot of feedback, despite his tight schedule. Many thanks to him, also for giving me this great assignment. I had fun with it.

Dr. Sandjai Bhulai is an assistant professor in applied probability (Operations Research) at the VU and a researcher at the PNA2 department of the CWI. My thanks to him for his tremendous support and for correcting this thesis.

Ir. Gerard Hoekstra is a Ph.D. candidate and a researcher at the PNA2 department of the CWI. This thesis is a resource for his Casimir funded project at the CWI, which is supervised by prof. Rob van der Mei. It is named "Analysis of distribution strategies for concurrent access in wireless communication networks." My appreciation for Gerard's comments.

It was really educational and a pleasure working with them. I am also grateful to other staff members of the CWI who helped me, in particular *Ms. Wemke van der Weij* for setting up Extend for me and teaching me some of the basics. I also want to mention *Ms. Regina Egorova*, with whom I shared an office at the CWI the longest. It was great for having her as a roommate. Furthermore, I want to give my gratitudes to *Mr. René Bekker* for being the second reader of this thesis and everyone who helped me during my study at the VU, in particular *dr. Freek van Schagen* for his supervision. And finally, I want to thank the *Lord* for making this all possible.

Ik vond het een leuke en leerzame stageopdracht. Bedankt!

Ronny Gunawan

Amsterdam, May 2008

Summary

Today's mobile networks provide full coverage in both densely populated and rural areas, and consequently, any given location is typically covered by multiple mobile networks, while at the same time the use of multi-antenna devices is gaining momentum. This phenomenon of so-called concurrent access opens up many possibilities for splitting traffic streams over different mobile networks in order to increase capacity and to improve performance.

We analyse two types of models with parallel processor sharing servers. In the first model, arriving jobs have access to only one of the processor sharing servers with a certain probability (the server selection model). In the second model, each job is split into parts and distributed over parallel processor sharing servers (the job split model). First, we analyse the optimal server selection probability and then the optimal job-split factor. We compare the models to each other. We thereby use analytical mathematical results and validate the model using simulation.

Contents

Chapter 1

Introduction

1.1 About the Institute

The Center for Mathematics and Computer Science (CWI), or "Centrum voor Wiskunde en Informatica" in Dutch, is the national research institute for mathematics and computer science in the Netherlands, located at the Science Park in Amsterdam. The CWI performs frontier research in mathematics and computer science and transfers new knowledge in these fields to society in general, and trade and industry in particular. It is one of the founding members of the European Research Consortium for Informatics and Mathematics (ERCIM). The ERCIM comprises research institutes in 17 different European countries with altogether more than 10,000 active researchers in the fields of computer science and applied mathematics.

The CWI was founded in 1946. It was originally called the Mathematical Center, or "Mathematisch Centrum" in Dutch, but it changed in 1983 to its current name due to the strong computer science component in its research.

1.2 Motivation for Concurrent Access

Today, there are multiple mobile networks commercially available, namely General Packet Radio Service (GPRS), Enhanced Data Rates for GSM Evolution (EDGE), Universal Mobile Telecommunications System (UMTS) and High-Speed Downlink Packet Access (HSDPA) [1] and mobile devices that support multiple networks. However, they are only capable of utilising a single network at the time. Moreover, it is often the case that these networks overlap each other in terms of coverage, especially in densely populated areas. Since wireless networks usually provide limited bandwidth and are subject to significant interference, the user-perceived performance of these networks is less than acceptable in many cases. To overcome this problem, a promising new technique, called *Concurrent Access (CA)*, allows end-user terminals to utilise multiple networks simultaneously, thereby increasing the available amount of bandwidth and providing robustness of the application to the end user.

(a) Samsung SGH-i600. (b) Palm Treo 750. (c) E-Ten X800 Glofiish.

Figure 1.1: Examples of mobile devices that support multiple networks (HSDPA, UMTS, EDGE and GPRS).

The main objective is to devise and evaluate simple but effective methods to realise the potential benefits of CA in wireless communication networks. These methods strive for optimal distribution and re-assembly of the traffic streams along the different paths between the distribution server and the terminal. The method that we consider in this thesis is the study of mathematical models that represent these networks, Processor Sharing servers in particular. We model the download requests as foreground jobs, we consider the required service time as a representation for the requested download time and we let the sojourn time be a model for the actual download time. We also introduce dummy download requests, in order to model the fact that the capacity of the networks is not always fully

[1] Interesting articles on wireless networks, UMTS and HSDPA in particular, are [2, 4, 5, 10, 11, 12, 13, 14, 15].

available. We refer to these requests as background jobs.

1.3 Structure of the Thesis

We consider models with two Processor Sharing servers and three traffic streams. One stream has access to both servers, while the other two have access to one server only. Jobs that may use both servers are called foreground jobs, as opposed to background jobs. We have two types of models. The server selection models let each foreground job go to one of the servers. The job split models, on the other hand, divide each job in two parts and then let each part go to one of the servers. The first objectives are to find an optimal server selection probability in the server selection model and an optimal split factor in the job split model. The other objective is to show which type of model is the better one by looking at the mean sojourn time of these jobs. Next to these static models, there is an additional dynamic model of the server selection strategy, constructed by means of Markov Decision chains and we compare this to its static counterpart by the mean sojourn times of the jobs.

We perform the analyses by means of mathematical theory, accompanied with simulation results. The simulations are done by means of a software program called Extend. Extend gives a unique identification number for each block and lets the corresponding seed parameter be equal to this number, which justifies the randomness of the model. We also use a self-made application in order to transform the raw output data.

Chapter 2

Preliminaries

The models that we are going to develop, require mathematical knowledge at university level, queueing theory in particular. This theory is partly summarised in this chapter.

2.1 Probability Theory

We mainly consider continuous stochastic variables X that possess a cumulative distribution function F_X and a probability density function f_X on the *probability space* $(\Omega, \mathcal{A}, P)$. The *cumulative distribution function* F_X is defined as:

$$F_X(x) := P(X \leq x) = P(\{y : X(y) \leq x\}) = \int_{-\infty}^{x} dF_X(y), \quad x \in \mathbb{R},$$

where P is the *probability measure*. The cumulative distribution function possesses this property:

$$\int_{-\infty}^{\infty} dF_X(x) = 1.$$

The *probability density function* f_X, is a function that is related to the cumulative distribution function as follows:

$$F_X(x) = \int_{-\infty}^{x} f_X(t)\, dt.$$

The *n-th moment* is defined as:

$$E[X^n] := \int X^n dP = \int_{-\infty}^{\infty} x^n \, dF_X(x).$$

The first moment is also called the *expectation*, or the *mean*. The *variance* is defined as:

$$\mathrm{Var}[X] := E[X^2] - (E[X])^2.$$

and the *squared coefficient of variation* is defined as:

$$c_X^2 := \frac{\mathrm{Var}[X]}{(E[X])^2}.$$

2.1.1 Exponential Distribution

A stochastic variable X is *exponentially* distributed with rate μ, denoted by $\mathrm{Exp}(\mu)$, if its probability density is:

$$f_X(x) = \mu e^{-\mu x}, \quad x \geq 0.$$

Its mean, variance and squared coefficient of variation are:

$$E[X] = \frac{1}{\mu}, \quad \mathrm{Var}[X] = \frac{1}{\mu^2}, \quad c_X^2 = 1.$$

An important property of an exponentially distributed random variable X is the *memoryless property.* This property states that for all $x \geq 0$ and $t \geq 0$:

$$P(X > t + x | X > t) = P(X > x) = e^{-\mu x}.$$

Suppose that X_1 and X_2 are independent exponentially distributed with rate μ_1, μ_2. Then the maximum $Y := \max\{X_1, X_2\}$ has the following mean:

$$E[Y] = \frac{1}{\mu_1} + \frac{1}{\mu_2} - \frac{1}{\mu_1 + \mu_2}.$$

Proof:

$$F_Y(x) := P(Y < x) = P(X_1 < x, X_2 < x) \overset{indep}{=} P(X_1 < x_1)P(X_2 < x_2)$$
$$= (1 - e^{-\mu_1 x})(1 - e^{-\mu_2 x}) = 1 - e^{-\mu_1 x} - e^{-\mu_2 x} + e^{-(\mu_1+\mu_2)x}$$
$$f_Y(x) = F_Y'(x) = \mu_1 e^{-\mu_1 x} + \mu_2 e^{-\mu_2 x} - (\mu_1 + \mu_2)e^{-(\mu_1+\mu_2)x}.$$

The proof can be completed by means of the definition of the expectation. □

If $X_1, X_2, ..., X_n$ are independent exponentially distributed with rate $\mu_1, \mu_2, ..., \mu_n$, then the probability that X_i is the smallest is as follows (cf. Boxma [6]):

$$P(X_i = \min_{j=1,2,\ldots,n}\{X_j\}) = \frac{\mu_i}{\mu_1 + \mu_2 + ... + \mu_n}.$$

2.1.2 Hyperexponential Distribution

A stochastic variable X has a *hyperexponential* distribution if it is exponential with rate μ_i with probability p_i, for $i = 1, \dots, k$. This is denoted by $H_k(p_1, \dots, p_k; \mu_1, \dots, \mu_k)$. Obviously, $\sum_{i=1}^k p_i = 1$. Its probability density is:

$$f_X(x) = \sum_{i=1}^{k} p_i \mu_i e^{-\mu_i x}, \qquad x \geq 0,$$

and its mean and second moment are:

$$E[X] = \sum_{i=1}^{k} \frac{p_i}{\mu_i}, \qquad E[X^2] = \sum_{i=1}^{k} \frac{2p_i}{\mu_i^2}.$$

For $k = 2$, we define $p := p_1 = 1 - p_2$. Its probability density then reduces to:

$$f_X(x) = p\mu_1 e^{-\mu_1 x} + (1-p)\mu_2 e^{-\mu_2 x}, \qquad x > 0,$$

with mean and second moment:

$$E[X] = \frac{p}{\mu_1} + \frac{1-p}{\mu_2}, \qquad E[X^2] = \frac{2p}{\mu_1^2} + \frac{2(1-p)}{\mu_2^2}.$$

It is common to add another condition, known as the balanced means:

$$\frac{p}{\mu_1} = \frac{1-p}{\mu_2}.$$

Then its mean and second moment reduce to:

$$\begin{aligned} E[X] &= \frac{2p}{\mu_1} = \frac{2(1-p)}{\mu_2}, \\ E[X^2] &= \frac{2p}{\mu_1^2} + \frac{2p^2}{1-p}\frac{1}{\mu_1^2} = \frac{2(1-p)}{\mu_2^2} + \frac{2(1-p)^2}{p}\frac{1}{\mu_2^2}, \end{aligned} \tag{2.1}$$

and its squared coefficient of variation is:

$$c_X^2 = \frac{1 - 2p(1-p)}{2p(1-p)} \geq 1. \tag{2.2}$$

Equivalently:

$$p = \frac{1}{2} \pm \frac{1}{2}\sqrt{\frac{c_X^2 - 1}{c_X^2 + 1}}. \tag{2.3}$$

Proof of Equation (2.1):

$$\frac{p}{\mu_1} = \frac{1-p}{\mu_2} \Leftrightarrow \mu_2 = \frac{1-p}{p}\mu_1$$

$$E[X^2] = \frac{2p}{\mu_1^2} + \frac{2(1-p)}{\mu_2^2} = \frac{2p}{\mu_1^2} + 2(1-p)\frac{p^2}{(1-p)^2}\frac{1}{\mu_1^2}$$

$$= \frac{2p}{\mu_1^2} + \frac{2p^2}{1-p}\frac{1}{\mu_1^2}. \qquad \square$$

This can be proven in terms of μ_2 in a similar way.

Proof of Equation (2.2):

$$c_X^2 = \frac{E[X^2] - (E[X])^2}{(E[X])^2} = \frac{\frac{2p}{\mu_1^2} + \frac{2p^2}{1-p}\frac{1}{\mu_1^2} - \frac{4p^2}{\mu_1^2}}{\frac{4p^2}{\mu_1^2}}$$

$$= \frac{2p + \frac{2p^2}{1-p} - 4p^2}{4p^2} = \frac{1 + \frac{p}{1-p} - 2p}{2p}$$

$$= \frac{(1-p) + p - 2p(1-p)}{2p(1-p)} = \frac{1-2p(1-p)}{2p(1-p)}. \qquad \square$$

It can be similarly shown in terms of μ_2.

Proof of Equation (2.3):

$$c_X^2 = \frac{1-2p(1-p)}{2p(1-p)}$$

$$1 - 2p(1-p) = 2p(1-p)c_X^2$$

$$1 - 2p + 2p^2 = (2p - 2p^2)c_X^2$$

$$2(1+c_X^2)p^2 - 2(1+c_X^2)p + 1 = 0$$

$$p = \frac{2(1+c_X^2) \pm \sqrt{4(1+c_X^2)^2 - 4 * 2(1+c_X^2)}}{4(1+c_X^2)}$$

$$p = \frac{1}{2} \pm \frac{2\sqrt{(1+c_X^2)^2 - 2(1+c_X^2)}}{4(1+c_X^2)}$$

$$p = \frac{1}{2} \pm \frac{1}{2}\sqrt{1 - \frac{2}{1+c_X^2}} = \frac{1}{2} \pm \frac{1}{2}\sqrt{\frac{c_X^2 - 1}{c_X^2 + 1}}. \qquad \square$$

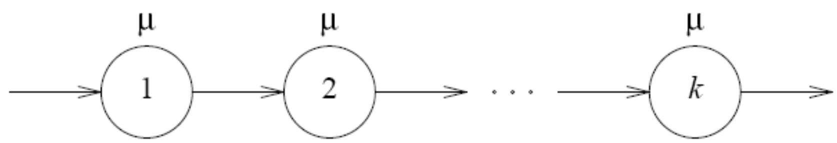


Figure 2.1: Phase diagram for the $E_k(\mu)$ distribution.

Consequently, $c_X^2 \geq 1$. Note that X is exponentially distributed with rate $\mu_1 = \mu_2$ if $p = \frac{1}{2}$.

Sometimes the parameter p is substituted for the parameter s, which satisfies $0 \leq s \leq \frac{1}{2}$. They are related as follows:

$$s := \begin{cases} p, & p \leq \frac{1}{2}, \\ 1 - p, & p > \frac{1}{2}. \end{cases}$$

2.1.3 Erlang Distribution

A stochastic variable X is *Erlang-k* distributed if it is the sum of k variables that are independent exponentially distributed with rate μ. This is denoted by $E_k(\mu)$, or briefly E_k. Its probability density is:

$$\mu \frac{(\mu x)^{k-1}}{(k-1)!} e^{-\mu x}, \quad x \geq 0,$$

and its mean, variance and squared coefficient of variation are:

$$E[X] = \frac{k}{\mu}, \qquad \text{Var}[X] = \frac{k}{\mu^2}, \qquad c_X^2 = \frac{1}{k}.$$

The parameter μ is called the scale parameter and k is the shape parameter. A phase diagram of the $E_k(\mu)$ distribution is shown in Figure 2.1. Note that X is exponentially distributed with rate μ if $k = 1$.

2.2 Poisson Process

A *stochastic process* is a collection of stochastic variables $\{N(t) : t \geq 0\}$. A *counting process* is a stochastic process, where $N(t)$ is the number of events up to time t for $t \geq 0$. A counting process where the events are arrivals is typically called an *arrival process*. A counting process is a *homogeneous Poisson process*, or simply *Poisson process*, with rate λ if $N(t)$ has a Poisson distribution with rate λt for $t \geq 0$ (cf. Bhulai and Koole [3]):

$$P(N(t) = n) = \frac{(\lambda t)^n}{n!} e^{-\lambda t}, \quad t \geq 0, n = 0, 1, 2, \ldots$$

Equivalently, if $X_1, X_2, \ldots$ is the corresponding sequence of inter-event times, then its elements are independent and identically exponentially distributed with rate λ:

$$f_{X_i}(x) = \lambda e^{-\lambda x}, \quad x \geq 0, i = 1, 2, \ldots$$

A Poisson process $\{N(t) : t \geq 0\}$ is *inhomogeneous* if its rate $\lambda(t)$ is time dependent. Obviously, the inter-event times are not independent in general, since:

$$P(X_{i+1} > t | X_i = s) = e^{-\int_s^{s+t} \lambda(x)\, dx}, \quad s < t.$$

In practice, the rates are constant in certain intervals.

2.3 Markov Decision Chain

A *Markov process* is a stochastic process $\{N(t) : t \geq 0\}$ that possesses the *Markov property*, i.e., for all $t, h \geq 0$

$$P(N(t+h) = y | N(u) = x(u), \forall u \leq t) = P(N(t+h) = y | N(t) = x(t)).$$

An example of a Markov process is the Poisson process. A Markov process $\{N(t) : t \geq 0\}$ where $t \in \mathbb{Z}_+$ is called a *Markov chain*.

A *Markov decision chain* $X_0, X_1, X_2, \ldots$ is a Markov chain that possesses a value function $V_n(X_n)$ with the following property:

$$V_{n+1}(x) = \min_a \{c(x,a) + \sum_y p(x,a,y) V_n(y)\},$$

where $c(x,a)$ is some cost function and $p(x,a,y)$ is a transition probability for action a. We condition this with $V_0(x) = 0$ for all x. Now, we let $V(x)$ be related to $V_n(x)$ as follows:

$$\lim_{n \to \infty} (V_n(x) - V_n(y)) = V(x) - V(y).$$

Then $V(x)$ can also be written iteratively:

$$V(x) + g = \min_a \{c(x,a) + \sum_y p(x,a,y)V(y)\},$$

where g is the following:

$$g = \lim_{n\to\infty} (V_{n+1}(x) - V_n(x)), \quad \forall x.$$

The value function V can be determined by means of an application that implements the following pseudo code:

Let $x \in \{0, 1, \ldots, N\}$, $E(x) \supset \{y | p(x, y) > 0\}$, and ε some small number (e.g., 10^{-6}).

```
Vector V[0,1,...,N], V'[0,1,...,N]
Float min, max
V = 0
do {
    V' = V
    for(x ∈ {0,1,...,N}) { % iterate
        V(x) = r(x)
        for(y ∈ E(x)) V(x) = V(x) + p(x,y)V'(y) }
    max = −10^10
    min = 10^10
    for(x ∈ {0,1,...,N}) { % compute span(V − V')
        if(V(x) − V'(x) < min) min = V(x) − V'(x)
        if(V(x) − V'(x) > max) max = V(x) − V'(x) } }
while(max − min > ε)
```

The optimal policy R^* is defined as follows:

$$R^*(x) = \arg\min_a \{c(x,a) + \sum_y p(x,a,y)V(y)\}.$$

Bhulai and Koole [3] discuss Markov decision chains in more detail.

2.4 Queueing Systems

Queueing systems are mathematical systems in which entities like customers or jobs wait in a queue before one or more servers, where each one can only serve one entity at the time. The servers are independent from each other. These queueing systems are classified as $X/Y/c$, known as the Kendall notation. X and Y represent the distribution of the interarrival times and the service times, e.g., exponential (M), deterministic (D) and general (G), and c is the number of servers. Suppose that λ is the arrival rate and β is the mean service time. Then $\rho := \lambda\beta$ is called the occupation rate, which must be less than one in order to keep the system stable. The occupation rate is the fraction of time the server is busy. Boxma [6] describes some of these models in great detail.

2.5 Processor Sharing Systems

A Processor Sharing system, or PS system, is a queueing system where arriving entities do not wait in a queue, but they go straight to the server. Each entity thereby gets an equal amount of server capacity. We consider jobs in an $M/G/1$ PS system. In this model we let jobs arrive according to a Poisson process with rate λ. They have a required service time B that is generally distributed with mean β. Then the occupation rate is $\rho := \lambda\beta$. The sojourn time S has the following mean:

$$E[S] = \frac{\beta}{1-\rho}. \tag{2.4}$$

Note here that the sojourn time does not depend on the distribution of the service time, but only on its mean β. The second moment of the sojourn time is related to the second moment of the service time as follows (cf. Van den Berg [1]):

$$(1-\rho)^2 E[S^2] \sim (1+\rho)E[B^2], \quad \rho \to 1. \tag{2.5}$$

Consequently, for $\rho \approx 1$:

$$\begin{aligned} c_S^2 = \frac{Var[S]}{(E[S])^2} = \frac{E[S^2]}{(E[S])^2} - 1 \approx \frac{(1+\rho)\frac{E[B^2]}{(1-\rho)^2}}{\left(\frac{E[B]}{1-\rho}\right)^2} - 1 = \frac{(1+\rho)E[B^2]}{(E[B])^2} - 1 \\ = (1+\rho)(c_B^2+1) - 1 \approx 2(c_B^2+1) - 1 = 2c_B^2 + 1. \end{aligned} \tag{2.6}$$

Suppose now that there are two Poisson arrival streams where the k-th stream has rate λ_k and has jobs with mean service time of β_k. Then the occupation rate of the k-th stream is $\rho_k := \lambda_k \beta_k$. The condition $\rho := \rho_1 + \rho_2 < 1$ holds, in order to keep the system stable.

The sojourn time S_k of jobs from the k-th arrival stream has mean

$$E[S_k] = \frac{\beta_k}{1-\rho}, \qquad k = 1, 2.$$

Proof. Let N_k denote the number of jobs from the k-th stream. Then the joint number of jobs (N_1, N_2) has the following distribution (cf. Cheung [7]):

$$p_{N_1,N_2}(n_1, n_2) := P(N_1 = n_1, N_2 = n_2) = (1-\rho)\binom{n_1 + n_2}{n_2}\rho_1^{n_1}\rho_2^{n_2}.$$

The marginal number of jobs N_k has a geometric distribution

$$p_{N_k}(i) := P(n_k = i) = (1 - p_k)p_k^i, \qquad i = 0, 1, 2, \ldots; k = 1, 2,$$

with success probabilities

$$p_1 := \frac{\rho_1}{1-\rho_2}, \qquad p_2 := \frac{\rho_2}{1-\rho_1}.$$

The number of jobs N_k from the first arrival stream has the following mean:

$$E[N_1] = \frac{\rho_1}{1-\rho},$$

since:

$$E[N_1] = \frac{p_1}{1-p_1} = \frac{\frac{\rho_1}{1-\rho_2}}{1 - \frac{\rho_1}{1-\rho_2}} = \frac{\frac{\rho_1}{1-\rho_2}}{\frac{1-\rho}{1-\rho_2}} = \frac{\rho_1}{1-\rho}.$$

It can be similarly shown that the number of jobs N_2 from the second arrival stream has the following mean:

$$E[N_2] = \frac{\rho_2}{1-\rho}.$$

Then the sojourn time S_k of jobs from the k-th arrival stream can be obtained by applying Little's formula. □

Coffman, Muntz and Trotter [8] provide more information on M/M/1 Processor Sharing servers. Detailed analysis of M/G/1 Processor Sharing servers can be found in Van den Berg's thesis [1]. Cohen [9] describes a generalised version of the Processor Sharing server.

Chapter 3

Server Selection Model

3.1 Static Model

We consider a queueing model with two procesor sharing servers and three traffic streams. Jobs that have access to both servers are called foreground jobs, or type 0 jobs. Type i background jobs, or simply type i jobs, only have access to server i. We let the foreground jobs go to server i with probability q_i, where $q_1 = q = 1 - q_2$. q is the *selection parameter*, which is *symmetrical* if:

$$q_i(\rho_1, \rho_2) = 1 - q_i(\rho_2, \rho_1).$$

A symmetrical selection parameter q has the following property:

$$q_i(\rho, \rho) = \frac{1}{2}.$$

Type j jobs arrive according to a Poisson process with rate $\lambda_j > 0$ and the *service time* B_j has a general distribution with mean $\beta_j > 0$, for $j = 0, 1, 2$. The occupation rate ρ_j of type j jobs is $\rho_j := \lambda_j \beta_j$. The occupation rate ρ_{S_i} at server i is:

$$\rho_{S_i} := q_i \rho_0 + \rho_i, \quad i = 1, 2. \tag{3.1}$$

We add the condition

$$\rho_{S_i} < 1, \quad i = 1, 2, \tag{3.2}$$

in order to keep the system stable. In other words:

$$q\rho_0 + \rho_1 < 1,$$
$$(1 - q)\rho_0 + \rho_2 < 1,$$

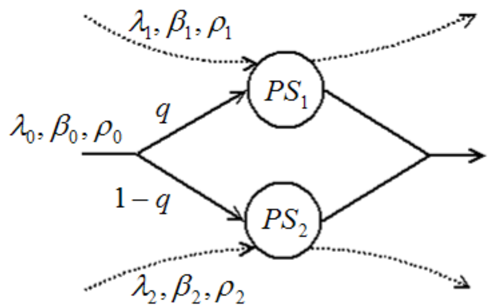


Figure 3.1: Server Selection: Foreground job uses PS server i only with probability q_i. Each server has one background traffic stream.

or equivalently:

$$q < \frac{1-\rho_1}{\rho_0} =: U \tag{3.3}$$

$$q > 1 - \frac{1-\rho_2}{\rho_0} =: L, \tag{3.4}$$

A selection parameter q is *stable* if $q \in (L, U)$. It is *feasible* if it is both symmetrical and stable. The system is *stable* if both servers are, which is the case if:

$$\rho_0 < (1-\rho_1) + (1-\rho_2),$$

Proof:

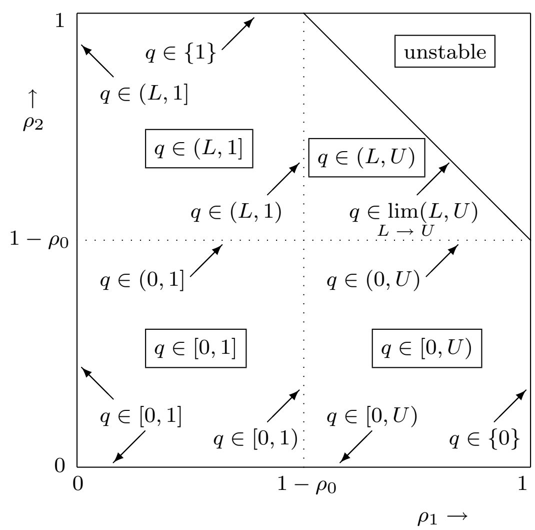
$$\frac{1-\rho_1}{\rho_0} > 1 - \frac{1-\rho_2}{\rho_0}$$
$$1-\rho_1 > \rho_0 - (1-\rho_2)$$
$$\rho_0 < (1-\rho_1) + (1-\rho_2).$$ □

We let S_j be the sojourn time of type j jobs. The sojourn time S_i of type i background jobs has the following mean:

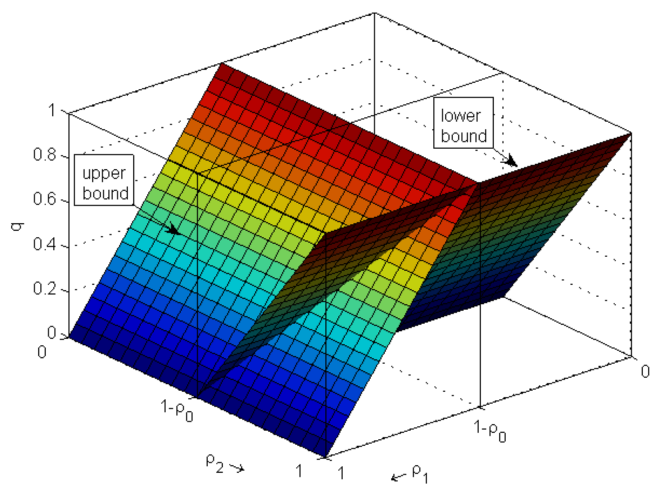
$$E[S_i] = \frac{\beta_i}{1 - q_i\rho_0 - \rho_i},$$

If Z is the server a foreground job goes to, then the sojourn time S_i of foreground jobs given $Z = i$, has the following mean:

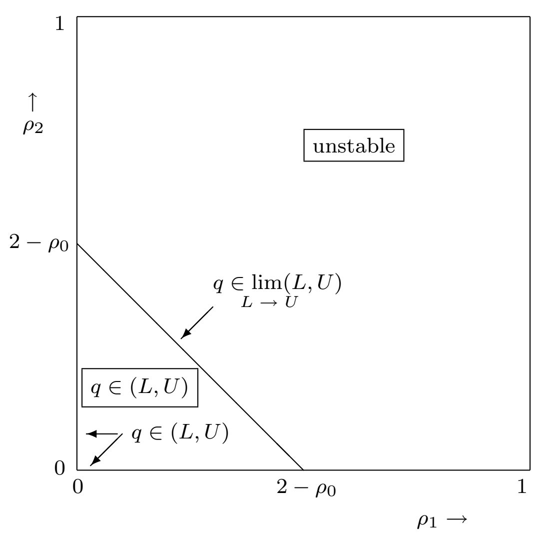
$$E[S_0|Z = i] = \frac{\beta_0}{1-\rho_{Si}}.$$



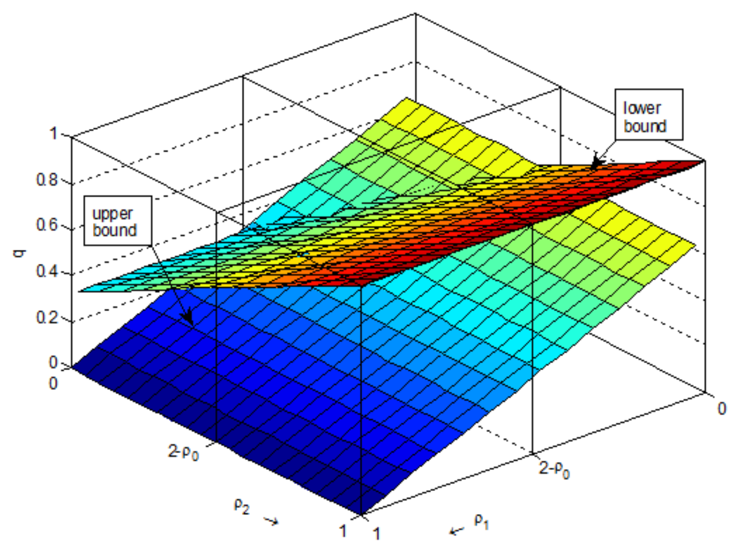
(a) $0 < \rho_0 \leq 1$.



(b) $0 < \rho_0 \leq 1$.



(c) $1 \leq \rho_0 < 2$.



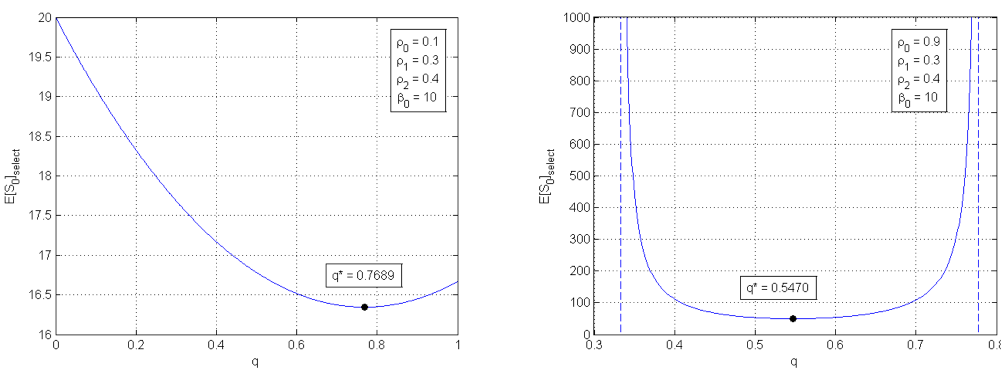
(d) $1 \leq \rho_0 < 2$.

Figure 3.2: Stable selection parameter q.

In general, the mean sojourn time of foreground jobs is:

$$
\begin{aligned}
E[S_0]_{\text{select}} &= \sum_{i=1}^{2} E[S_0|Z=i]P(Z=i) \\
&= \sum_{i=1}^{2} q_i \frac{\beta_0}{1-q_i\rho_0-\rho_i}.
\end{aligned}
$$

We let f be the mean sojourn time $E[S_0]$ of foreground jobs as a function of the selection parameter q. The optimal selection probability q^* is the one where f is minimal with respect to q.



(a) $\rho_0 = 0.1$. (b) $\rho_0 = 0.9$.

Figure 3.3: Mean sojourn time $E[S_0]_{\text{select}}$ of foreground jobs as a function of the selection parameter q.

Then:

$$
\begin{aligned}
f'(q) &= \frac{\beta_0(1-q\rho_0-\rho_1)+q\beta_0\rho_0}{(1-q\rho_0-\rho_1)^2} + \frac{-\beta_0(1-(1-q)\rho_0-\rho_2)-(1-q)\beta_0\rho_0}{(1-(1-q)\rho_0-\rho_2)^2} \\
&= \frac{\beta_0(1-\rho_1)}{(1-q\rho_0-\rho_1)^2} + \frac{-\beta_0(1-\rho_2)}{(1-(1-q)\rho_0-\rho_2)^2} \\
&= \beta_0 \frac{(1-\rho_1)(1-(1-q)\rho_0-\rho_2)^2-(1-\rho_2)(1-q\rho_0-\rho_1)^2}{(1-q\rho_0-\rho_1)^2(1-(1-q)\rho_0-\rho_2)^2}
\end{aligned}
$$

$$
\begin{aligned}
&f'(q) = 0 \Leftrightarrow (1-\rho_1)(1-(1-q)\rho_0-\rho_2)^2 = (1-\rho_2)(1-q\rho_0-\rho_1)^2 \\
&(1-\rho_1)\left\{(1-\rho_2)^2 - 2(1-q)(1-\rho_2)\rho_0 + (1-q)^2\rho_0^2\right\} \\
&\qquad = (1-\rho_2)\left\{(1-\rho_1)^2 - 2q(1-\rho_1)\rho_0 + q^2\rho_0^2\right\} \\
&2q(1-\rho_1)(1-\rho_2)\rho_0 + (1-q)^2\rho_0^2(1-\rho_1) + 2q(1-\rho_1)(1-\rho_2)\rho_0 \\
&\qquad - q^2\rho_0^2(1-\rho_2) + (1-\rho_1)(1-\rho_2)^2 - 2(1-\rho_2)\rho_0(1-\rho_1) \\
&\qquad - (1-\rho_2)(1-\rho_1)^2 = 0 \\
&2q(1-\rho_1)(1-\rho_2)\rho_0 + (-2q+q^2)\rho_0^2(1-\rho_1) + 2q(1-\rho_1)(1-\rho_2)\rho_0 \\
&\qquad - q^2\rho_0^2(1-\rho_2) + (1-\rho_1)(1-\rho_2)^2 - 2(1-\rho_2)\rho_0(1-\rho_1) \\
&\qquad + \rho_0^2(1-\rho_1) - (1-\rho_2)(1-\rho_1)^2 = 0.
\end{aligned}
$$

Thus, $f'(q) = 0$ if and only if:

$$aq^2 + bq + c = 0, \tag{3.5}$$

where

$$
\begin{cases}
a = \rho_0^2((1-\rho_1)-(1-\rho_2)), \\
b = 4(1-\rho_1)(1-\rho_2)\rho_0 - 2\rho_0^2(1-\rho_1), \\
c = (1-\rho_1)(1-\rho_2)^2 - (1-\rho_2)(1-\rho_1)^2 - 2(1-\rho_1)(1-\rho_2)\rho_0 + \rho_0^2(1-\rho_1).
\end{cases}
$$

If $\rho_1 = \rho_2$, then Equation (3.5) has only one solution, namely

$$q = \frac{1}{2}, \tag{3.6}$$

because:

$$
\begin{aligned}
&(4(1-\rho_1)^2\rho_0 - 2\rho_0^2(1-\rho_1))q + (1-\rho_1)^3 - (1-\rho_1)^3 - 2(1-\rho_1)^2\rho_0 \\
&\qquad + \rho_0^2(1-\rho_1) = 0 \\
&(2(2(1-\rho_1)^2 - (1-\rho_1)\rho_0)q = 2(1-\rho_1)^2 - (1-\rho_1)\rho_0).
\end{aligned}
$$

which is a minimum. Moreover:

$$
\begin{aligned}
f'(q) &= \frac{\beta_0(1-\rho_1)}{(1-q\rho_0-\rho_1)^2} - \frac{\beta_0(1-\rho_1)}{(1-(1-q)\rho_0-\rho_1)^2} \\
\lim_{q\to 0} f'(q) &= \frac{\beta_0}{1-\rho_1} - \frac{\beta_0(1-\rho_1)}{((1-\rho_1)-\rho_0)^2} = \beta_0\frac{((1-\rho_1)-\rho_0)^2-(1-\rho_1)^2}{(1-\rho_1)((1-\rho_1)-\rho_0)^2} \\
&= \beta_0\frac{\rho_0^2 - 2\rho_0(1-\rho_1)}{(1-\rho_1)((1-\rho_1)-\rho_0)^2} = \beta_0\rho_0\frac{\rho_0-(1-\rho_1)-(1-\rho_2)}{(1-\rho_1)((1-\rho_1)-\rho_0)^2} < 0,
\end{aligned}
$$

since the stability condition $\rho_0 < (1-\rho_1) + (1-\rho_2)$ holds. It can be similarly proven that:

$$\lim_{q \to 1} f'(q) > 0.$$

Thus $q^* = \frac{1}{2}$ is a minimum. If $\rho_1 \neq \rho_2$, then f has two extrema:

$$q_\pm = \frac{-b \pm \sqrt{D}}{2a},$$

where D is the discriminant. Now:

$$\begin{aligned}
D &= 16\rho_0^2(1-\rho_1)^2(1-\rho_2)^2 - 16\rho_0^3(1-\rho_1)^2(1-\rho_2) + 4\rho_0^4(1-\rho_1)^2 \\
&\quad - 4\rho_0^2((1-\rho_1)-(1-\rho_2))((1-\rho_1)(1-\rho_2)^2 - (1-\rho_2)(1-\rho_1)^2) \\
&\quad + 8\rho_0^3((1-\rho_1)-(1-\rho_2))(1-\rho_1)(1-\rho_2) - 4\rho_0^4((1-\rho_1)-(1-\rho_2))(1-\rho_1) \\
&= 16\rho_0^2(1-\rho_1)^2(1-\rho_2)^2 - 16\rho_0^3(1-\rho_1)^2(1-\rho_2) + 4\rho_0^4(1-\rho_1)^2 \\
&\quad - 4\rho_0^2(1-\rho_1)^2(1-\rho_2)^2 + 4\rho_0^2(1-\rho_1)(1-\rho_2)^3 + 4\rho_0^2(1-\rho_1)^3(1-\rho_2) \\
&\quad - 4\rho_0^2(1-\rho_1)^2(1-\rho_2)^2 + 8\rho_0^3(1-\rho_1)^2(1-\rho_2) - 8\rho_0^3(1-\rho_1)(1-\rho_2)^2 \\
&\quad - 4\rho_0^4(1-\rho_1)^2 + 4\rho_0^4(1-\rho_1)(1-\rho_2) \\
&= 8\rho_0^2(1-\rho_1)^2(1-\rho_2)^2 - 8\rho_0^3(1-\rho_1)^2(1-\rho_2) + 4\rho_0^2(1-\rho_1)(1-\rho_2)^3 \\
&\quad + 4\rho_0^2(1-\rho_1)^3(1-\rho_2) - 8\rho_0^3(1-\rho_1)(1-\rho_2)^2 + 4\rho_0^4(1-\rho_1)(1-\rho_2) \\
&= 4\rho_0^2(1-\rho_1)(1-\rho_2)(2(1-\rho_1)(1-\rho_2) - 2\rho_0((1-\rho_1)+(1-\rho_2)) \\
&\quad + (1-\rho_2)^2 + (1-\rho_1)^2 + \rho_0^2) \\
&= 4\rho_0^2(1-\rho_1)(1-\rho_2)(((1-\rho_1)+(1-\rho_2))^2 - 2\rho_0((1-\rho_1)+(1-\rho_2)) + \rho_0^2) \\
&= 4\rho_0^2(1-\rho_1)(1-\rho_2)((1-\rho_1)+(1-\rho_2)-\rho_0)^2.
\end{aligned}$$

Thus the two extrema are:

$$q_+ = \frac{(1-\rho_1)(\rho_0 - 2(1-\rho_2)) + ((1-\rho_1)+(1-\rho_2)-\rho_0)\sqrt{(1-\rho_1)(1-\rho_2)}}{\rho_0((1-\rho_1)-(1-\rho_2))}. \tag{3.7}$$

and

$$q_- = \frac{(1-\rho_1)(\rho_0 - 2(1-\rho_2)) - ((1-\rho_1)+(1-\rho_2)-\rho_0)\sqrt{(1-\rho_1)(1-\rho_2)}}{\rho_0((1-\rho_1)-(1-\rho_2))}.$$

Now, $q_+ < U$, because:

$$
\begin{aligned}
q_+ &< \frac{(1-\rho_1)(\rho_0 - 2(1-\rho_2)) + ((1-\rho_1)+(1-\rho_2)-\rho_0)\sqrt{(1-\rho_1)^2}}{\rho_0((1-\rho_1)-(1-\rho_2))} \\
&= \frac{(1-\rho_1)((1-\rho_1)-(1-\rho_2))}{\rho_0((1-\rho_1)-(1-\rho_2))} = U.
\end{aligned}
$$

Note that the inequality holds for both $(1-\rho_1) < (1-\rho_2)$ and $(1-\rho_1) > (1-\rho_2)$. Moreover, it can be similarly shown that $q_+ > L$, thus q_+ is stable. $q_- > U$ if $(1-\rho_1) < (1-\rho_2)$, since:

$$
\begin{aligned}
q_- &> \frac{(1-\rho_1)(\rho_0 - 2(1-\rho_2)) - ((1-\rho_1)+(1-\rho_2)-\rho_0)\sqrt{(1-\rho_1)^2}}{\rho_0((1-\rho_1)-(1-\rho_2))} \\
&> \frac{(1-\rho_1)(\rho_0 - 2(1-\rho_2)) + ((1-\rho_1)+(1-\rho_2)-\rho_0)\sqrt{(1-\rho_1)^2}}{\rho_0((1-\rho_1)-(1-\rho_2))} = U,
\end{aligned}
$$

It can be similarly shown that $q_- < L$ if $(1-\rho_1) > (1-\rho_2)$. Thus q_- is unstable. Hence q_+ is the only stable extremum. Moreover:

$$
\lim_{q \to L} f'(q) = -\infty < 0, \quad \lim_{q \to U} f'(q) = +\infty > 0.
$$

Thus q_+ is a minimum in (L, U). Thus:

$$
q^* = \begin{cases} 0, & q_+ < 0, \\ q_+, & q_+ \in [0,1], \\ 1, & q_+ > 1, \end{cases}
$$

since $q^* \in [0,1]$.

The optimal selection parameter q^* as a function of ρ_2 is continuous in $\rho_2 = \rho_1$, since:

$$
\begin{aligned}
\lim_{\rho_2 \to \rho_1} q^* &= \lim_{\rho_2 \to \rho_1} \frac{(1-\rho_1)(\rho_0 - 2(1-\rho_2)) + ((1-\rho_1)+(1-\rho_2)-\rho_0)\sqrt{(1-\rho_1)(1-\rho_2)}}{\rho_0((1-\rho_1)-(1-\rho_2))} \\
&\overset{\text{L'Hôpital}}{=} \lim_{\rho_2 \to \rho_1} \frac{2(1-\rho_1) - \sqrt{(1-\rho_1)(1-\rho_2)} + ((1-\rho_1)+(1-\rho_2)-\rho_0)\frac{-(1-\rho_1)}{2\sqrt{(1-\rho_1)(1-\rho_2)}}}{\rho_0} \\
&= \frac{2(1-\rho_1) - (1-\rho_1) + (2(1-\rho_1)-\rho_0)\frac{-(1-\rho_1)}{2(1-\rho_1)}}{\rho_0} = \frac{(1-\rho_1) + (-(1-\rho_1) + \frac{1}{2}\rho_0)}{\rho_0} = \frac{1}{2}.
\end{aligned}
$$

Note that the opimtal selection parameter q^* does not depend on the distribution of the service time B_0. It is obvious that q^* is stable if $q_+ \in [0, 1]$. It is also stable for $q_+ < 0$, since:

$$q^* = 0 < \frac{1 - \rho_1}{\rho_0},$$
$$q^* = 0 > q_+ > 1 - \frac{1 - \rho_2}{\rho_0}.$$

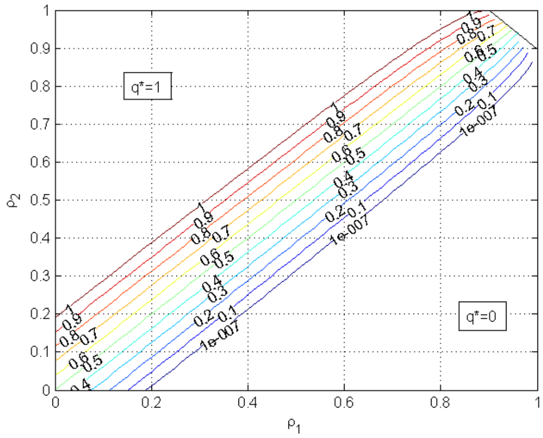
and it can be similarly shown that it is also stable for $q_+ > 1$. It is also obvious that q^* is symmetrical. Hence q^* is feasible. Figure 3.4 and table 3.1 show plots and numerical values of q^* for different values of the occupation rates ρ_j, for $j = 0, 1, 2$.

3.2 Markov Decision Model

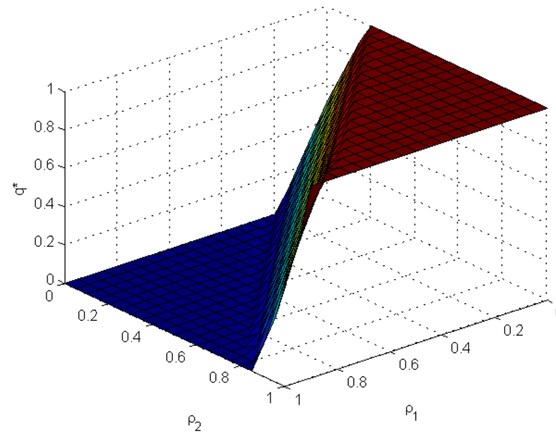
We can improve the above model by deciding for each foreground job which server it goes to, based on the number x_i of jobs upon arrival by modelling the joint number of jobs x_{ij} with characterisation j in server i as a Markov decision chain. The model is service time distribution specific. From this we can deduce optimal policies $R^*(\mathbf{x})$ of this model, telling us which server each arriving foreground job goes to, depending on the number of jobs x_i in server i upon arrival and the occupation rates ρ_j of each job type. We can determine value functions V by implementing the pseudo code mentioned in Section 2.3 for finite state spaces. Our model, however, has an infinite state space. So we have to approximate our model by a model where server i can only handle at most m jobs simultaneously, where m is large. Arriving foreground jobs that find only one of the servers full, go to the server that is not full. If they find both servers full, then they are blocked. Arriving type i background jobs that find server i full, are also blocked.

3.2.1 Exponential Service Times

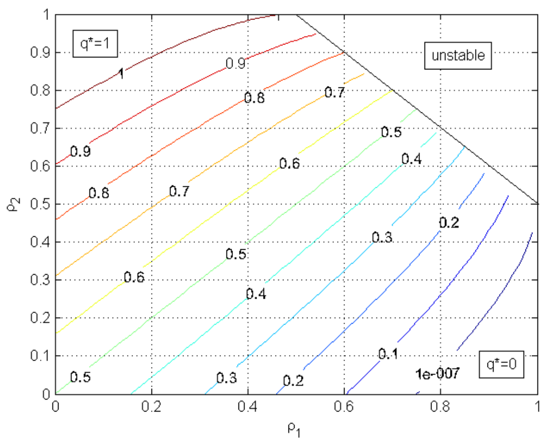
Suppose that the service times B_j are independent exponentially distributed with rate μ, for $j = 0, 1, 2$. Then we choose each arriving foreground job to go to the server that leads to the smallest number of jobs in the long run. So if the total number of jobs in server i is x_i, then the state of the system



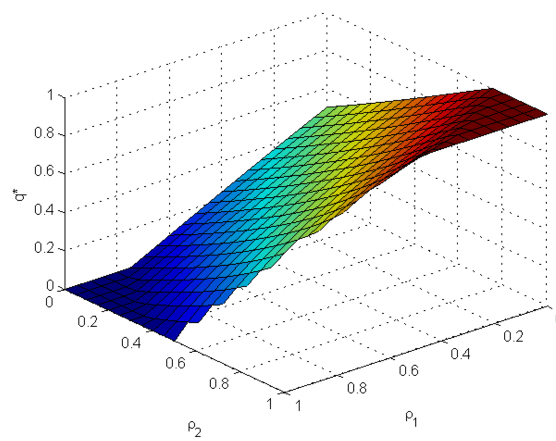
(a) $\rho_0 = 0.1$.



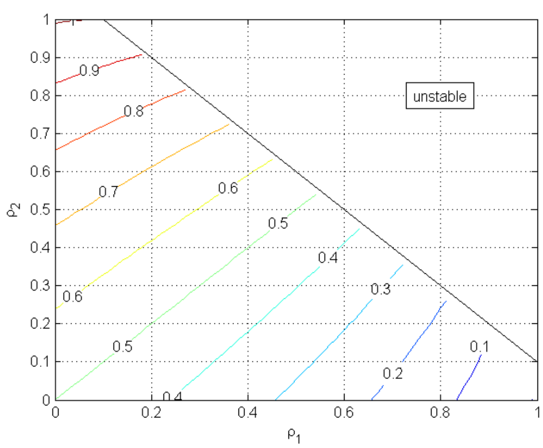
(b) $\rho_0 = 0.1$.



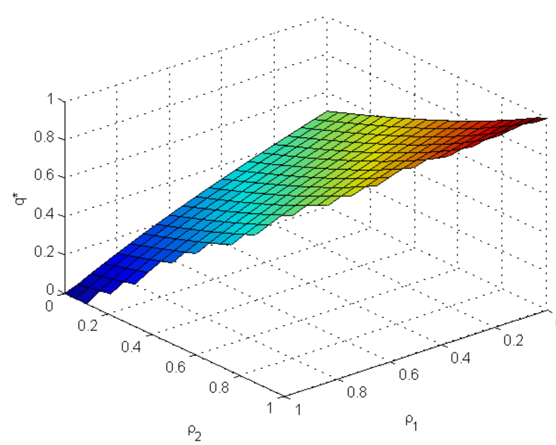
(c) $\rho_0 = 0.5$.



(d) $\rho_0 = 0.5$.



(e) $\rho_0 = 0.9$.



(f) $\rho_0 = 0.9$.

Figure 3.4: Plots of the optimal selection q^*.

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	0.5	0.763	1	1	1	1	1	1	1	1	1
0.1	0.237	0.5	0.7645	1	1	1	1	1	1	1	1
0.2	0	0.2355	0.5	0.7664	1	1	1	1	1	1	1
0.3	0	0	0.2336	0.5	0.7689	1	1	1	1	1	1
0.4	0	0	0	0.2311	0.5	0.7723	1	1	1	1	1
0.5	0	0	0	0	0.2277	0.5	0.7771	1	1	1	1
0.6	0	0	0	0	0	0.2229	0.5	0.7846	1	1	1
0.7	0	0	0	0	0	0	0.2154	0.5	0.798	1	1
0.8	0	0	0	0	0	0	0	0.202	0.5	0.8284	1
0.9	0	0	0	0	0	0	0	0	0.1716	0.5	1
1	0	0	0	0	0	0	0	0	0	0	-

(a) $\rho_0 = 0.1$

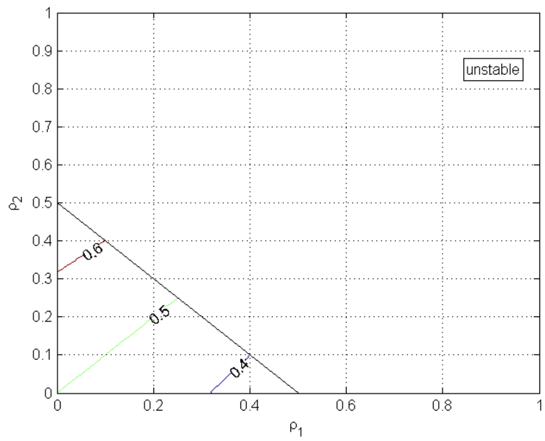
ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	0.5	0.5631	0.6276	0.6933	0.7603	0.8284	0.8974	0.9662	1	1	1
0.1	0.4369	0.5	0.5647	0.631	0.699	0.7687	0.84	0.9124	0.9845	1	1
0.2	0.3724	0.4353	0.5	0.5666	0.6354	0.7064	0.7799	0.8558	0.9333	1	1
0.3	0.3067	0.369	0.4334	0.5	0.5692	0.6413	0.7166	0.7956	0.8787	0.9646	1
0.4	0.2397	0.301	0.3646	0.4308	0.5	0.5727	0.6495	0.7314	0.8196	0.916	1
0.5	0.1716	0.2313	0.2936	0.3587	0.4273	0.5	0.5777	0.6619	0.755	0.8618	-
0.6	0.1026	0.16	0.2201	0.2834	0.3505	0.4223	0.5	0.5856	0.6828	-	-
0.7	0.0338	0.0876	0.1442	0.2044	0.2686	0.3381	0.4144	0.5	-	-	-
0.8	0	0.0155	0.0667	0.1213	0.1804	0.245	0.3172	-	-	-	-
0.9	0	0	0	0.0354	0.084	0.1382	-	-	-	-	-
1	0	0	0	0	0	-	-	-	-	-	-

(b) $\rho_0 = 0.5$

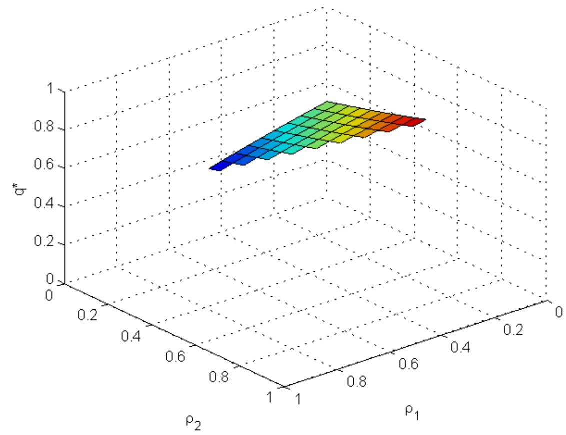
ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	0.5	0.5409	0.5832	0.6271	0.6728	0.7206	0.7708	0.824	0.8808	0.9423	1
0.1	0.4591	0.5	0.5425	0.5867	0.633	0.6817	0.7333	0.7887	0.849	0.9167	-
0.2	0.4168	0.4575	0.5	0.5444	0.5912	0.6407	0.6936	0.7511	0.8148	-	-
0.3	0.3729	0.4133	0.4556	0.5	0.547	0.5971	0.6512	0.7106	-	-	-
0.4	0.3272	0.367	0.4088	0.453	0.5	0.5505	0.6055	-	-	-	-
0.5	0.2794	0.3183	0.3593	0.4029	0.4495	0.5	-	-	-	-	-
0.6	0.2292	0.2667	0.3064	0.3488	0.3945	-	-	-	-	-	-
0.7	0.176	0.2113	0.2489	0.2894	-	-	-	-	-	-	-
0.8	0.1192	0.151	0.1852	-	-	-	-	-	-	-	-
0.9	0.0577	0.0833	-	-	-	-	-	-	-	-	-
1	0	-	-	-	-	-	-	-	-	-	-

(c) $\rho_0 = 0.9$

Table 3.1: Numerical values of the optimal selection q^*.



(g) $\rho_0 = 1.5$.



(h) $\rho_0 = 1.5$.

Figure 3.4: Plots of the optimal selection q^*.

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	0.5	0.5298	0.5611	0.5941	0.6291	-	-	-	-	-	-
0.1	0.4702	0.5	0.5314	0.5646	-	-	-	-	-	-	-
0.2	0.4389	0.4686	0.5	-	-	-	-	-	-	-	-
0.3	0.4059	0.4354	-	-	-	-	-	-	-	-	-
0.4	0.3709	-	-	-	-	-	-	-	-	-	-
0.5	-	-	-	-	-	-	-	-	-	-	-
0.6	-	-	-	-	-	-	-	-	-	-	-
0.7	-	-	-	-	-	-	-	-	-	-	-
0.8	-	-	-	-	-	-	-	-	-	-	-
0.9	-	-	-	-	-	-	-	-	-	-	-
1	-	-	-	-	-	-	-	-	-	-	-

(d) $\rho_0 = 1.5$

Table 3.1: Numerical values of the optimal selection q^*.

is the joint number of jobs $\mathbf{x} = (x_1, x_2)$ in each server. The cost $c(\mathbf{x})$ is a scaled sum of the jobs in the servers after the event occurs:

$$c(\mathbf{x}) = \frac{1}{\gamma}(x_1 + x_2).$$

where

$$\gamma = \sum_{j=0}^{2} \lambda_j + 2\mu.$$

Now, we let the system start in some state $\mathbf{s}$ with zero cost, i.e.,

$$V_0(\mathbf{s}) = 0.$$

Then we write the value function iteratively as follows:

$$V_{n+1}(\mathbf{x}) = c(\mathbf{x}) + \min_i \sum_{\mathbf{y}} V_n(\mathbf{y}) p(\mathbf{x}, i, \mathbf{y}),$$

where $p(\mathbf{x}, i, \mathbf{y})$ is the transition probability from state $\mathbf{x}$ to state $\mathbf{y}$ if either a job goes to server i or leaves it.

Now, the residual service time of a job in server i is exponentially distributed with rate μ/x_i, since the server capacity is equally shared among the jobs. The minimum of x_i residual service times in server i, which are all exponentially distributed with rate μ/x_i, is exponentially distributed with rate μ. As a consequence, the transition rate due to a departing job is also μ. It is obvious that the transition rate due to the arrival of a type j job is λ_j, for $j = 0, 1, 2$. The transition probability due to some kind of event is the probability that some event comes before all other possible events. Since the residual event times are all exponential, this transition probability is the rate of this event, divided by the sum of the rates of all possible kind of events.

Recall that $\mathbf{e}_i$ is a standard basis vector, i.e., a unit vector where the i-th entry is one and all other entries are zero. Then we can write the value function iteratively as follows, if both servers are not empty:

$$V_{n+1}(\mathbf{x}) = \frac{x_1 + x_2}{\gamma} + \frac{\lambda_0}{\gamma} \min_{i=1,2} \{V_n(\mathbf{x} + \mathbf{e}_i)\} + \sum_{i=1}^{2} \left(\frac{\lambda_i}{\gamma} V_n(\mathbf{x} + \mathbf{e}_i) + \frac{\mu}{\gamma} V_n(\mathbf{x} - \mathbf{e}_i) \right). \tag{3.8}$$

The optimal policy is then:

$$R^*(\mathbf{x}) = \arg\min_{i=1,2} \{V(\mathbf{x}+\mathbf{e}_i)\}.$$

3.2.2 Hyperexponential Service Times

Suppose now that the service times B_j are independent hyperexponentially distributed, for $j = 0, 1, 2$. More precisely, they are exponentially distributed with rate μ_j with probability p_j, for $j = 1, 2$. Jobs with service rate μ_j are typically called service type j jobs. We choose the service type of jobs to be unknown upon arrival. We let X be a 2×2 matrix, where the (i, j)-th entry x_{ij} is the number of jobs of service type j in server i, for $i, j = 1, 2$. The cost $c(X)$ is a scaled sum of the jobs in the servers after the event occurs:

$$c(X) = \frac{1}{\gamma}\sum_{i=1}^{2}\sum_{j=1}^{2} x_{ij},$$

where

$$\gamma = \sum_{j=0}^{2}\lambda_j + 2\mu_1 + 2\mu_2.$$

Now, we let the system start in some state S with zero cost, i.e.,

$$V_0(S) = 0.$$

If we let E_{ij} be a matrix where the (i, j)-th entry is one and all other entries are zero, then we can write the value function iteratively as follows, if jobs of all service types are present in both servers:

$$\begin{aligned} V_{n+1}(X) = {} & \frac{1}{\gamma}\sum_{i=1}^{2}\sum_{j=1}^{2} x_{ij} + \frac{\lambda_0}{\gamma}\min_{i=1,2}\left\{\sum_{j=1}^{2} p_j V_n(X+E_{ij})\right\} \\ & + \sum_{i=1}^{2}\sum_{j=1}^{2}\left(\frac{p_j\lambda_i}{\gamma}V_n(X+E_{ij}) + \frac{x_{ij}}{x_{i1}+x_{i2}}\frac{\mu_j}{\gamma}V_n(X-E_{ij})\right) \\ & + \frac{1}{\gamma}\left(\gamma - \sum_{j=0}^{2}\lambda_j - \sum_{i=1}^{2}\sum_{j=1}^{2}\frac{x_{ij}}{x_{i1}+x_{i2}}\mu_j\right)V_n(X). \end{aligned} \tag{3.9}$$

The value function V can be written iteratively in a similar manner in case at least one of the job types is absent or at least one of the servers is empty.

If x_i is the number of jobs in server i, then $x_i = x_{i1} + x_{i2}$. The optimal policy is then:

$$R^*(\mathbf{x}) = \arg\min_{i=1,2} \left\{ \sum_{j=1}^{2} p_j V(X + E_{ij}) \right\}.$$

3.2.3 Erlang Service Times

Now, we let the service times B_j be Erlang-2 distributed with rate $\mu/2$, for $j = 0, 1, 2$. Jobs that have this property go through two phases in series, where each phase is exponentially distributed with rate μ. We therefore consider a four-dimensional Markov chain, where we let X be a 2×2 matrix, where the (i, j)-th entry x_{ij} is the number of jobs in the j-th phase of server i, for $i, j = 1, 2$. The cost $c(X)$ is again a scaled sum of the jobs in the servers after the event occurs:

$$c(X) = \frac{1}{\gamma} \sum_{i=1}^{2} \sum_{j=1}^{2} x_{ij},$$

where

$$\gamma = \sum_{j=0}^{2} \lambda_j + 2\mu.$$

We again let the system start in some state S with zero cost, i.e.,

$$V_0(S) = 0.$$

Then we can write the value function iteratively as follows, if jobs are present in both phases of both servers:

$$V_{n+1}(X) = \frac{1}{\gamma} \sum_{i=1}^{2} \sum_{j=1}^{2} x_{ij} + \frac{\lambda_0}{\gamma} \min_{i=1,2} \{V_n(X + E_{i1})\} + \sum_{i=1}^{2} \frac{\lambda_i}{\gamma} V_n(X + E_{i1})$$
$$+ \sum_{i=1}^{2} \left(\frac{x_{i1}}{x_{i1} + x_{i2}} \frac{\mu}{\gamma} V_n(X - E_{i1} + E_{i2}) + \frac{x_{i2}}{x_{i1} + x_{i2}} \frac{\mu}{\gamma} V_n(X - E_{i2}) \right).$$

The value function V can be written iteratively in a similar manner in case at least one of the job types is absent or at least one of the servers is empty.

The optimal policy is then:

$$R^*(\mathbf{x}) = \arg\min_{i=1,2} V(X + E_{i1}).$$

3.2.4 Optimal Policies

We obtain the numerical values of the optimal policies by means of implementing the pseudo-code mentioned in section 2.3. If we let M be defined as the fraction of possible states $\mathbf{x}$, in which the optimal policy $R^*(\mathbf{x})$ is the server with the least number of jobs, i.e.:

$$M := \lim_{N\to\infty} \frac{1}{N} \#\{\mathbf{x} : R^*(\mathbf{x}) = \arg\min_{i=1,2}\{x_i\} | \#\{\mathbf{x}\} = N\},$$

then it appears that for fixed occupation rates ρ_0, ρ_1 and ρ_2, M increases if exponential service times are substituted for hyperexponential service times, because there is a possibility that a job with a hyperexponential service time has a higher service rate than a job with an exponential service time has. The higher the squared coefficient of variation $c^2_{B_j}$ of the service time is for $j = 0, 1, 2$, the higher it becomes. On the other hand, for fixed occupation rates ρ_0, ρ_1 and ρ_2, M decreases if exponential service times are substituted for Erlang-2 service times, because there is a possibility that a job with an Erlang-2 service time has a smaller residual service rate than a job with an exponential service rate has. This occurs when the first job is in phase 2. Table 3.2 shows these observations.

3.3 Comparison of the Models

We compare the mean sojourn times $E[S_0]$ of foreground jobs in the static model with the optimal selection parameter q^* to the Markov Decision model with optimal policies for *exponential*, *Erlang-2* and *hyperexponential* service times with mean $\beta_j = 10$ seconds and the latter has coefficient of variation $c^2_{B_j} \in \{2, 4, 16\}$ for $j = 0, 1, 2$. We perform this for each combination of the occupation rates

$$(\rho_0, \rho_1, \rho_2) \in \{0.1, 0.5, 0.9\} \times \{0, 0.1, \ldots, 0.9\}^2$$

by means of their relative difference:

$$\frac{E[S_0]_{\text{static}}(q^*) - E[S_0]_{\text{Markov}}}{E[S_0]_{\text{static}}(q^*)} \cdot 100\%.$$

We thereby use a simulation program called Extend by building some models and run them for all mentioned values of ρ_j for $j = 0, 1, 2$ in order to determine the mean sojourn times $E[S_0]_{\text{Markov}}$ of foreground jobs in the Markov decision model with optimal policies, which we then smoothen by means of regression. It appears that it is positive in all cases, meaning that the mean sojourn time $E[S_0]$ of foreground jobs decreases when we substitute the static model by the Markov decision model. The higher the occupation rates ρ_i differ from each other, for $i = 1, 2$, the smaller the relative difference becomes. This is, because if $\rho_i \to 1$, for some $i = 1, 2$, then $q^* \to i - 1$, while $R^*(\mathbf{x}) \to i - 1$. In other words, all jobs go to server $i - 1$ in both cases, thus proving the increasing resemblance between the models. So their relative difference goes to zero. Also, the higher the occupation rate ρ_0, the higher the relative difference becomes. This is, because a higher ρ_0 means more foreground jobs in the system that influence the mean sojourn time $E[S_0]$. Appendix Tables A.1 - A.5 show these observations.

$x_1 \backslash x_2$	0	1	2	3	4	5	6	7	8	9
0	-	1	1	1	1	1	1	1	1	1
1	2	-	1	1	1	1	1	1	1	1
2	2	2	-	1	1	1	1	1	1	1
3	2	2	2	-	1	1	1	1	1	1
4	2	2	2	2	-	1	1	1	1	1
5	2	2	2	2	2	-	1	1	1	1
6	2	2	2	2	2	2	-	1	1	1
7	2	2	2	2	2	2	2	-	1	1
8	2	2	2	2	2	2	2	2	-	1
9	2	2	2	2	2	2	2	2	2	-

(a) $\rho_0 = 0.1, \rho_1 = 0.1, \rho_2 = 0.1$,
$B_j \sim exp, H_2, E_2,\ j = 0, 1, 2$

$x_1 \backslash x_2$	0	1	2	3	4	5	6	7	8	9
0	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1	1
3	2	1	1	1	1	1	1	1	1	1
4	2	1	1	1	1	1	1	1	1	1
5	2	1	1	1	1	1	1	1	1	1
6	2	1	1	1	1	1	1	1	1	1
7	2	1	1	1	1	1	1	1	1	1
8	2	2	1	1	1	1	1	1	1	1
9	2	2	1	1	1	1	1	1	1	1

(b) $\rho_0 = 0.1, \rho_1 = 0.1, \rho_2 = 0.5$,
$B_j \sim E_2, c^2_{B_j} = 0.5,\ j = 0, 1, 2$

$x_1 \backslash x_2$	0	1	2	3	4	5	6	7	8	9
0	1	1	1	1	1	1	1	1	1	1
1	2	1	1	1	1	1	1	1	1	1
2	2	1	1	1	1	1	1	1	1	1
3	2	2	1	1	1	1	1	1	1	1
4	2	2	1	1	1	1	1	1	1	1
5	2	2	2	1	1	1	1	1	1	1
6	2	2	2	1	1	1	1	1	1	1
7	2	2	2	2	1	1	1	1	1	1
8	2	2	2	2	2	1	1	1	1	1
9	2	2	2	2	2	1	1	1	1	1

(c) $\rho_0 = 0.1, \rho_1 = 0.1, \rho_2 = 0.5$,
$B_j \sim exp, c^2_{B_j} = 1,\ j = 0, 1, 2$

$x_1 \backslash x_2$	0	1	2	3	4	5	6	7	8	9
0	1	1	1	1	1	1	1	1	1	1
1	2	1	1	1	1	1	1	1	1	1
2	2	2	1	1	1	1	1	1	1	1
3	2	2	1	1	1	1	1	1	1	1
4	2	2	2	1	1	1	1	1	1	1
5	2	2	2	1	1	1	1	1	1	1
6	2	2	2	2	1	1	1	1	1	1
7	2	2	2	2	2	1	1	1	1	1
8	2	2	2	2	2	1	1	1	1	1
9	2	2	2	2	2	2	1	1	1	1

(d) $\rho_0 = 0.1, \rho_1 = 0.1, \rho_2 = 0.5$,
$B_j \sim H_2, c^2_{B_j} = 2,\ j = 0, 1, 2$

$x_1 \backslash x_2$	0	1	2	3	4	5	6	7	8	9
0	1	1	1	1	1	1	1	1	1	1
1	2	1	1	1	1	1	1	1	1	1
2	2	2	1	1	1	1	1	1	1	1
3	2	2	1	1	1	1	1	1	1	1
4	2	2	2	1	1	1	1	1	1	1
5	2	2	2	2	1	1	1	1	1	1
6	2	2	2	2	1	1	1	1	1	1
7	2	2	2	2	2	1	1	1	1	1
8	2	2	2	2	2	2	1	1	1	1
9	2	2	2	2	2	2	2	1	1	1

(e) $\rho_0 = 0.1, \rho_1 = 0.1, \rho_2 = 0.5$,
$B_j \sim H_2, c^2_{B_j} = 4,\ j = 0, 1, 2$

$x_1 \backslash x_2$	0	1	2	3	4	5	6	7	8	9
0	1	1	1	1	1	1	1	1	1	1
1	2	1	1	1	1	1	1	1	1	1
2	2	2	1	1	1	1	1	1	1	1
3	2	2	1	1	1	1	1	1	1	1
4	2	2	2	1	1	1	1	1	1	1
5	2	2	2	2	1	1	1	1	1	1
6	2	2	2	2	2	1	1	1	1	1
7	2	2	2	2	2	2	1	1	1	1
8	2	2	2	2	2	2	1	1	1	1
9	2	2	2	2	2	2	2	1	1	1

(f) $\rho_0 = 0.1, \rho_1 = 0.1, \rho_2 = 0.5$,
$B_j \sim H_2, c^2_{B_j} = 16,\ j = 0, 1, 2$

Table 3.2: Examples of optimal policies $R^*(\mathbf{x})$.

Chapter 4

Job Split Model

4.1 Static Model

We consider a queueing model with two processor sharing servers, as before. However, the foreground jobs do not go to one of the servers only. Instead, they use both servers simultaneously, in the sense that a fraction α_i of the service time goes to server i, where $\alpha_1 = \alpha = 1 - \alpha_2$. α is the *split parameter*, which is *symmetrical* if:

$$\alpha_i(\rho_1, \rho_2) = 1 - \alpha_i(\rho_2, \rho_1).$$

A symmetrical split parameter α has the following property:

$$\alpha_i(\rho, \rho) = \frac{1}{2}.$$

We again let type j jobs arrive according to a Poisson process with rate $\lambda_j > 0$, each having a generally distributed *service time* B_j with mean $\beta_j > 0$, for $j = 0, 1, 2$. The occupation rate ρ_j of type j jobs is $\rho_j := \lambda_j \beta_j$, for $j = 0, 1, 2$. Then the service time $B_{0,i}$ of the partial foreground job that goes to server i has mean $\beta_{0,i}$, where

$$\beta_{0,i} := \alpha_i \beta_0, \quad i = 1, 2. \tag{4.1}$$

The occupation rate $\rho_{0,i}$ of the partial foreground job that goes to server i is

$$\rho_{0,i} := \alpha_i \rho_0, \quad i = 1, 2,$$

where $\rho_0 = \lambda_0 \beta_0$ for $i = 1, 2$. Equivalently, $\rho_{0,i} := \lambda_0 \beta_{0,i}$.

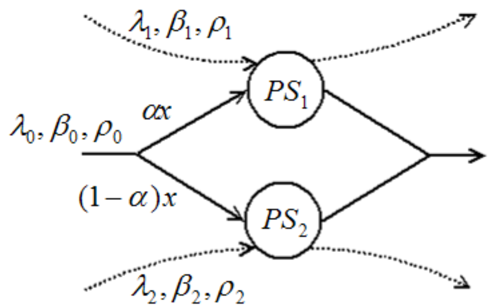


Figure 4.1: Job Split: Foreground job requiring x service time uses both PS servers simultaneously. Each server has one background traffic stream.

The occupation rate ρ_{S_i} in server i is then

$$\rho_{S_i} := \alpha_i \rho_0 + \rho_i, \quad i = 1, 2. \tag{4.2}$$

which has to be smaller than one in order to keep the system stable. As we similarly have seen in the server selection models:

$$\alpha < \frac{1-\rho_1}{\rho_0} =: U$$
$$\alpha > 1 - \frac{1-\rho_2}{\rho_0} =: L,$$

A split parameter α is *stable* if $\alpha \in (L, U)$. It is *feasible* if it is symmetrical and stable. The system is *stable* if both servers are, which is the case if:

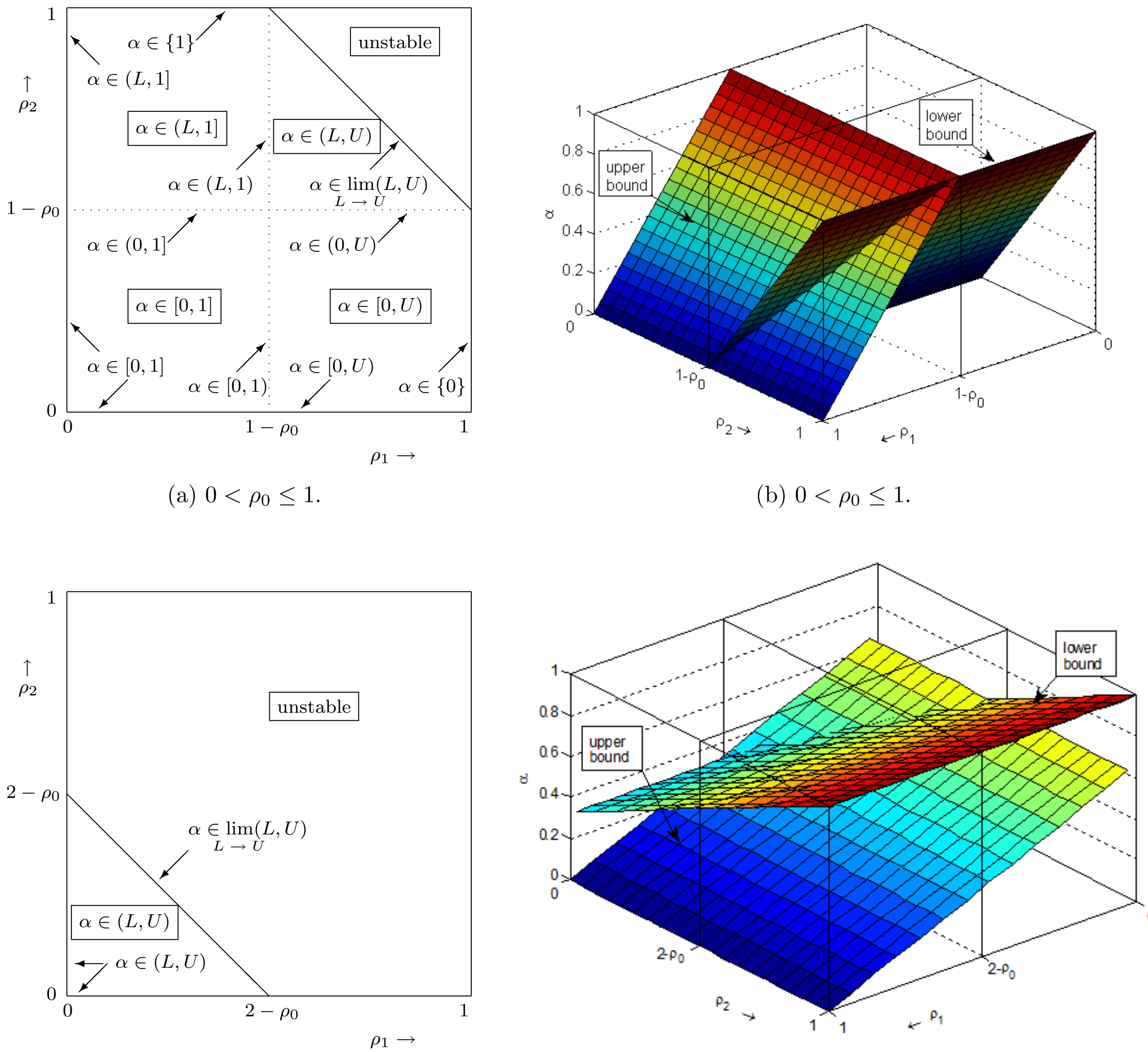
$$\rho_0 < (1-\rho_1) + (1-\rho_2),$$

as we have similarly seen in the server selection model.
The sojourn time S_i of type i background jobs has the following mean:

$$E[S_i] = \frac{\beta_i}{1 - \alpha_i \rho_0 - \rho_i}, \quad i = 1, 2. \tag{4.3}$$

Note that it depends on the split parameter α.
The sojourn time $S_{0,i}$ of the partial foreground job that goes to server i has the following mean:

$$E[S_{0,i}] = \frac{\beta_{0,i}}{1-\rho_{Si}} = \frac{\alpha_i \beta_0}{1 - \alpha_i \rho_0 - \rho_i}. \quad i = 1, 2$$



(a) $0 < \rho_0 \leq 1$.

(b) $0 < \rho_0 \leq 1$.

(c) $1 \leq \rho_0 < 2$.

(d) $1 \leq \rho_0 < 2$.

Figure 4.2: Stable split parameter α.

and the sojourn time of a whole foreground job then has the following mean:

$$E[S_0] = E[S_0](\alpha) = E[\max_{i=1,2}\{S_{0,i}\}].$$

α^* is the one that minimizes the expected sojourn time of a job:

$$\alpha^* = \min_{\alpha}\{E[S_0](\alpha)\}.$$

Finding the optimal split α^* analytically is rather difficult, because there is no simple expression available for the expected sojourn time of a job. We can solve this by approximating this by other expressions that are analytically available and where an optimal split, say $\hat{\alpha}^*$, can be calculated easily. Then we simulate the original model to find an optimal split α and we compare the approximated model to the simulated model.

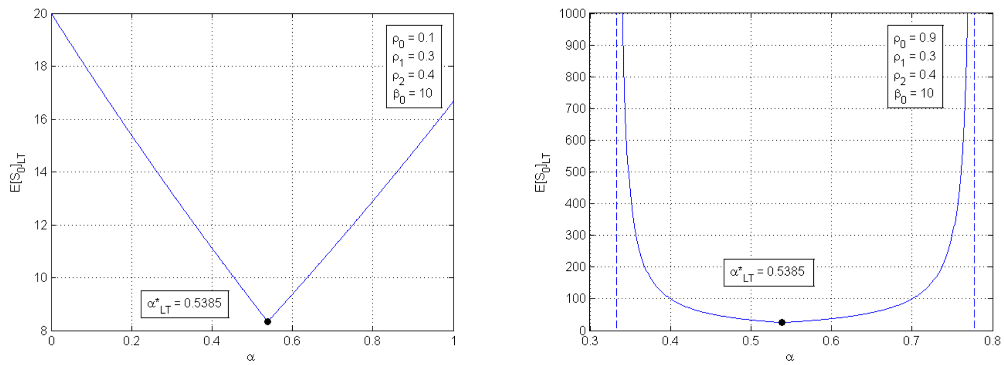
4.2 Light Traffic Approximation

When the occupation rates ρ_{S_i} in server i are close to zero for all i, we can approximate the sojourn times $S_{0,i}$ of each part of the foreground job by their respective service times $B_{0,i}$. Consequently, we can approximate the mean sojourn time $E[S_0]$ of a whole foreground job as follows:

$$\begin{aligned} E[S_0] &= E[\max_{i=1,2}\{S_{0,i}\}] \approx E[\max_{i=1,2}\{B_{0,i}\}] = E[\max_{i=1,2}\{\alpha_i B_0\}] = E[\max_{i=1,2}\{\alpha_i\}B_0] \\ &= \max_{i=1,2}\{\alpha_i\}E[B_0] = \max_{i=1,2}\{\alpha_i E[B_0]\} = \max_{i=1,2}\{E[\alpha_i B_0]\} = \max_{i=1,2}\{E[B_{0,i}]\} \\ &\approx \max_{i=1,2}\{E[S_{0,i}]\} = E[S_0]_{LT}. \end{aligned}$$

Computing α^*_{LT}, i.e. the split parameter that minimizes $E[S_0]_{LT}$, is somewhat easier, because it has an exact expression.

Now, we let f be the LT approximation of the mean sojourn time $E[S_0]_{LT}$ of a foreground job and we let f_i be the mean sojourn time $E[S_{0,3-i}]$ of the part of the foreground job that goes to server $3-i$ as a function of the split parameter α. Then $f(\alpha) = f_1(\alpha)$ if $f_2(\alpha) \leq f_1(\alpha)$. We can rewrite this as



(a) $\rho_0 = 0.1$.

(b) $\rho_0 = 0.9$.

Figure 4.3: The light traffic approximated mean sojourn time $E[S_0]_{LT}$ of foreground jobs as a function of the split parameter α.

follows:

$$
\begin{aligned}
& f_2(\alpha) \le f_1(\alpha) \\
& \frac{\alpha\beta_0}{1-\alpha\rho_0-\rho_1} \le \frac{(1-\alpha)\beta_0}{1-(1-\alpha)\rho_0-\rho_2} \\
& \frac{\alpha\beta_0}{1-\alpha\rho_0-\rho_1} - \frac{(1-\alpha)\beta_0}{1-(1-\alpha)\rho_0-\rho_2} \le 0 \\
& \frac{\alpha\beta_0(1-(1-\alpha)\rho_0-\rho_2)-(1-\alpha)\beta_0(1-\alpha\rho_0-\rho_1)}{(1-\alpha\rho_0-\rho_1)(1-(1-\alpha)\rho_0-\rho_2)} \le 0 \\
& \alpha\beta_0(1-(1-\alpha)\rho_0-\rho_2)-(1-\alpha)\beta_0(1-\alpha\rho_0-\rho_1) \le 0
\end{aligned}
$$

$$
\begin{aligned}
& \alpha(1-(1-\alpha)\rho_0-\rho_2)-(1-\alpha)(1-\alpha\rho_0-\rho_1) \le 0 \\
& \alpha(1-\rho_2)-(1-\alpha)(1-\rho_1) \le 0 \\
& \alpha((1-\rho_1)+(1-\rho_2))-(1-\rho_1) \le 0 \\
& \alpha \le \frac{1-\rho_1}{(1-\rho_1)+(1-\rho_2)}.
\end{aligned}
$$

Thus we define f as:

$$
f(\alpha) = \max_i\{f_i(\alpha)\} = \begin{cases} f_1(\alpha), & \alpha \le \alpha_0, \\ f_2(\alpha), & \alpha > \alpha_0. \end{cases}
$$

where

$$\alpha_0 = \frac{1-\rho_1}{(1-\rho_1)+(1-\rho_2)}.$$

Now,

$$f_1(\alpha) = \frac{(1-\alpha)\beta_0}{1-\rho_{S_2}} = \frac{(1-\alpha)\beta_0}{1-(1-\alpha)\rho_0-\rho_2}$$

$$f_1'(\alpha) = \frac{-\beta_0(1-(1-\alpha)\rho_0-\rho_2)-(1-\alpha)\beta_0\rho_0}{(1-(1-\alpha)\rho_0-\rho_2)^2} = \frac{-\beta_0(1-\rho_2)}{(1-(1-\alpha)\rho_0-\rho_2)^2} < 0,$$

so f_1 is decreasing and

$$f_2(\alpha) = \frac{\alpha\beta_0}{1-\rho_{S_1}} = \frac{\alpha\beta_0}{1-\alpha\rho_0-\rho_1}$$

$$f_2'(\alpha) = \frac{\beta_0(1-\alpha\rho_0-\rho_1)+\alpha\beta_0\rho_0}{(1-\alpha\rho_0-\rho_1)^2} = \frac{\beta_0(1-\rho_1)}{(1-\alpha\rho_0-\rho_1)^2} > 0,$$

so f_2 is increasing. Both functions have α_0 as its edge minimum. Thus it appears that the optimal split α^*_{LT} of the LT approximation is:

$$\alpha^*_{LT} = \frac{1-\rho_1}{(1-\rho_1)+(1-\rho_2)}.$$

Note here that it does not depend on the distribution of the service time B_0, nor does it depend on ρ_0. It is trivial that it is symmetrical. It is stable, because:

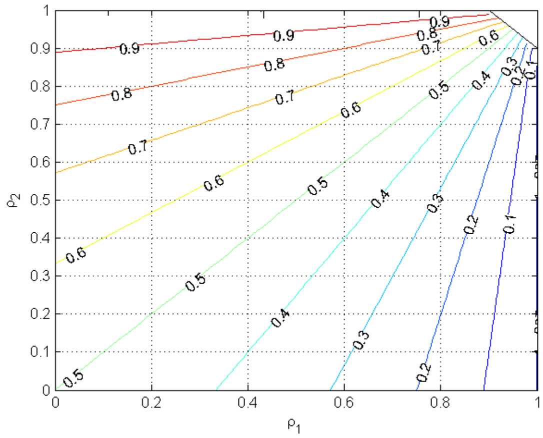
$$\alpha^*_{LT} = \frac{1-\rho_1}{(1-\rho_1)+(1-\rho_2)} < U,$$

since $\rho_0 < (1-\rho_1)+(1-\rho_2)$ and it can be similarly shown that $\alpha^*_{LT} > L$. Hence α^*_{LT} is feasible. Figure 4.4 and table 4.1 show plots and numerical values of α^*_{LT} for different values of the occupation rates ρ_j, for $j = 0, 1, 2$.

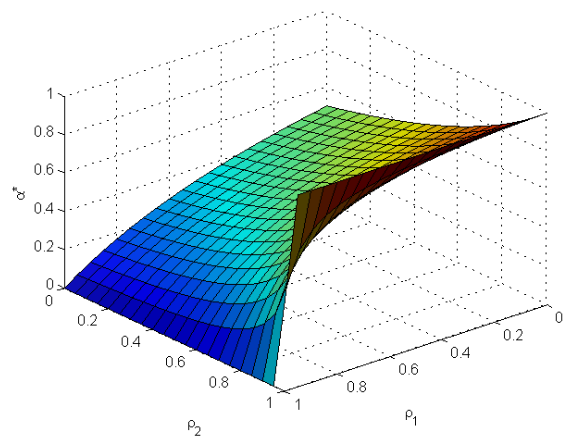
4.3 Heavy Traffic Approximation

When the occupation rates ρ_{S_i} in server i are close to one for all i, we can approximate the squared coefficient of variation $c^2_{S_{0,i}}$ of the sojourn time of part i of a foreground job as follows (cf. Equation (2.6)):

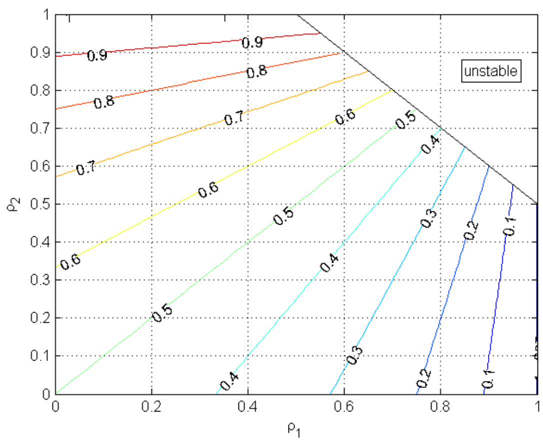
$$c^2_{S_{0,i}} \approx 2c^2_{B_{0,i}} + 1 = 2c^2_{\alpha_i B_0} + 1 = 2c^2_{B_0} + 1.$$



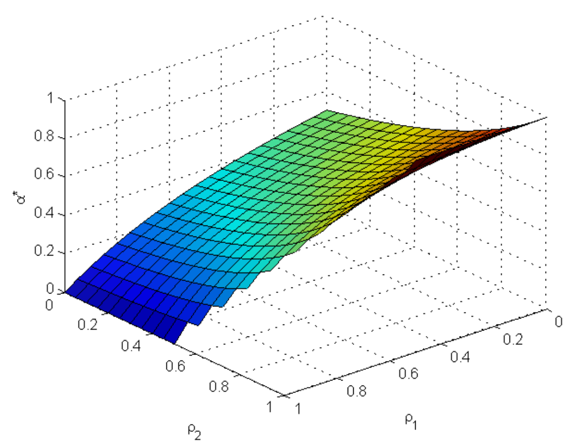
(a) $\rho_0 = 0.1$.



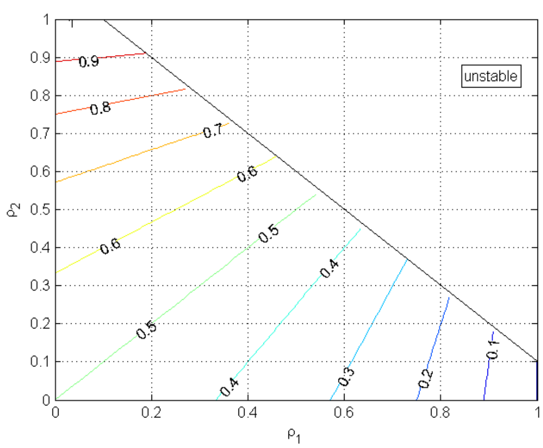
(b) $\rho_0 = 0.1$.



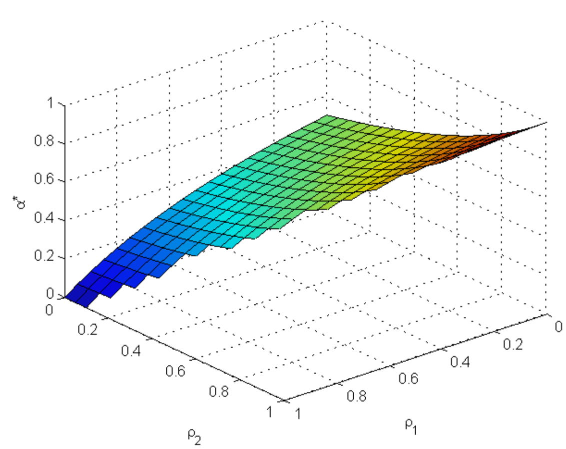
(c) $\rho_0 = 0.5$.



(d) $\rho_0 = 0.5$.

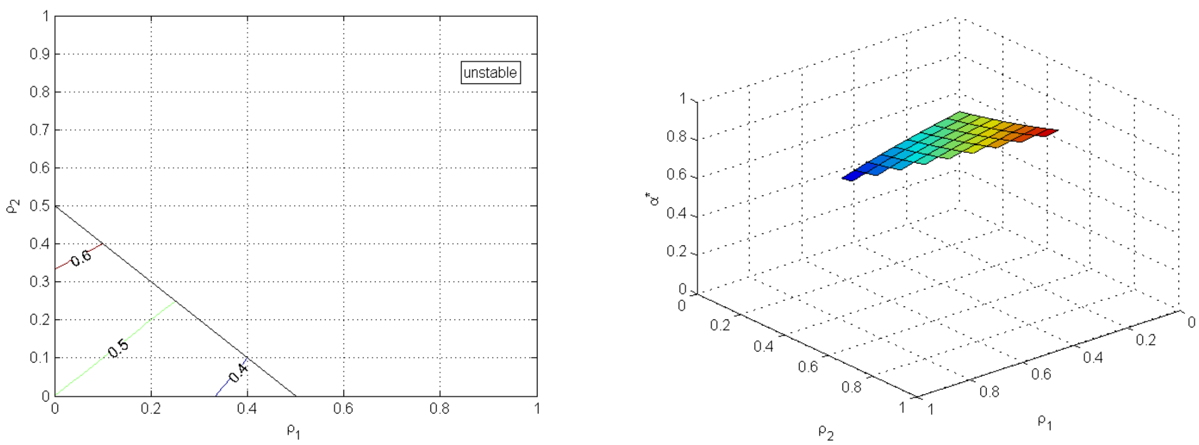


(e) $\rho_0 = 0.9$.



(f) $\rho_0 = 0.9$.

Figure 4.4: Plots of the light traffic optimal split α^*_{LT}.

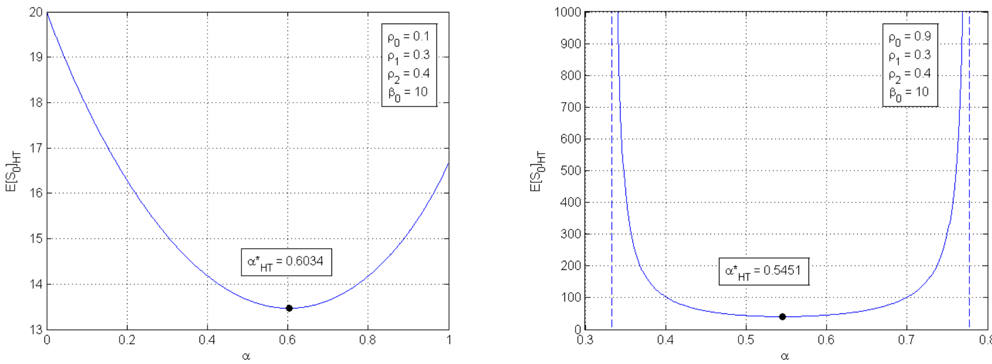


(g) $\rho_0 = 1.5$. (h) $\rho_0 = 1.5$.

Figure 4.4: Plots of the light traffic optimal split α^*_{LT}.

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	0.5	0.5263	0.5556	0.5882	0.625	0.6667	0.7143	0.7692	0.8333	0.9091	1
0.1	0.4737	0.5	0.5294	0.5625	0.6	0.6429	0.6923	0.75	0.8182	0.9	1
0.2	0.4444	0.4706	0.5	0.5333	0.5714	0.6154	0.6667	0.7273	0.8	0.8889	1
0.3	0.4118	0.4375	0.4667	0.5	0.5385	0.5833	0.6364	0.7	0.7778	0.875	1
0.4	0.375	0.4	0.4286	0.4615	0.5	0.5455	0.6	0.6667	0.75	0.8571	1
0.5	0.3333	0.3571	0.3846	0.4167	0.4545	0.5	0.5556	0.625	0.7143	0.8333	1
0.6	0.2857	0.3077	0.3333	0.3636	0.4	0.4444	0.5	0.5714	0.6667	0.8	1
0.7	0.2308	0.25	0.2727	0.3	0.3333	0.375	0.4286	0.5	0.6	0.75	1
0.8	0.1667	0.1818	0.2	0.2222	0.25	0.2857	0.3333	0.4	0.5	0.6667	1
0.9	0.0909	0.1	0.1111	0.125	0.1429	0.1667	0.2	0.25	0.3333	0.5	1
1	0	0	0	0	0	0	0	0	0	0	-

Table 4.1: Numerical values of the light traffic optimal split α^*_{LT}.



(a) $\rho_0 = 0.1$. (b) $\rho_0 = 0.9$.

Figure 4.5: The heavy traffic approximated mean sojourn time $E[S_0]_{HT}$ of foreground jobs as a function of the split parameter α for exponential service times B_j, $j = 0, 1, 2$.

Note that it is greater or equal to one. Thus for all service time distributions, we can approximate the distribution of the sojourn time $S_{0,i}$ of part i of a foreground job by a $H_2(p_i; \mu_{i1}, \mu_{i2})$ distribution, where the parts are independent from each other. We thereby determine μ_{ij} as follows:

$$E[S_{0,i}] = \frac{\alpha_i \beta_0}{1 - \alpha_i \rho_0 - \rho_i} = \frac{2p_{ij}}{\mu_{ij}}$$

$$\mu_{ij} = 2p_{ij} \frac{1 - \alpha_i \rho_0 - \rho_i}{\alpha_i \beta_0},$$

where $p_{i1} = p_i = 1 - p_{i2}$. Hence we approximate the mean sojourn time $E[S_0]$ of foreground jobs as follows:

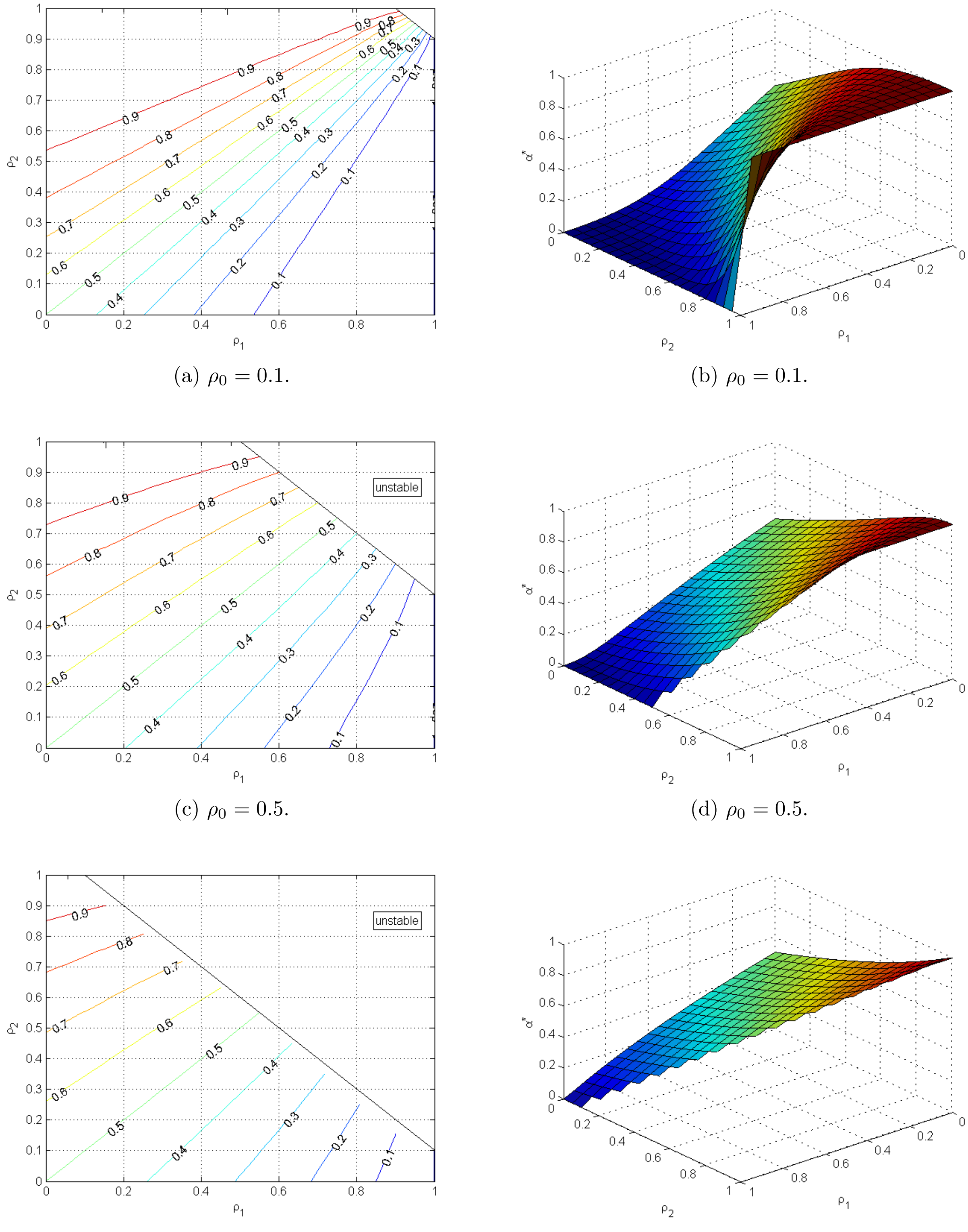
$$E[S_0]_{HT} = E[\max_{i=1,2}\{S_{0,i}\}]_{HT} = \sum_{i=1}^{2} \sum_{j=1}^{2} p_{1i} p_{2j} \left(\frac{1}{\mu_{1i}} + \frac{1}{\mu_{2j}} - \frac{1}{\mu_{1i} + \mu_{2j}} \right)$$

$$= \sum_{i=1}^{2} \sum_{j=1}^{2} p_{1i} p_{2j} \left(\frac{\alpha \beta_0}{2p_{1i}(1 - \alpha\rho_0 - \rho_1)} + \frac{(1-\alpha)\beta_0}{2p_{2j}(1 - (1-\alpha)\rho_0 - \rho_2)} \right.$$

$$\left. - \frac{1}{\frac{2p_{1i}(1-\alpha\rho_0-\rho_1)}{\alpha\beta_0} + \frac{2p_{2j}(1-(1-\alpha)\rho_0-\rho_2)}{(1-\alpha)\beta_0}} \right)$$

$$= \frac{\beta_0}{2} \sum_{i=1}^{2} \sum_{j=1}^{2} \left(\frac{\alpha p_{2j}}{1 - \alpha\rho_0 - \rho_1} + \frac{(1-\alpha)p_{1i}}{1 - (1-\alpha)\rho_0 - \rho_2} - \frac{1}{\frac{1-\alpha\rho_0-\rho_1}{\alpha p_{2j}} + \frac{1-(1-\alpha)\rho_0-\rho_2}{(1-\alpha)p_{1i}}} \right).$$

Note that the mean sojourn time $E[S_0]_{HT}$ depends on the distribution of the service time. This also holds for its optimal split α^*_{HT}, which we cannot determine explicitly. However, we can determine it numerically by means of a mathematical software program, such as Maple. Figure 4.6 and Table 4.2 show plots and numerical values of α^*_{HT} for *exponential* service times B_j and different values of the occupation rates ρ_j, for $j = 0, 1, 2$. However, they are similar for *Erlang-2*, *deterministic* and *hyperexponential* service times B_j, where the latter one has squared coefficient of variation $c^2_{B_j} \in \{2, 4, 16\}$, for $j = 0, 1, 2$.



(a) $\rho_0 = 0.1$.

(b) $\rho_0 = 0.1$.

(c) $\rho_0 = 0.5$.

(d) $\rho_0 = 0.5$.

(e) $\rho_0 = 0.9$.

(f) $\rho_0 = 0.9$.

Figure 4.6: Plots of the heavy traffic optimal split α^*_{HT} for exponential service times B_j, $j = 0, 1, 2$.

$\rho_1 \backslash \rho_2$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	0.5	0.5767	0.6572	0.7379	0.8141	0.8803	0.9318	0.9669	0.9875	0.9973	1
0.1	0.4233	0.5	0.5839	0.6721	0.7598	0.8408	0.9076	0.9551	0.9833	0.9965	1
0.2	0.3428	0.4161	0.5	0.5926	0.6901	0.7861	0.8715	0.9367	0.9766	0.9953	1
0.3	0.2621	0.3279	0.4074	0.5	0.6034	0.7123	0.8179	0.9066	0.9654	0.9932	1
0.4	0.1859	0.2402	0.3099	0.3966	0.5	0.617	0.7405	0.857	0.9449	0.9894	1
0.5	0.1197	0.1592	0.2139	0.2877	0.383	0.5	0.6349	0.7776	0.9052	0.9814	1
0.6	0.0682	0.0924	0.1285	0.1821	0.2595	0.3651	0.5	0.6598	0.8289	0.9618	1
0.7	0.0331	0.0449	0.0633	0.0934	0.143	0.2224	0.3402	0.5	0.6972	0.9052	1
0.8	0.0125	0.0167	0.0234	0.0346	0.0551	0.0948	0.1711	0.3028	0.5	0.7638	1
0.9	0.0027	0.0035	0.0047	0.0068	0.0106	0.0186	0.0382	0.0948	0.2362	0.5	1
1	0	0	0	0	0	0	0	0	0	0	-

(a) $\rho_0 = 0.1$

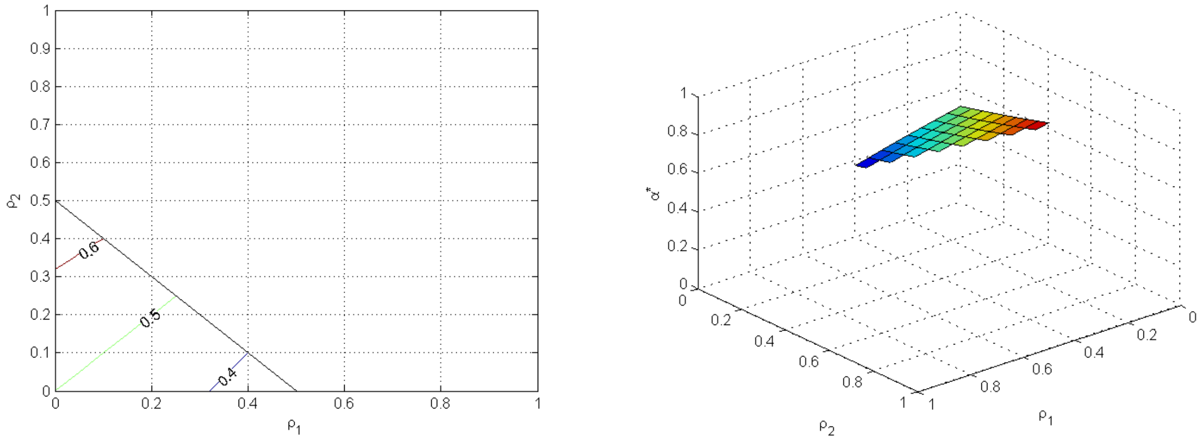
$\rho_1 \backslash \rho_2$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	0.5	0.5476	0.5978	0.6507	0.7061	0.7638	0.8232	0.8829	0.9396	0.984	1
0.1	0.4524	0.5	0.5507	0.6046	0.6617	0.7219	0.7851	0.8505	0.916	0.9742	1
0.2	0.4022	0.4493	0.5	0.5544	0.6127	0.6749	0.7413	0.8117	0.8854	0.9579	1
0.3	0.3493	0.3954	0.4456	0.5	0.5589	0.6226	0.6916	0.7663	0.8472	0.9332	1
0.4	0.2939	0.3383	0.3873	0.4411	0.5	0.5645	0.6354	0.7137	0.8009	0.8991	1
0.5	0.2362	0.2781	0.3251	0.3774	0.4355	0.5	0.572	0.6529	0.7456	0.8552	-
0.6	0.1768	0.2149	0.2587	0.3084	0.3646	0.428	0.5	0.5826	0.6797	-	-
0.7	0.1171	0.1495	0.1883	0.2337	0.2863	0.3471	0.4174	0.5	-	-	-
0.8	0.0604	0.084	0.1146	0.1528	0.1991	0.2544	0.3203	-	-	-	-
0.9	0.016	0.0258	0.0421	0.0668	0.1009	0.1448	-	-	-	-	-
1	0	0	0	0	0	-	-	-	-	-	-

(b) $\rho_0 = 0.5$

$\rho_1 \backslash \rho_2$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	0.5	0.5367	0.5755	0.6166	0.6602	0.7067	0.7565	0.8102	0.8685	0.9329	1
0.1	0.4633	0.5	0.539	0.5805	0.6248	0.6724	0.7238	0.7798	0.8419	0.9126	-
0.2	0.4245	0.461	0.5	0.5417	0.5866	0.635	0.6879	0.7461	0.8118	-	-
0.3	0.3834	0.4195	0.4583	0.5	0.5451	0.5943	0.6483	0.7086	-	-	-
0.4	0.3398	0.3752	0.4134	0.4549	0.5	0.5495	0.6045	-	-	-	-
0.5	0.2933	0.3276	0.365	0.4057	0.4505	0.5	-	-	-	-	-
0.6	0.2435	0.2762	0.3121	0.3517	0.3955	-	-	-	-	-	-
0.7	0.1898	0.2202	0.2539	0.2914	-	-	-	-	-	-	-
0.8	0.1315	0.1581	0.1882	-	-	-	-	-	-	-	-
0.9	0.0671	0.0874	-	-	-	-	-	-	-	-	-
1	0	-	-	-	-	-	-	-	-	-	-

(c) $\rho_0 = 0.9$

Table 4.2: Numerical values of the heavy traffic optimal split α^*_{HT} for *exponential* service times B_j, $j = 0, 1, 2$.



(g) $\rho_0 = 1.5$. (h) $\rho_0 = 1.5$.

Figure 4.6: Plots of the heavy traffic optimal split α^*_{HT} for *exponential* service times.

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	0.5	0.5291	0.5601	0.593	0.6284	-	-	-	-	-	-
0.1	0.4709	0.5	0.531	0.5642	-	-	-	-	-	-	-
0.2	0.4399	0.469	0.5	-	-	-	-	-	-	-	-
0.3	0.407	0.4358	-	-	-	-	-	-	-	-	-
0.4	0.3716	-	-	-	-	-	-	-	-	-	-
0.5	-	-	-	-	-	-	-	-	-	-	-
0.6	-	-	-	-	-	-	-	-	-	-	-
0.7	-	-	-	-	-	-	-	-	-	-	-
0.8	-	-	-	-	-	-	-	-	-	-	-
0.9	-	-	-	-	-	-	-	-	-	-	-
1	-	-	-	-	-	-	-	-	-	-	-

(d) $\rho_0 = 1.5$

Table 4.2: Numerical values of the heavy traffic optimal split α^*_{HT} for *exponential* service times B_j, $j = 0, 1, 2$.

4.4 Composed Optimal Split

The actual optimal split α^* of our model must have the following properties:

$$\alpha^* \to \begin{cases} \alpha^*_{LT}, & \rho_{S_i} \to 0, i = 1, 2, \\ \alpha^*_{HT}, & \rho_{S_i} \to 1, i = 1, 2. \end{cases}$$

This composed optimal split possesses these properties:

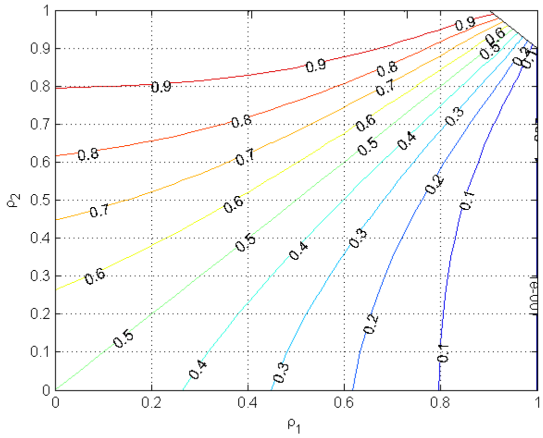
$$\begin{aligned} \alpha^*_{CP} &= (1 - \frac{\rho_{S_1} + \rho_{S_2}}{2})\alpha^*_{LT} + \frac{\rho_{S_1} + \rho_{S_2}}{2}\alpha^*_{HT} \\ &= (1 - \frac{\rho_0 + \rho_1 + \rho_2}{2})\alpha^*_{LT} + \frac{\rho_0 + \rho_1 + \rho_2}{2}\alpha^*_{HT}. \end{aligned}$$

Note that it depends on the distribution of the service time. Figure 4.7 and Table 4.3 show plots and numerical values of α^*_{CP} for *exponential* service times B_j and different values of the occupation rates ρ_j, for $j = 0, 1, 2$. However, they are similar for *Erlang-2*, *deterministic* and *hyperexponential* service times B_j, where the latter one has squared coefficient of variation $c^2_{B_j} \in \{2, 4, 16\}$, for $j = 0, 1, 2$.

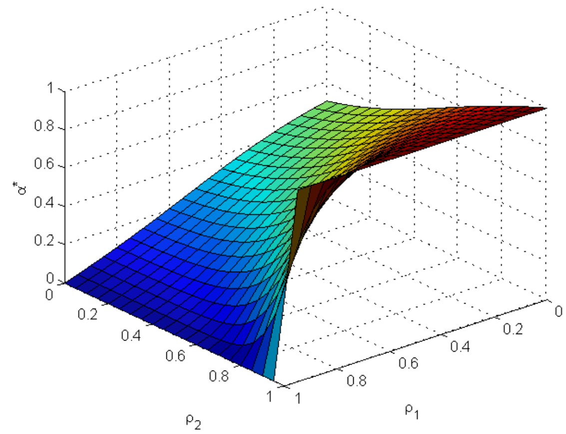
4.5 Comparison of the Optimal Splits

We want to verify the accuracy of the mentioned analytical optimal splits. Therefore, we simulate the mean sojourn times, we smoothen them by means of regression and extract the optimal split from these outcomes. We thereby use Extend again. We build some models in Extend and run them for different values of α and ρ_i for $i = 1, 2$ and fixed values of β_i for $i = 1, 2$. More precisely, $\alpha \in \{0, 0.01, 0.02, \ldots, 1\}$, $\rho_0 \in \{0.1, 0.5, 0.9\}$, $\rho_i \in \{0, 0.1, 0.2, \ldots, 0.9\}$ for $i = 1, 2$ and $\beta_j = 10$ seconds for $j = 0, 1, 2$. We consider six distributions of the service times B_j for $j = 0, 1, 2$: exponential, hyperexponential with squared coefficient of variation $c^2_{B_j} \in \{2, 4, 16\}$, Erlang-2 and deterministic for $j = 0, 1, 2$. We then smoothen the simulated mean sojourn times by means of regression and compare them in case the split α is an analytic optimal split to the case where the split α is an analytic optimal split. We perform this for exponential, Erlang-2 and hyperexponential service times with $c^2 \in \{2, 4, 16\}$ and with mean $\beta_j = 10$ for $j = 0, 1, 2$ for each combination of the occupation rates

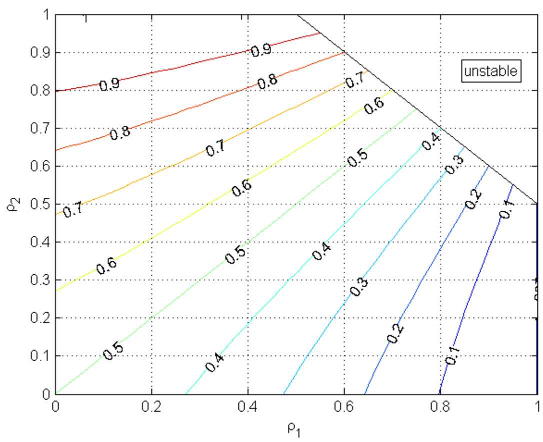
$$(\rho_0, \rho_1, \rho_2) \in \{0.1, 0.5, 0.9\} \times \{0, 0.1, \ldots, 0.9\}^2$$



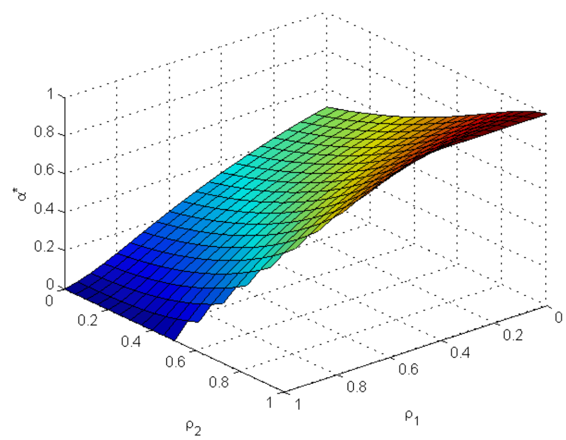
(a) $\rho_0 = 0.1$.



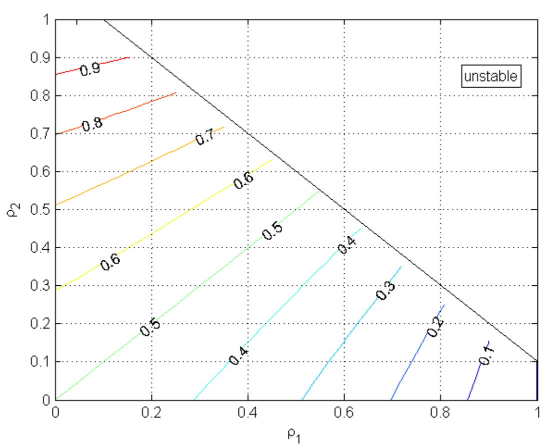
(b) $\rho_0 = 0.1$.



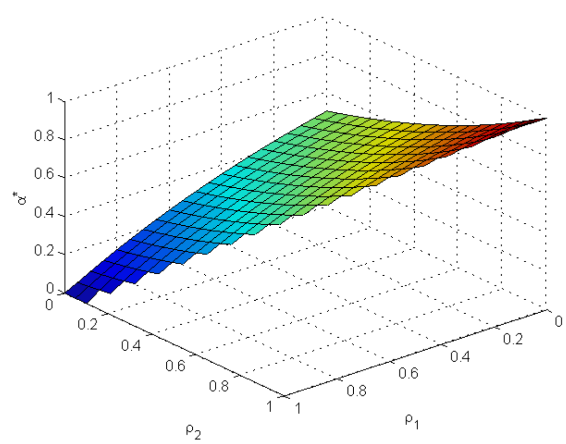
(c) $\rho_0 = 0.5$.



(d) $\rho_0 = 0.5$.



(e) $\rho_0 = 0.9$.



(f) $\rho_0 = 0.9$.

Figure 4.7: Plots of the composed optimal split α^*_{CP} for *exponential* service times.

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	0.5	0.5314	0.5708	0.6182	0.6723	0.7307	0.7904	0.8483	0.9027	0.9532	1
0.1	0.4686	0.5	0.5403	0.5899	0.648	0.7121	0.7784	0.8423	0.9007	0.9531	1
0.2	0.4292	0.4597	0.5	0.5511	0.613	0.6837	0.7589	0.832	0.8972	0.9527	1
0.3	0.3818	0.4101	0.4489	0.5	0.5644	0.6414	0.7271	0.8136	0.8904	0.9518	1
0.4	0.3277	0.352	0.387	0.4356	0.5	0.5812	0.6773	0.7808	0.8767	0.9497	1
0.5	0.2693	0.2879	0.3163	0.3586	0.4188	0.5	0.6032	0.7242	0.848	0.9444	1
0.6	0.2096	0.2216	0.2411	0.2729	0.3227	0.3968	0.5	0.6333	0.7884	0.9294	1
0.7	0.1517	0.1577	0.168	0.1864	0.2192	0.2758	0.3667	0.5	0.6777	0.8819	1
0.8	0.0973	0.0993	0.1028	0.1096	0.1233	0.152	0.2116	0.3223	0.5	0.7541	1
0.9	0.0468	0.0469	0.0473	0.0482	0.0503	0.0556	0.0706	0.1181	0.2459	0.5	1
1	0	0	0	0	0	0	0	0	0	0	-

(a) $\rho_0 = 0.1$

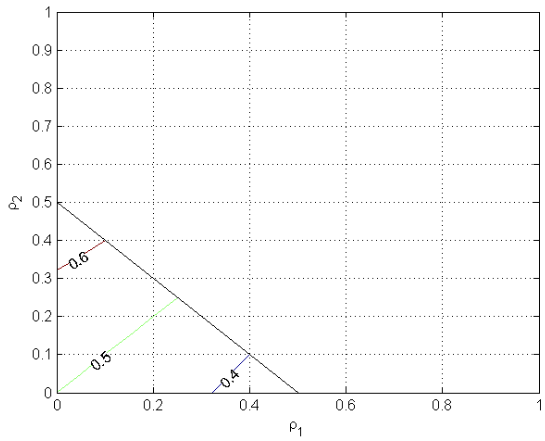
ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	0.5	0.5327	0.5703	0.6132	0.6615	0.7152	0.7742	0.8374	0.9024	0.9615	1
0.1	0.4673	0.5	0.5379	0.5814	0.6308	0.6863	0.748	0.8153	0.8867	0.9557	1
0.2	0.4297	0.4621	0.5	0.5439	0.5941	0.6511	0.7152	0.7864	0.864	0.9441	1
0.3	0.3868	0.4186	0.4561	0.5	0.5507	0.6089	0.675	0.7497	0.8333	0.9244	1
0.4	0.3385	0.3692	0.4059	0.4493	0.5	0.5588	0.6266	0.7043	0.7932	0.8949	1
0.5	0.2848	0.3137	0.3489	0.3911	0.4412	0.5	0.5687	0.6488	0.7424	0.8541	-
0.6	0.2258	0.252	0.2848	0.325	0.3734	0.4313	0.5	0.5815	0.679	-	-
0.7	0.1626	0.1847	0.2136	0.2503	0.2957	0.3512	0.4185	0.5	-	-	-
0.8	0.0976	0.1133	0.136	0.1667	0.2068	0.2576	0.321	-	-	-	-
0.9	0.0385	0.0443	0.0559	0.0756	0.1051	0.1459	-	-	-	-	-
1	0	0	0	0	0	-	-	-	-	-	-

(b) $\rho_0 = 0.5$

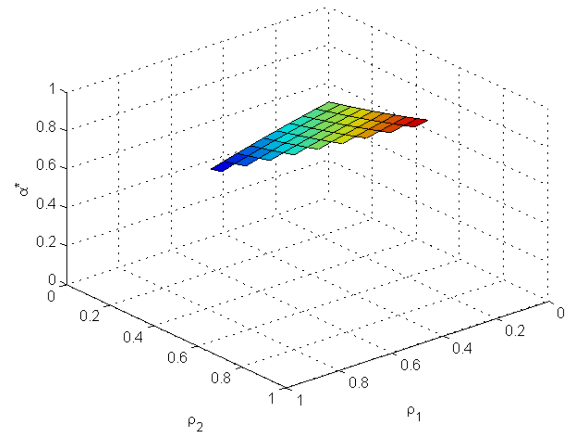
ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	0.5	0.5315	0.5665	0.6053	0.6479	0.6947	0.746	0.802	0.8633	0.9305	1
0.1	0.4685	0.5	0.5352	0.5742	0.6174	0.665	0.7175	0.7754	0.8395	0.912	-
0.2	0.4335	0.4648	0.5	0.5392	0.5828	0.6311	0.6847	0.7443	0.8112	-	-
0.3	0.3947	0.4258	0.4608	0.5	0.5438	0.5926	0.6471	0.7082	-	-	-
0.4	0.3521	0.3826	0.4172	0.4562	0.5	0.5491	0.6043	-	-	-	-
0.5	0.3053	0.335	0.3689	0.4074	0.4509	0.5	-	-	-	-	-
0.6	0.254	0.2825	0.3153	0.3529	0.3957	-	-	-	-	-	-
0.7	0.198	0.2246	0.2557	0.2918	-	-	-	-	-	-	-
0.8	0.1367	0.1605	0.1888	-	-	-	-	-	-	-	-
0.9	0.0695	0.088	-	-	-	-	-	-	-	-	-
1	0	-	-	-	-	-	-	-	-	-	-

(c) $\rho_0 = 0.9$

Table 4.3: Numerical values of the composed optimal split α^*_{CP} for *exponential* service times.



(g) $\rho_0 = 1.5$.



(h) $\rho_0 = 1.5$.

Figure 4.7: Plots of the composed optimal split α^*_{CP} for *exponential* service times.

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	0.5	0.5286	0.5594	0.5925	0.6282	-	-	-	-	-	-
0.1	0.4714	0.5	0.5309	0.5641	-	-	-	-	-	-	-
0.2	0.4406	0.4691	0.5	-	-	-	-	-	-	-	-
0.3	0.4075	0.4359	-	-	-	-	-	-	-	-	-
0.4	0.3718	-	-	-	-	-	-	-	-	-	-
0.5	-	-	-	-	-	-	-	-	-	-	-
0.6	-	-	-	-	-	-	-	-	-	-	-
0.7	-	-	-	-	-	-	-	-	-	-	-
0.8	-	-	-	-	-	-	-	-	-	-	-
0.9	-	-	-	-	-	-	-	-	-	-	-
1	-	-	-	-	-	-	-	-	-	-	-

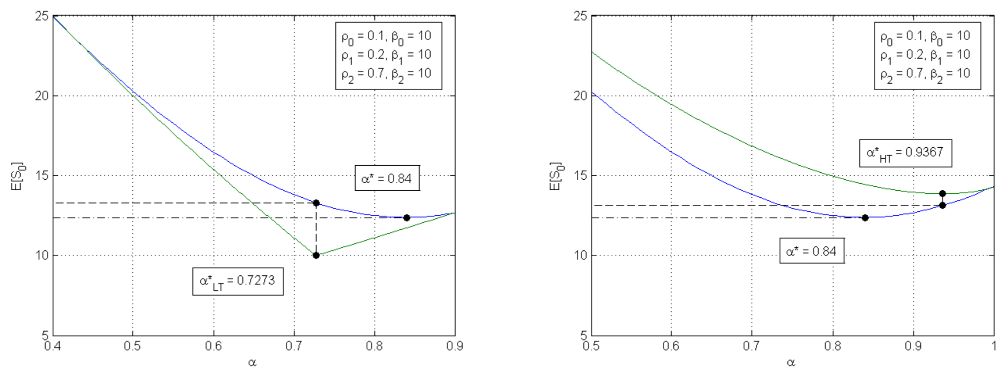
(d) $\rho_0 = 1.5$

Table 4.3: Numerical values of the composed optimal split α^*_{CP} for *exponential* service times.

by means of their relative difference:

$$\frac{E[S_0](\alpha^*_{\text{analytic}}) - E[S_0](\alpha^*_{\text{actual}})}{E[S_0](\alpha^*_{\text{actual}})} \cdot 100\%.$$

We thereby consider these analytic optimal splits: the light traffic optimal split α^*_{LT}, the heavy traffic optimal split α^*_{HT} and the composed optimal split α^*_{CP}.



(a) Light traffic optimal split α^*_{LT}. (b) Heavy traffic optimal split α^*_{HT}.

Figure 4.8: Comparison of the analytic optimal splits α^*_{LT} and α^*_{HT} with the actual optimal split α^* for exponential service times B_j, $j = 0, 1, 2$.

It appears that the light traffic optimal split α^*_{LT} is indeed closest to the actual optimal split for small occupation rates ρ_{S_i} in each server i. It is also apparent that the heavy traffic optimal split α^*_{LT} is indeed closest to the actual optimal split for large occupation rates ρ_{S_i} in each server i. Moreover, we find that the relative difference between the latter optimal split and the approximate actual optimal split is less than 1% for all occupation rates ρ_j for $j = 0, 1, 2$, in contrast to the other analytic optimal splits. Thus the composed optimal split approximates the actual optimal split very well. Note that the relative differences must go to zero in case $\rho_2 \rightarrow \rho_1$, because of the symmetry of the optimal splits or $\rho_0 \rightarrow (1 - \rho_1) + (1 - \rho_2)$, because of the stability of the optimal splits. Appendix Tables A.6 - A.23 show these observations.

Chapter 5

Assessment of the Models

5.1 Comparison of the Static Models

We compare the static job split model to the static server selection model by means of their mean sojourn times $E[S_0]$ of foreground jobs to each other with respect to their optimal parameters, which we already simulated and smoothened for the job split model and we have them analytically available in the server selection model. We perform this for *exponential*, *Erlang-2*, *deterministic* and *hyperexponential* service times with $c^2 \in \{2, 4, 16\}$ and with mean $\beta_0 = 10$ seconds for each combination of the occupation rates

$$(\rho_0, \rho_1, \rho_2) \in \{0.1, 0.5, 0.9\} \times \{0, 0.1, \ldots, 0.9\}^2$$

by means of their relative difference:

$$\frac{E[S_0]_{\text{select}}(q^*) - E[S_0]_{\text{split}}(\alpha^*)}{E[S_0]_{\text{select}}(q^*)} \cdot 100\%.$$

It appears that it is positive in all cases, meaning that the mean sojourn time $E[S_0]$ of foreground jobs decreases when we substitute the server selection model by the job split model. The higher the occupation rates ρ_i for $i = 1, 2$ differ from each other, the smaller the relative difference becomes. Also, the higher the occupation rate ρ_0, the higher the relative difference becomes. The reason for the latter one is, because a higher ρ_0 means more foreground jobs in the system that influence the mean sojourn time $E[S_0]$. Note also that if the occupation rates ρ_{S_i} go to zero for some

$i = 1, 2$, then:

$$E[S_{0,i}]_{\text{select}} \to \sum_{i=1}^{2} q_i E[B_0],$$
$$E[S_{0,i}]_{\text{split}} \to \max_{i=1,2}\{\alpha_i\} E[B_0].$$

Consequently, if both go to zero, then $\rho_2 \to \rho_1$ and so $q_i, \alpha_i \to \frac{1}{2}$, in which case:

$$E[S_{0,i}]_{\text{split}} \to \frac{1}{2} E[S_{0,i}]_{\text{select}}.$$

In other words, the relative difference goes to 50%. Appendix Tables A.24 - A.29 show these observations.

5.2 Conclusions

We found in our research that the *composed optimal split* is the one that approximates the actual optimal split very well. We also found that we can improve the static server selection model by the dynamic server selection model constructed by Markov Decision chains. We discovered that the largest improvement occurs when the occupation rate ρ_0 is large and the occupation rates ρ_i do not differ much from each other, for $i = 1, 2$. And finally, we have seen that the mean sojourn time decreases if we substitute the server selection model for the job split model. We again found that the largest improvement occurs when the occupation rate ρ_0 is large and the occupation rates ρ_i do not differ much from each other, for $i = 1, 2$. Although this last finding is based on static models, it already shows the potential benefits of Concurrent Access for mobile operators, where they can utilise their current network capacity in a much more efficient way. It is a relatively small adjustment in their systems for a relatively high gain.

5.3 Topics of Further Research

In order to make the models more realistic, we can elaborate this research with the following models:

- models where the occupation rates ρ_j are estimated per foreground job arrival based on some time interval in the past,
- job split model where each foreground job is split according to the number of jobs in each server, which is modelled as a Markov decision chain,
- Markov decision models for both server selection models and job split models, whose optimal policy depend on the job characterisation, which are service types for hyperexponential service times and phases for Erlang distributions,
- models with inhomogeneous Poisson arrivals and non-Poisson arrivals,
- models with more than two Processor Sharing servers.

Appendix A

Relative Differences

The appendix contains tables of relative differences. Section A.1 shows relative differences between these server selection models: the static and the Markov decision model, i.e.:

$$\frac{E[S_0]_{\text{static}}(q^*) - E[S_0]_{\text{Markov}}}{E[S_0]_{\text{static}}(q^*)} \cdot 100\%,$$

Section A.2 displays relative differences between the actual optimal split and an analytic optimal split, i.e.:

$$\frac{E[S_0](\alpha^*_{\text{analytic}}) - E[S_0](\alpha^*_{\text{actual}})}{E[S_0](\alpha^*_{\text{actual}})} \cdot 100\%,$$

and Section A.3 shows relative differences between these static models: the server selection model and the job split model, i.e.:

$$\frac{E[S_0]_{\text{select}}(q^*) - E[S_0]_{\text{split}}(\alpha^*)}{E[S_0]_{\text{select}}(q^*)} \cdot 100\%.$$

A.1 Server Selection Models

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	4.4%	3.9%	1.9%	0.9%	0.4%	0.1%	0.0%	0.0%	0.0%	0.0%
0.1	3.9%	4.6%	4.0%	2.3%	1.2%	0.5%	0.1%	0.0%	0.0%	0.0%
0.2	1.9%	4.0%	4.8%	4.2%	2.5%	1.3%	0.5%	0.1%	0.0%	0.0%
0.3	0.9%	2.3%	4.2%	5.1%	4.4%	2.6%	1.2%	0.4%	0.1%	0.0%
0.4	0.4%	1.2%	2.5%	4.4%	5.6%	4.8%	2.6%	1.1%	0.3%	0.0%
0.5	0.1%	0.5%	1.3%	2.6%	4.8%	6.3%	5.3%	2.6%	0.8%	0.1%
0.6	0.0%	0.1%	0.5%	1.2%	2.6%	5.3%	7.4%	6.1%	2.4%	0.3%
0.7	0.0%	0.0%	0.1%	0.4%	1.1%	2.6%	6.1%	9.4%	7.4%	1.7%
0.8	0.0%	0.0%	0.0%	0.1%	0.3%	0.8%	2.4%	7.4%	13.3%	9.8%
0.9	0.0%	0.0%	0.0%	0.0%	0.0%	0.1%	0.3%	1.7%	9.8%	23.4%

(a) $\rho_0 = 0.1$

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	17.4%	18.6%	18.5%	17.2%	14.6%	10.4%	5.7%	2.4%	0.6%	0.0%
0.1	18.6%	18.2%	19.2%	18.5%	17.9%	15.3%	10.9%	5.4%	1.9%	0.2%
0.2	18.5%	19.2%	19.4%	20.4%	20.4%	19.3%	16.7%	11.5%	4.8%	0.8%
0.3	17.2%	19.2%	20.4%	21.1%	22.2%	22.4%	21.5%	19.0%	12.4%	3.1%
0.4	14.6%	17.9%	20.4%	22.2%	23.4%	24.9%	25.5%	25.3%	23.2%	22.5%
0.5	10.4%	15.3%	19.3%	22.4%	24.9%	26.9%	29.1%	30.8%	32.4%	32.7%
0.6	5.7%	10.9%	16.7%	21.5%	25.5%	29.1%	32.3%	36.1%	39.8%	-
0.7	2.4%	5.4%	11.5%	19.0%	25.3%	30.8%	36.1%	40.9%	-	-
0.8	0.6%	1.9%	4.8%	12.4%	23.2%	32.4%	39.8%	-	-	-
0.9	0.0%	0.2%	0.8%	3.1%	22.5%	32.7%	-	-	-	-

(b) $\rho_0 = 0.5$

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	26.6%	27.9%	28.8%	29.2%	29.3%	29.1%	28.6%	27.5%	24.7%	14.8%
0.1	27.9%	28.3%	29.8%	30.8%	31.6%	32.2%	32.8%	33.5%	34.2%	34.1%
0.2	28.8%	29.8%	30.6%	32.4%	33.8%	35.2%	36.8%	38.9%	41.6%	-
0.3	29.2%	30.8%	32.4%	33.8%	36.0%	38.2%	40.8%	43.7%	-	-
0.4	29.3%	31.6%	33.8%	36.0%	38.3%	41.5%	44.4%	-	-	-
0.5	29.1%	32.2%	35.2%	38.2%	41.5%	44.5%	-	-	-	-
0.6	28.6%	32.8%	36.8%	40.8%	44.4%	-	-	-	-	-
0.7	27.5%	33.5%	38.9%	43.7%	-	-	-	-	-	-
0.8	24.7%	34.2%	41.6%	-	-	-	-	-	-	-
0.9	14.8%	34.1%	-	-	-	-	-	-	-	-

(c) $\rho_0 = 0.9$

Table A.1: Relative differences between $E[S_0]_{\text{static}}(q^*)$ and $E[S_0]_{\text{Markov}}$ for *exponential* service times B_j, $j = 0, 1, 2$.

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	4.3%	4.1%	1.8%	1.1%	0.4%	0.1%	0.0%	0.0%	0.0%	0.0%
0.1	4.1%	4.7%	3.9%	2.2%	1.0%	0.4%	0.1%	0.0%	0.0%	0.0%
0.2	1.8%	3.9%	4.8%	4.2%	2.6%	1.4%	0.4%	0.1%	0.0%	0.0%
0.3	1.1%	2.2%	4.2%	5.2%	4.5%	2.7%	1.0%	0.4%	0.1%	0.0%
0.4	0.4%	1.0%	2.6%	4.5%	5.6%	4.9%	2.6%	0.9%	0.4%	0.0%
0.5	0.1%	0.4%	1.4%	2.7%	4.9%	6.2%	5.2%	2.8%	0.7%	0.2%
0.6	0.0%	0.1%	0.4%	1.0%	2.6%	5.2%	7.4%	6.3%	2.4%	0.4%
0.7	0.0%	0.0%	0.1%	0.4%	0.9%	2.8%	6.3%	9.4%	7.3%	1.5%
0.8	0.0%	0.0%	0.0%	0.1%	0.4%	0.7%	2.4%	7.3%	13.5%	9.9%
0.9	0.0%	0.0%	0.0%	0.0%	0.0%	0.2%	0.4%	1.5%	9.9%	23.3%

(a) $\rho_0 = 0.1$

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	17.4%	18.7%	18.5%	17.2%	14.7%	10.4%	5.7%	2.4%	0.6%	0.0%
0.1	18.7%	18.3%	19.4%	19.3%	18.1%	15.4%	10.9%	5.4%	1.8%	0.2%
0.2	18.5%	19.4%	19.2%	20.5%	20.5%	19.4%	16.7%	11.5%	4.8%	0.8%
0.3	17.2%	19.3%	20.5%	21.1%	22.3%	22.2%	21.4%	18.8%	12.4%	3.1%
0.4	14.7%	18.1%	20.5%	22.3%	23.4%	24.8%	25.7%	25.4%	23.1%	22.5%
0.5	10.4%	15.4%	19.4%	22.2%	24.8%	26.7%	29.3%	30.7%	32.5%	32.7%
0.6	5.7%	10.9%	16.7%	21.4%	25.7%	29.3%	32.4%	36.1%	39.9%	-
0.7	2.4%	5.4%	11.5%	18.8%	25.4%	30.7%	36.1%	40.8%	-	-
0.8	0.6%	1.8%	4.8%	12.4%	23.1%	32.5%	39.9%	-	-	-
0.9	0.0%	0.2%	0.8%	3.1%	22.5%	32.7%	-	-	-	-

(b) $\rho_0 = 0.5$

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	26.5%	28.1%	28.7%	29.3%	29.4%	29.1%	28.6%	27.5%	24.7%	14.8%
0.1	28.1%	28.5%	29.6%	31.0%	31.7%	32.2%	32.8%	33.5%	34.2%	34.1%
0.2	28.7%	29.6%	30.5%	32.4%	33.9%	35.3%	36.9%	38.9%	41.6%	-
0.3	29.3%	31.0%	32.4%	33.8%	35.9%	38.3%	40.7%	43.6%	-	-
0.4	29.4%	31.7%	33.9%	35.9%	38.1%	41.6%	44.4%	-	-	-
0.5	29.1%	32.2%	35.3%	38.3%	41.6%	44.6%	-	-	-	-
0.6	28.6%	32.8%	36.9%	40.7%	44.4%	-	-	-	-	-
0.7	27.5%	33.5%	38.9%	43.6%	-	-	-	-	-	-
0.8	24.7%	34.2%	41.6%	-	-	-	-	-	-	-
0.9	14.8%	34.1%	-	-	-	-	-	-	-	-

(c) $\rho_0 = 0.9$

Table A.2: Relative differences between $E[S_0]_{\text{static}}(q^*)$ and $E[S_0]_{\text{Markov}}$ for *Erlang-2* service times B_j, $j = 0, 1, 2$.

$\rho_1 \backslash \rho_2$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	4.6%	3.9%	2.0%	0.9%	0.4%	0.1%	0.0%	0.0%	0.0%	0.0%
0.1	3.9%	4.5%	4.1%	2.3%	1.1%	0.6%	0.1%	0.0%	0.0%	0.0%
0.2	2.0%	4.1%	5.0%	4.4%	2.7%	1.1%	0.5%	0.1%	0.0%	0.0%
0.3	0.9%	2.3%	4.4%	5.0%	4.3%	2.5%	1.1%	0.2%	0.1%	0.0%
0.4	0.4%	1.1%	2.7%	4.3%	5.7%	4.7%	2.6%	1.2%	0.3%	0.0%
0.5	0.1%	0.6%	1.1%	2.5%	4.7%	6.2%	5.2%	2.4%	0.8%	0.1%
0.6	0.0%	0.1%	0.5%	1.1%	2.6%	5.2%	7.4%	6.1%	2.3%	0.5%
0.7	0.0%	0.0%	0.1%	0.2%	1.2%	2.4%	6.1%	9.5%	7.5%	1.5%
0.8	0.0%	0.0%	0.0%	0.1%	0.3%	0.8%	2.3%	7.5%	13.2%	9.7%
0.9	0.0%	0.0%	0.0%	0.0%	0.0%	0.1%	0.5%	1.5%	9.7%	23.5%

(a) $\rho_0 = 0.1$

$\rho_1 \backslash \rho_2$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	17.5%	18.7%	18.3%	17.3%	14.7%	10.4%	5.7%	2.4%	0.6%	0.0%
0.1	18.7%	18.2%	19.3%	19.0%	18.1%	15.2%	10.9%	5.4%	1.9%	0.2%
0.2	18.3%	19.3%	19.6%	20.4%	20.3%	19.4%	16.9%	11.4%	4.8%	0.8%
0.3	17.3%	19.0%	20.4%	21.2%	22.3%	22.3%	21.4%	19.0%	12.4%	3.1%
0.4	14.7%	18.1%	20.3%	22.3%	23.2%	24.8%	25.5%	25.4%	23.1%	22.6%
0.5	10.4%	15.2%	19.4%	22.3%	24.8%	27.0%	29.1%	31.0%	32.6%	32.7%
0.6	5.7%	10.9%	16.9%	21.4%	25.5%	29.1%	32.4%	36.2%	39.8%	-
0.7	2.4%	5.4%	11.4%	19.0%	25.4%	31.0%	36.2%	40.8%	-	-
0.8	0.6%	1.9%	4.8%	12.4%	23.1%	32.6%	39.8%	-	-	-
0.9	0.0%	0.2%	0.8%	3.1%	22.6%	32.7%	-	-	-	-

(b) $\rho_0 = 0.5$

$\rho_1 \backslash \rho_2$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	26.7%	28.1%	28.8%	29.3%	29.2%	29.2%	28.6%	27.5%	24.7%	14.8%
0.1	28.1%	28.4%	29.8%	30.9%	31.5%	32.3%	32.8%	33.5%	34.2%	34.1%
0.2	28.8%	29.8%	30.7%	32.4%	34.0%	35.2%	36.7%	38.9%	41.7%	-
0.3	29.3%	30.9%	32.4%	33.9%	36.2%	38.1%	40.9%	43.6%	-	-
0.4	29.2%	31.5%	34.0%	36.2%	38.4%	41.5%	44.3%	-	-	-
0.5	29.2%	32.3%	35.2%	38.1%	41.5%	44.6%	-	-	-	-
0.6	28.6%	32.8%	36.7%	40.9%	44.3%	-	-	-	-	-
0.7	27.5%	33.5%	38.9%	43.6%	-	-	-	-	-	-
0.8	24.7%	34.2%	41.7%	-	-	-	-	-	-	-
0.9	14.8%	34.1%	-	-	-	-	-	-	-	-

(c) $\rho_0 = 0.9$

Table A.3: Relative differences between $E[S_0]_{\text{static}}(q^*)$ and $E[S_0]_{\text{Markov}}$ for *hyperexponential* service times B_j with $c_{B_j}^2 = 2$, $j = 0, 1, 2$.

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	4.5%	4.1%	1.8%	0.9%	0.4%	0.1%	0.0%	0.0%	0.0%	0.0%
0.1	4.1%	4.7%	3.8%	2.4%	1.4%	0.3%	0.1%	0.0%	0.0%	0.0%
0.2	1.8%	3.8%	5.0%	4.2%	2.6%	1.3%	0.4%	0.1%	0.0%	0.0%
0.3	0.9%	2.4%	4.2%	5.0%	4.6%	2.8%	1.2%	0.4%	0.0%	0.0%
0.4	0.4%	1.4%	2.6%	4.6%	5.6%	4.7%	2.5%	0.9%	0.4%	0.0%
0.5	0.1%	0.3%	1.3%	2.8%	4.7%	6.4%	5.1%	2.8%	0.9%	0.2%
0.6	0.0%	0.1%	0.4%	1.2%	2.5%	5.1%	7.4%	6.0%	2.4%	0.4%
0.7	0.0%	0.0%	0.1%	0.4%	0.9%	2.8%	6.0%	9.3%	7.5%	1.5%
0.8	0.0%	0.0%	0.0%	0.0%	0.4%	0.9%	2.4%	7.5%	13.5%	9.8%
0.9	0.0%	0.0%	0.0%	0.0%	0.0%	0.2%	0.4%	1.5%	9.8%	23.5%

(a) $\rho_0 = 0.1$

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	17.5%	18.6%	18.4%	17.3%	14.6%	10.4%	5.7%	2.4%	0.6%	0.0%
0.1	18.6%	18.2%	19.4%	19.0%	17.7%	15.2%	10.9%	5.4%	1.9%	0.2%
0.2	18.4%	19.4%	19.6%	20.3%	20.5%	19.4%	16.6%	11.4%	4.8%	0.8%
0.3	17.3%	19.0%	20.3%	21.2%	22.2%	22.5%	21.6%	19.0%	12.4%	3.2%
0.4	14.6%	17.7%	20.5%	22.2%	23.5%	24.9%	25.5%	25.3%	23.1%	22.6%
0.5	10.4%	15.2%	19.4%	22.5%	24.9%	26.8%	29.1%	30.7%	32.5%	32.7%
0.6	5.7%	10.9%	16.6%	21.6%	25.5%	29.1%	32.4%	36.0%	39.9%	-
0.7	2.4%	5.4%	11.4%	19.0%	25.3%	30.7%	36.0%	41.1%	-	-
0.8	0.6%	1.9%	4.8%	12.4%	23.1%	32.5%	39.9%	-	-	-
0.9	0.0%	0.2%	0.8%	3.2%	22.6%	32.7%	-	-	-	-

(b) $\rho_0 = 0.5$

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	26.7%	27.7%	28.8%	29.1%	29.3%	29.1%	28.6%	27.5%	24.7%	14.7%
0.1	27.7%	28.4%	29.8%	30.7%	31.8%	32.1%	32.8%	33.5%	34.2%	34.1%
0.2	28.8%	29.8%	30.5%	32.5%	33.9%	35.4%	36.9%	38.9%	41.6%	-
0.3	29.1%	30.7%	32.5%	33.7%	35.8%	38.1%	40.6%	43.8%	-	-
0.4	29.3%	31.8%	33.9%	35.8%	38.3%	41.4%	44.4%	-	-	-
0.5	29.1%	32.1%	35.4%	38.1%	41.4%	44.5%	-	-	-	-
0.6	28.6%	32.8%	36.9%	40.6%	44.4%	-	-	-	-	-
0.7	27.5%	33.5%	38.9%	43.8%	-	-	-	-	-	-
0.8	24.7%	34.2%	41.6%	-	-	-	-	-	-	-
0.9	14.7%	34.1%	-	-	-	-	-	-	-	-

(c) $\rho_0 = 0.9$

Table A.4: Relative differences between $E[S_0]_{\text{static}}(q^*)$ and $E[S_0]_{\text{Markov}}$ for *hyperexponential* service times B_j with $c^2_{B_j} = 4$, $j = 0, 1, 2$.

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	4.6%	4.1%	2.0%	1.0%	0.5%	0.1%	0.0%	0.0%	0.0%	0.0%
0.1	4.1%	4.8%	4.2%	2.2%	1.2%	0.3%	0.1%	0.0%	0.0%	0.0%
0.2	2.0%	4.2%	4.7%	4.0%	2.5%	1.4%	0.4%	0.1%	0.0%	0.0%
0.3	1.0%	2.2%	4.0%	5.3%	4.3%	2.5%	1.0%	0.4%	0.1%	0.0%
0.4	0.5%	1.2%	2.5%	4.3%	5.4%	4.9%	2.8%	1.0%	0.4%	0.0%
0.5	0.1%	0.3%	1.4%	2.5%	4.9%	6.2%	5.3%	2.7%	0.6%	0.0%
0.6	0.0%	0.1%	0.4%	1.0%	2.8%	5.3%	7.2%	6.2%	2.3%	0.5%
0.7	0.0%	0.0%	0.1%	0.4%	1.0%	2.7%	6.2%	9.5%	7.6%	1.6%
0.8	0.0%	0.0%	0.0%	0.1%	0.4%	0.6%	2.3%	7.6%	13.2%	9.7%
0.9	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.5%	1.6%	9.7%	23.5%

(a) $\rho_0 = 0.1$

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	17.6%	18.5%	18.3%	17.1%	14.6%	10.4%	5.7%	2.4%	0.6%	0.0%
0.1	18.5%	18.4%	19.1%	19.1%	18.0%	15.5%	10.9%	5.4%	1.8%	0.2%
0.2	18.3%	19.1%	19.4%	20.3%	20.6%	19.5%	16.8%	11.5%	4.8%	0.8%
0.3	17.1%	19.1%	20.3%	21.2%	22.2%	22.3%	21.5%	19.0%	12.4%	3.1%
0.4	14.6%	18.0%	20.6%	22.2%	23.3%	25.0%	25.7%	25.5%	23.2%	22.5%
0.5	10.4%	15.5%	19.5%	22.3%	25.0%	27.0%	29.2%	30.6%	32.3%	32.7%
0.6	5.7%	10.9%	16.8%	21.5%	25.7%	29.2%	32.4%	36.0%	39.6%	-
0.7	2.4%	5.4%	11.5%	19.0%	25.5%	30.6%	36.0%	41.1%	-	-
0.8	0.6%	1.8%	4.8%	12.4%	23.2%	32.3%	39.6%	-	-	-
0.9	0.0%	0.2%	0.8%	3.1%	22.5%	32.7%	-	-	-	-

(b) $\rho_0 = 0.5$

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	26.8%	27.9%	28.6%	29.2%	29.3%	29.1%	28.6%	27.5%	24.7%	14.8%
0.1	27.9%	28.3%	29.6%	30.9%	31.5%	32.3%	32.8%	33.5%	34.2%	34.1%
0.2	28.6%	29.6%	30.8%	32.3%	33.7%	35.2%	36.7%	38.9%	41.6%	-
0.3	29.2%	30.9%	32.3%	33.9%	36.1%	38.0%	40.7%	43.8%	-	-
0.4	29.3%	31.5%	33.7%	36.1%	38.3%	41.5%	44.5%	-	-	-
0.5	29.1%	32.3%	35.2%	38.0%	41.5%	44.6%	-	-	-	-
0.6	28.6%	32.8%	36.7%	40.7%	44.5%	-	-	-	-	-
0.7	27.5%	33.5%	38.9%	43.8%	-	-	-	-	-	-
0.8	24.7%	34.2%	41.6%	-	-	-	-	-	-	-
0.9	14.8%	34.1%	-	-	-	-	-	-	-	-

(c) $\rho_0 = 0.9$

Table A.5: Relative differences between $E[S_0]_{\text{static}}(q^*)$ and $E[S_0]_{\text{Markov}}$ for *hyperexponential* service times B_j with $c^2_{B_j} = 16,\ j = 0, 1, 2$.

A.2 Optimal Splits in Job Split Model

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.0%	0.0%	0.0%	0.1%	0.8%	2.1%	5.5%	9.6%	17.1%	20.0%
0.1	0.0%	0.0%	0.0%	0.2%	0.5%	1.9%	4.2%	10.7%	18.2%	21.9%
0.2	0.0%	0.0%	0.0%	0.2%	0.3%	1.5%	3.3%	7.5%	18.7%	22.9%
0.3	0.1%	0.2%	0.2%	0.0%	0.1%	0.6%	2.4%	5.1%	15.3%	23.5%
0.4	0.8%	0.5%	0.3%	0.1%	0.0%	0.4%	1.6%	4.5%	9.5%	24.1%
0.5	2.1%	1.9%	1.5%	0.6%	0.4%	0.0%	0.4%	2.9%	6.3%	19.6%
0.6	5.5%	4.2%	3.3%	2.4%	1.6%	0.4%	0.0%	0.8%	4.5%	13.4%
0.7	9.6%	10.7%	7.5%	5.1%	4.5%	2.9%	0.8%	0.0%	2.5%	12.6%
0.8	17.1%	18.2%	18.7%	15.3%	9.5%	6.3%	4.5%	2.5%	0.0%	4.6%
0.9	20.0%	21.9%	22.9%	23.5%	24.1%	19.6%	13.4%	12.6%	4.6%	0.0%

(a) Light traffic optimal split α^*_{LT}.

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.0%	2.0%	5.7%	10.5%	9.7%	8.7%	8.1%	6.1%	4.3%	0.5%
0.1	2.0%	0.0%	1.6%	4.5%	7.4%	7.4%	7.2%	6.1%	4.9%	1.3%
0.2	5.7%	1.6%	0.0%	0.8%	3.8%	5.5%	6.5%	6.4%	6.0%	1.6%
0.3	10.5%	4.5%	0.8%	0.0%	1.1%	3.2%	4.9%	6.8%	6.1%	1.8%
0.4	9.7%	7.4%	3.8%	1.1%	0.0%	0.5%	2.1%	3.5%	6.1%	1.8%
0.5	8.7%	7.4%	5.5%	3.2%	0.5%	0.0%	0.9%	2.0%	6.4%	3.7%
0.6	8.1%	7.2%	6.5%	4.9%	2.1%	0.9%	0.0%	0.6%	2.5%	1.8%
0.7	6.1%	6.1%	6.4%	6.8%	3.5%	2.0%	0.6%	0.0%	0.3%	0.9%
0.8	4.3%	4.9%	6.0%	6.1%	6.1%	6.4%	2.5%	0.3%	0.0%	0.2%
0.9	0.5%	1.3%	1.6%	1.8%	1.8%	3.7%	1.8%	0.9%	0.2%	0.0%

(b) Heavy traffic optimal split α^*_{HT}.

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.0%	0.0%	0.0%	0.1%	0.1%	0.2%	0.4%	0.5%	0.4%	0.4%
0.1	0.0%	0.0%	0.0%	0.0%	0.1%	0.1%	0.3%	0.5%	0.8%	0.3%
0.2	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.2%	0.4%	0.9%	0.4%
0.3	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.1%	0.2%	0.5%	0.5%
0.4	0.1%	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.2%	0.4%	0.6%
0.5	0.2%	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.1%	0.4%	0.5%
0.6	0.4%	0.3%	0.2%	0.1%	0.0%	0.0%	0.0%	0.1%	0.1%	0.3%
0.7	0.5%	0.5%	0.4%	0.2%	0.2%	0.1%	0.1%	0.0%	0.0%	0.1%
0.8	0.4%	0.8%	0.9%	0.5%	0.4%	0.4%	0.1%	0.0%	0.0%	0.0%
0.9	0.4%	0.3%	0.4%	0.5%	0.6%	0.5%	0.3%	0.1%	0.0%	0.0%

(c) Composed optimal split α^*_{CP}.

Table A.6: Relative differences between $E[S_0](\alpha^*_{\text{actual}})$ and $E[S_0](\alpha^*_{\text{analytic}})$ for *exponential* service times B_j, $j = 0, 1, 2$, and $\rho_0 = 0.1$.

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.0%	0.0%	0.0%	0.5%	0.8%	2.1%	3.3%	9.3%	16.8%	19.2%
0.1	0.0%	0.0%	0.0%	0.4%	0.5%	1.9%	4.2%	11.0%	17.2%	21.7%
0.2	0.0%	0.0%	0.0%	0.2%	0.3%	1.5%	3.5%	10.4%	18.9%	21.9%
0.3	0.5%	0.4%	0.2%	0.0%	0.1%	0.6%	2.4%	5.2%	13.1%	28.2%
0.4	0.8%	0.5%	0.3%	0.1%	0.0%	0.4%	1.5%	4.2%	9.1%	23.1%
0.5	2.1%	1.9%	1.5%	0.6%	0.4%	0.0%	0.4%	3.0%	6.6%	19.4%
0.6	3.3%	4.2%	3.5%	2.4%	1.5%	0.4%	0.0%	1.1%	4.1%	13.7%
0.7	9.3%	11.0%	10.4%	5.2%	4.2%	3.0%	1.1%	0.0%	2.8%	12.0%
0.8	16.8%	17.2%	18.9%	13.1%	9.1%	6.6%	4.1%	2.8%	0.0%	3.6%
0.9	19.2%	21.7%	21.9%	28.2%	23.1%	19.4%	13.7%	12.0%	3.6%	0.0%

(a) Light traffic optimal split α^*_{LT}.

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.0%	2.0%	5.6%	10.5%	9.7%	8.7%	6.9%	6.0%	0.6%	0.4%
0.1	2.0%	0.0%	1.6%	4.4%	7.4%	7.3%	7.3%	6.3%	3.8%	1.5%
0.2	5.6%	1.6%	0.0%	0.8%	3.8%	5.5%	6.3%	9.2%	4.1%	1.8%
0.3	10.5%	4.4%	0.8%	0.0%	1.0%	3.3%	4.8%	6.7%	5.3%	2.2%
0.4	9.7%	7.4%	3.8%	1.0%	0.0%	0.5%	2.3%	3.3%	6.4%	2.6%
0.5	8.7%	7.3%	5.5%	3.3%	0.5%	0.0%	0.8%	1.7%	6.7%	2.5%
0.6	6.9%	7.3%	6.3%	4.8%	2.3%	0.8%	0.0%	0.9%	2.3%	2.7%
0.7	6.0%	6.3%	9.2%	6.7%	3.3%	1.7%	0.9%	0.0%	0.5%	1.6%
0.8	0.6%	3.8%	4.1%	5.3%	6.4%	6.7%	2.3%	0.5%	0.0%	0.8%
0.9	0.4%	1.5%	1.8%	2.2%	2.6%	2.5%	2.7%	1.6%	0.8%	0.0%

(b) Heavy traffic optimal split α^*_{HT}.

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.0%	0.0%	0.0%	0.1%	0.2%	0.2%	0.4%	0.5%	0.4%	0.3%
0.1	0.0%	0.0%	0.0%	0.1%	0.1%	0.1%	0.3%	0.5%	0.8%	0.3%
0.2	0.0%	0.0%	0.0%	0.0%	0.1%	0.1%	0.2%	0.4%	0.9%	0.4%
0.3	0.1%	0.1%	0.0%	0.0%	0.0%	0.0%	0.2%	0.2%	0.6%	0.5%
0.4	0.2%	0.1%	0.1%	0.0%	0.0%	0.0%	0.1%	0.2%	0.5%	0.6%
0.5	0.2%	0.1%	0.1%	0.0%	0.0%	0.0%	0.0%	0.1%	0.4%	0.5%
0.6	0.4%	0.3%	0.2%	0.2%	0.1%	0.0%	0.0%	0.1%	0.1%	0.4%
0.7	0.5%	0.5%	0.4%	0.2%	0.2%	0.1%	0.1%	0.0%	0.0%	0.1%
0.8	0.4%	0.8%	0.9%	0.6%	0.5%	0.4%	0.1%	0.0%	0.0%	0.0%
0.9	0.3%	0.3%	0.4%	0.5%	0.6%	0.5%	0.4%	0.1%	0.0%	0.0%

(c) Composed optimal split α^*_{CP}.

Table A.7: Relative differences between $E[S_0](\alpha^*_{\text{actual}})$ and $E[S_0](\alpha^*_{\text{analytic}})$ for *Erlang-2* service times B_j, $j = 0, 1, 2$, and $\rho_0 = 0.1$.

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.0%	0.0%	0.0%	0.4%	0.7%	2.1%	5.5%	9.5%	17.6%	19.2%
0.1	0.0%	0.0%	0.0%	0.2%	0.5%	1.0%	4.2%	10.6%	17.6%	19.6%
0.2	0.0%	0.0%	0.0%	0.1%	0.3%	0.9%	3.2%	7.5%	18.9%	21.9%
0.3	0.4%	0.2%	0.1%	0.0%	0.1%	0.6%	2.6%	5.1%	14.8%	23.1%
0.4	0.7%	0.5%	0.3%	0.1%	0.0%	0.3%	1.6%	4.7%	9.1%	32.3%
0.5	2.1%	1.0%	0.9%	0.6%	0.3%	0.0%	0.2%	3.1%	5.8%	28.4%
0.6	5.5%	4.2%	3.2%	2.6%	1.6%	0.2%	0.0%	0.6%	4.5%	20.9%
0.7	9.5%	10.6%	7.5%	5.1%	4.7%	3.1%	0.6%	0.0%	2.0%	13.6%
0.8	17.6%	17.6%	18.9%	14.8%	9.1%	5.8%	4.5%	2.0%	0.0%	4.3%
0.9	19.2%	19.6%	21.9%	23.1%	32.3%	28.4%	20.9%	13.6%	4.3%	0.0%

(a) Light traffic optimal split α^*_{LT}.

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.0%	2.0%	5.7%	10.5%	9.7%	8.6%	8.0%	5.9%	4.7%	0.0%
0.1	2.0%	0.0%	1.6%	9.6%	9.7%	7.4%	7.3%	6.0%	5.6%	1.9%
0.2	5.7%	1.6%	0.0%	0.9%	3.8%	5.5%	6.4%	6.1%	5.9%	5.5%
0.3	10.5%	9.6%	0.9%	0.0%	1.1%	3.2%	4.8%	6.7%	5.9%	7.6%
0.4	9.7%	9.7%	3.8%	1.1%	0.0%	0.5%	2.0%	3.3%	5.8%	8.9%
0.5	8.6%	7.4%	5.5%	3.2%	0.5%	0.0%	0.8%	1.8%	5.6%	8.3%
0.6	8.0%	7.3%	6.4%	4.8%	2.0%	0.8%	0.0%	0.6%	3.1%	7.3%
0.7	5.9%	6.0%	6.1%	6.7%	3.3%	1.8%	0.6%	0.0%	0.5%	1.5%
0.8	4.7%	5.6%	5.9%	5.9%	5.8%	5.6%	3.1%	0.5%	0.0%	1.3%
0.9	0.0%	1.9%	5.5%	7.6%	8.9%	8.3%	7.3%	1.5%	1.3%	0.0%

(b) Heavy traffic optimal split α^*_{HT}.

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.0%	0.0%	0.1%	0.1%	0.2%	0.3%	0.4%	0.5%	0.4%	0.2%
0.1	0.0%	0.0%	0.0%	0.1%	0.1%	0.1%	0.3%	0.5%	0.8%	0.3%
0.2	0.1%	0.0%	0.0%	0.0%	0.1%	0.1%	0.2%	0.4%	0.9%	0.4%
0.3	0.1%	0.1%	0.0%	0.0%	0.0%	0.1%	0.1%	0.3%	0.6%	0.5%
0.4	0.2%	0.1%	0.1%	0.0%	0.0%	0.0%	0.1%	0.2%	0.4%	0.6%
0.5	0.3%	0.1%	0.1%	0.1%	0.0%	0.0%	0.0%	0.1%	0.4%	0.5%
0.6	0.4%	0.3%	0.2%	0.1%	0.1%	0.0%	0.0%	0.1%	0.1%	0.4%
0.7	0.5%	0.5%	0.4%	0.3%	0.2%	0.1%	0.1%	0.0%	0.1%	0.1%
0.8	0.4%	0.8%	0.9%	0.6%	0.4%	0.4%	0.1%	0.1%	0.0%	0.1%
0.9	0.2%	0.3%	0.4%	0.5%	0.6%	0.5%	0.4%	0.1%	0.1%	0.0%

(c) Composed optimal split α^*_{CP}.

Table A.8: Relative differences between $E[S_0](\alpha^*_{\text{actual}})$ and $E[S_0](\alpha^*_{\text{analytic}})$ for *deterministic* service times B_j, $j = 0, 1, 2$, and $\rho_0 = 0.1$.

$\rho_1 \backslash \rho_2$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.0%	0.0%	0.0%	0.1%	0.7%	2.2%	5.7%	9.8%	16.6%	19.1%
0.1	0.0%	0.0%	0.0%	0.2%	0.5%	1.9%	4.2%	11.0%	19.7%	21.8%
0.2	0.0%	0.0%	0.0%	0.2%	0.3%	1.4%	3.3%	7.4%	22.6%	22.1%
0.3	0.1%	0.2%	0.2%	0.0%	0.1%	0.6%	2.5%	5.4%	11.2%	25.1%
0.4	0.7%	0.5%	0.3%	0.1%	0.0%	0.4%	1.6%	4.3%	9.6%	24.5%
0.5	2.2%	1.9%	1.4%	0.6%	0.4%	0.0%	0.2%	3.2%	6.2%	20.2%
0.6	5.7%	4.2%	3.3%	2.5%	1.6%	0.2%	0.0%	0.8%	4.0%	12.7%
0.7	9.8%	11.0%	7.4%	5.4%	4.3%	3.2%	0.8%	0.0%	2.3%	12.1%
0.8	16.6%	19.7%	22.6%	11.2%	9.6%	6.2%	4.0%	2.3%	0.0%	3.8%
0.9	19.1%	21.8%	22.1%	25.1%	24.5%	20.2%	12.7%	12.1%	3.8%	0.0%

(a) Light traffic optimal split α^*_{LT}.

$\rho_1 \backslash \rho_2$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.0%	0.4%	5.7%	10.5%	9.7%	8.7%	8.1%	5.8%	4.7%	0.6%
0.1	0.4%	0.0%	1.6%	4.5%	7.4%	7.4%	7.1%	6.2%	5.3%	4.6%
0.2	5.7%	1.6%	0.0%	0.8%	3.8%	5.5%	6.4%	6.6%	6.0%	7.3%
0.3	10.5%	4.5%	0.8%	0.0%	1.1%	3.3%	4.9%	6.8%	6.3%	3.9%
0.4	9.7%	7.4%	3.8%	1.1%	0.0%	0.6%	2.2%	3.6%	6.6%	1.1%
0.5	8.7%	7.4%	5.5%	3.3%	0.6%	0.0%	1.0%	2.0%	6.2%	1.1%
0.6	8.1%	7.1%	6.4%	4.9%	2.2%	1.0%	0.0%	0.4%	1.9%	1.0%
0.7	5.8%	6.2%	6.6%	6.8%	3.6%	2.0%	0.4%	0.0%	0.3%	0.2%
0.8	4.7%	5.3%	6.0%	6.3%	6.6%	6.2%	1.9%	0.3%	0.0%	1.0%
0.9	0.6%	4.6%	7.3%	3.9%	1.1%	1.1%	1.0%	0.2%	1.0%	0.0%

(b) Heavy traffic optimal split α^*_{HT}.

$\rho_1 \backslash \rho_2$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.0%	0.0%	0.0%	0.1%	0.2%	0.2%	0.4%	0.5%	0.4%	0.2%
0.1	0.0%	0.0%	0.0%	0.1%	0.1%	0.1%	0.3%	0.5%	0.8%	0.3%
0.2	0.0%	0.0%	0.0%	0.0%	0.0%	0.1%	0.2%	0.5%	0.9%	0.4%
0.3	0.1%	0.1%	0.0%	0.0%	0.0%	0.0%	0.2%	0.2%	0.6%	0.5%
0.4	0.2%	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.2%	0.4%	0.6%
0.5	0.2%	0.1%	0.1%	0.0%	0.0%	0.0%	0.0%	0.1%	0.4%	0.5%
0.6	0.4%	0.3%	0.2%	0.2%	0.0%	0.0%	0.0%	0.1%	0.1%	0.3%
0.7	0.5%	0.5%	0.5%	0.2%	0.2%	0.1%	0.1%	0.0%	0.0%	0.1%
0.8	0.4%	0.8%	0.9%	0.6%	0.4%	0.4%	0.1%	0.0%	0.0%	0.1%
0.9	0.2%	0.3%	0.4%	0.5%	0.6%	0.5%	0.3%	0.1%	0.1%	0.0%

(c) Composed optimal split α^*_{CP}.

Table A.9: Relative differences between $E[S_0](\alpha^*_{\text{actual}})$ and $E[S_0](\alpha^*_{\text{analytic}})$ for *hyperexponential* service times B_j with $c^2_{B_j} = 2$, $j = 0, 1, 2$ and $\rho_0 = 0.1$.

$\rho_1 \backslash \rho_2$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.0%	0.0%	0.0%	0.1%	0.7%	2.1%	5.4%	9.4%	17.4%	17.6%
0.1	0.0%	0.0%	0.0%	0.2%	0.6%	1.9%	4.1%	7.9%	18.1%	18.3%
0.2	0.0%	0.0%	0.0%	0.1%	0.3%	1.4%	3.3%	7.3%	18.5%	23.0%
0.3	0.1%	0.2%	0.1%	0.0%	0.1%	0.5%	2.4%	5.2%	17.0%	26.9%
0.4	0.7%	0.6%	0.3%	0.1%	0.0%	0.4%	1.6%	4.5%	9.0%	26.4%
0.5	2.1%	1.9%	1.4%	0.5%	0.4%	0.0%	0.5%	2.8%	6.4%	22.3%
0.6	5.4%	4.1%	3.3%	2.4%	1.6%	0.5%	0.0%	0.6%	4.4%	13.8%
0.7	9.4%	7.9%	7.3%	5.2%	4.5%	2.8%	0.6%	0.0%	3.0%	13.6%
0.8	17.4%	18.1%	18.5%	17.0%	9.0%	6.4%	4.4%	3.0%	0.0%	3.3%
0.9	31.5%	18.3%	23.0%	26.9%	26.4%	22.3%	13.8%	13.6%	3.3%	0.0%

(a) Light traffic optimal split α^*_{LT}.

$\rho_1 \backslash \rho_2$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.0%	2.0%	5.7%	10.5%	9.7%	8.7%	8.1%	4.9%	4.8%	0.2%
0.1	2.0%	0.0%	1.6%	4.4%	7.5%	7.3%	7.2%	5.9%	5.8%	2.5%
0.2	5.7%	1.6%	0.0%	0.8%	3.8%	5.5%	6.7%	6.3%	5.8%	2.8%
0.3	10.5%	4.4%	0.8%	0.0%	1.0%	3.2%	5.0%	6.7%	10.1%	4.8%
0.4	9.7%	7.5%	3.8%	1.0%	0.0%	0.5%	2.2%	3.6%	6.1%	3.8%
0.5	8.7%	7.3%	5.5%	3.2%	0.5%	0.0%	1.0%	1.8%	5.1%	2.5%
0.6	8.1%	7.2%	6.7%	5.0%	2.2%	1.0%	0.0%	0.8%	2.1%	1.3%
0.7	4.9%	5.9%	6.3%	6.7%	3.6%	1.8%	0.8%	0.0%	0.2%	0.9%
0.8	4.8%	5.8%	5.8%	10.1%	6.1%	5.1%	2.1%	0.2%	0.0%	0.6%
0.9	0.2%	2.5%	2.8%	4.8%	3.8%	2.5%	1.3%	0.9%	0.6%	0.0%

(b) Heavy traffic optimal split α^*_{HT}.

$\rho_1 \backslash \rho_2$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.0%	0.0%	0.1%	0.1%	0.2%	0.2%	0.4%	0.4%	0.4%	0.2%
0.1	0.0%	0.0%	0.0%	0.0%	0.1%	0.2%	0.3%	0.5%	0.8%	0.3%
0.2	0.1%	0.0%	0.0%	0.0%	0.1%	0.1%	0.2%	0.5%	0.9%	0.4%
0.3	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.2%	0.2%	0.5%	0.5%
0.4	0.2%	0.1%	0.1%	0.0%	0.0%	0.0%	0.1%	0.2%	0.4%	0.6%
0.5	0.2%	0.2%	0.1%	0.0%	0.0%	0.0%	0.1%	0.1%	0.4%	0.5%
0.6	0.4%	0.3%	0.2%	0.2%	0.1%	0.1%	0.0%	0.1%	0.1%	0.3%
0.7	0.4%	0.5%	0.5%	0.2%	0.2%	0.1%	0.1%	0.0%	0.0%	0.1%
0.8	0.4%	0.8%	0.9%	0.5%	0.4%	0.4%	0.1%	0.0%	0.0%	0.1%
0.9	0.2%	0.3%	0.4%	0.5%	0.6%	0.5%	0.3%	0.1%	0.1%	0.0%

(c) Composed optimal split α^*_{CP}.

Table A.10: Relative differences between $E[S_0](\alpha^*_{\text{actual}})$ and $E[S_0](\alpha^*_{\text{analytic}})$ for *hyperexponential* service times B_j with $c^2_{B_j} = 4$, $j = 0, 1, 2$ and $\rho_0 = 0.1$.

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.0%	0.0%	0.0%	0.1%	0.8%	2.1%	3.3%	9.3%	13.7%	13.7%
0.1	0.0%	0.0%	0.0%	0.1%	0.6%	1.9%	4.3%	10.8%	13.8%	21.9%
0.2	0.0%	0.0%	0.0%	0.1%	0.3%	1.4%	3.5%	7.2%	18.6%	32.2%
0.3	0.1%	0.1%	0.1%	0.0%	0.1%	0.6%	2.3%	4.9%	15.9%	27.3%
0.4	0.8%	0.6%	0.3%	0.1%	0.0%	0.5%	1.6%	4.3%	15.4%	23.6%
0.5	2.1%	1.9%	1.4%	0.6%	0.5%	0.0%	0.2%	2.8%	6.8%	18.6%
0.6	3.3%	4.3%	3.5%	2.3%	1.6%	0.2%	0.0%	1.1%	4.2%	18.3%
0.7	9.3%	10.8%	7.2%	4.9%	4.3%	2.8%	1.1%	0.0%	2.2%	13.8%
0.8	13.7%	13.8%	18.6%	15.9%	15.4%	6.8%	4.2%	2.2%	0.0%	4.0%
0.9	13.7%	21.9%	32.2%	27.3%	23.6%	18.6%	18.3%	13.8%	4.0%	0.0%

(a) Light traffic optimal split α^*_{LT}.

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.0%	2.0%	5.7%	10.5%	9.7%	8.7%	5.8%	4.5%	4.0%	0.0%
0.1	2.0%	0.0%	1.6%	4.5%	7.4%	7.3%	7.1%	5.5%	4.4%	1.1%
0.2	5.7%	1.6%	0.0%	0.8%	3.8%	5.5%	6.5%	9.8%	5.9%	10.5%
0.3	10.5%	4.5%	0.8%	0.0%	1.1%	3.3%	4.8%	7.1%	6.9%	6.1%
0.4	9.7%	7.4%	3.8%	1.1%	0.0%	0.5%	2.2%	3.4%	3.6%	2.6%
0.5	8.7%	7.3%	5.5%	3.3%	0.5%	0.0%	0.8%	1.5%	1.7%	2.6%
0.6	5.8%	7.1%	6.5%	4.8%	2.2%	0.8%	0.0%	0.3%	0.9%	1.9%
0.7	4.5%	5.5%	9.8%	7.1%	3.4%	1.5%	0.3%	0.0%	0.2%	1.2%
0.8	4.0%	4.4%	5.9%	6.9%	3.6%	1.7%	0.9%	0.2%	0.0%	0.7%
0.9	0.0%	1.1%	10.5%	6.1%	2.6%	2.6%	1.9%	1.2%	0.7%	0.0%

(b) Heavy traffic optimal split α^*_{HT}.

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.0%	0.0%	0.0%	0.1%	0.2%	0.2%	0.4%	0.5%	0.4%	0.4%
0.1	0.0%	0.0%	0.0%	0.1%	0.1%	0.1%	0.3%	0.5%	0.8%	0.3%
0.2	0.0%	0.0%	0.0%	0.0%	0.0%	0.1%	0.2%	0.4%	0.9%	0.4%
0.3	0.1%	0.1%	0.0%	0.0%	0.0%	0.0%	0.1%	0.2%	0.6%	0.5%
0.4	0.2%	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.2%	0.4%	0.6%
0.5	0.2%	0.1%	0.1%	0.0%	0.0%	0.0%	0.0%	0.1%	0.4%	0.5%
0.6	0.4%	0.3%	0.2%	0.1%	0.0%	0.0%	0.0%	0.1%	0.1%	0.3%
0.7	0.5%	0.5%	0.4%	0.2%	0.2%	0.1%	0.1%	0.0%	0.0%	0.1%
0.8	0.4%	0.8%	0.9%	0.6%	0.4%	0.4%	0.1%	0.0%	0.0%	0.1%
0.9	0.4%	0.3%	0.4%	0.5%	0.6%	0.5%	0.3%	0.1%	0.1%	0.0%

(c) Composed optimal split α^*_{CP}.

Table A.11: Relative differences between $E[S_0](\alpha^*_{\text{actual}})$ and $E[S_0](\alpha^*_{\text{analytic}})$ for *hyperexponential* service times B_j with $c^2_{B_j} = 16$, $j = 0, 1, 2$ and $\rho_0 = 0.1$.

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.0%	0.0%	0.1%	0.2%	1.3%	1.8%	4.2%	8.7%	12.8%	39.4%
0.1	0.0%	0.0%	0.0%	0.2%	0.5%	1.5%	2.6%	6.3%	11.7%	25.5%
0.2	0.1%	0.0%	0.0%	0.1%	0.2%	1.0%	2.4%	5.8%	10.2%	11.6%
0.3	0.2%	0.2%	0.1%	0.0%	0.1%	0.6%	1.7%	4.4%	7.4%	9.6%
0.4	1.3%	0.5%	0.2%	0.1%	0.0%	0.2%	0.4%	2.5%	5.1%	9.2%
0.5	1.8%	1.5%	1.0%	0.6%	0.2%	0.0%	0.2%	1.7%	3.0%	8.0%
0.6	4.2%	2.6%	2.4%	1.7%	0.4%	0.2%	0.0%	0.1%	0.3%	-
0.7	8.7%	6.3%	5.8%	4.4%	2.5%	1.7%	0.1%	0.0%	-	-
0.8	12.8%	11.7%	10.2%	7.4%	5.1%	3.0%	0.3%	-	-	-
0.9	39.4%	25.5%	11.6%	9.6%	9.2%	8.0%	-	-	-	-

(a) Light traffic optimal split α^*_{LT}.

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.0%	0.6%	1.0%	2.3%	4.1%	3.7%	3.6%	2.2%	1.6%	0.1%
0.1	0.6%	0.0%	0.3%	0.7%	1.7%	2.1%	1.7%	1.7%	1.4%	0.7%
0.2	1.0%	0.3%	0.0%	0.1%	0.9%	0.9%	1.6%	1.5%	1.3%	0.8%
0.3	2.3%	0.7%	0.1%	0.0%	0.1%	0.3%	0.6%	0.5%	1.1%	1.8%
0.4	4.1%	1.7%	0.9%	0.1%	0.0%	0.0%	0.3%	0.4%	0.5%	0.8%
0.5	3.7%	2.1%	0.9%	0.3%	0.0%	0.0%	0.0%	0.1%	0.3%	0.6%
0.6	3.6%	1.7%	1.6%	0.6%	0.3%	0.0%	0.0%	0.1%	0.2%	-
0.7	2.2%	1.7%	1.5%	0.5%	0.4%	0.1%	0.1%	0.0%	-	-
0.8	1.6%	1.4%	1.3%	1.1%	0.5%	0.3%	0.2%	-	-	-
0.9	0.1%	0.7%	0.8%	1.8%	0.8%	0.6%	-	-	-	-

(b) Heavy traffic optimal split α^*_{HT}.

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.0%	0.0%	0.0%	0.1%	0.3%	0.1%	0.0%	0.0%	0.0%	0.0%
0.1	0.0%	0.0%	0.0%	0.0%	0.1%	0.1%	0.1%	0.0%	0.0%	0.0%
0.2	0.0%	0.0%	0.0%	0.0%	0.1%	0.1%	0.1%	0.0%	0.0%	0.0%
0.3	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.1%	0.0%	0.0%
0.4	0.3%	0.1%	0.1%	0.0%	0.0%	0.0%	0.0%	0.1%	0.1%	0.3%
0.5	0.1%	0.1%	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.2%	0.4%
0.6	0.0%	0.1%	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.2%	-
0.7	0.0%	0.0%	0.0%	0.1%	0.1%	0.0%	0.0%	0.0%	-	-
0.8	0.0%	0.0%	0.0%	0.0%	0.1%	0.2%	0.2%	-	-	-
0.9	0.0%	0.0%	0.0%	0.0%	0.3%	0.4%	-	-	-	-

(c) Composed optimal split α^*_{CP}.

Table A.12: Relative differences between $E[S_0](\alpha^*_{\text{actual}})$ and $E[S_0](\alpha^*_{\text{analytic}})$ for *exponential* service times B_j, $j = 0, 1, 2$, and $\rho_0 = 0.5$.

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.0%	0.1%	0.2%	0.3%	1.3%	1.8%	4.2%	8.7%	12.8%	39.4%
0.1	0.1%	0.0%	0.1%	0.3%	0.6%	1.7%	2.6%	6.3%	11.7%	25.5%
0.2	0.2%	0.1%	0.0%	0.0%	0.2%	1.1%	2.5%	5.8%	10.1%	11.6%
0.3	0.3%	0.3%	0.0%	0.0%	0.2%	0.6%	1.5%	4.5%	7.4%	9.6%
0.4	1.3%	0.6%	0.2%	0.2%	0.0%	0.1%	0.5%	2.6%	5.0%	9.2%
0.5	1.8%	1.7%	1.1%	0.6%	0.1%	0.0%	0.1%	1.6%	3.1%	8.1%
0.6	4.2%	2.6%	2.5%	1.5%	0.5%	0.1%	0.0%	0.0%	0.1%	-
0.7	8.7%	6.3%	5.8%	4.5%	2.6%	1.6%	0.0%	0.0%	-	-
0.8	12.8%	11.7%	10.1%	7.4%	5.0%	3.1%	0.1%	-	-	-
0.9	39.4%	25.5%	11.6%	9.6%	9.2%	8.1%	-	-	-	-

(a) Light traffic optimal split α^*_{LT}.

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.0%	0.4%	1.0%	2.4%	3.9%	3.7%	3.6%	2.2%	1.6%	0.2%
0.1	0.4%	0.0%	0.3%	0.9%	1.5%	2.3%	1.9%	1.7%	1.4%	0.7%
0.2	1.0%	0.3%	0.0%	0.3%	0.8%	0.9%	1.7%	1.5%	1.3%	0.8%
0.3	2.4%	0.9%	0.3%	0.0%	0.0%	0.2%	0.4%	0.5%	1.1%	1.8%
0.4	3.9%	1.5%	0.8%	0.0%	0.0%	0.1%	0.2%	0.3%	0.5%	0.8%
0.5	3.7%	2.3%	0.9%	0.2%	0.1%	0.0%	0.0%	0.1%	0.4%	0.5%
0.6	3.6%	1.9%	1.7%	0.4%	0.2%	0.0%	0.0%	0.2%	0.3%	-
0.7	2.2%	1.7%	1.5%	0.5%	0.3%	0.1%	0.2%	0.0%	-	-
0.8	1.6%	1.4%	1.3%	1.1%	0.5%	0.4%	0.3%	-	-	-
0.9	0.2%	0.7%	0.8%	1.8%	0.8%	0.5%	-	-	-	-

(b) Heavy traffic optimal split α^*_{HT}.

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.0%	0.0%	0.0%	0.2%	0.4%	0.1%	0.0%	0.0%	0.0%	0.0%
0.1	0.0%	0.0%	0.0%	0.0%	0.1%	0.1%	0.1%	0.1%	0.0%	0.0%
0.2	0.0%	0.0%	0.0%	0.0%	0.1%	0.1%	0.1%	0.1%	0.0%	0.0%
0.3	0.2%	0.0%	0.0%	0.0%	0.0%	0.0%	0.1%	0.1%	0.0%	0.0%
0.4	0.4%	0.1%	0.1%	0.0%	0.0%	0.0%	0.1%	0.1%	0.1%	0.3%
0.5	0.1%	0.1%	0.1%	0.0%	0.0%	0.0%	0.0%	0.1%	0.2%	0.4%
0.6	0.0%	0.1%	0.1%	0.1%	0.1%	0.0%	0.0%	0.0%	0.2%	-
0.7	0.0%	0.1%	0.1%	0.1%	0.1%	0.1%	0.0%	0.0%	-	-
0.8	0.0%	0.0%	0.0%	0.0%	0.1%	0.2%	0.2%	-	-	-
0.9	0.0%	0.0%	0.0%	0.0%	0.3%	0.4%	-	-	-	-

(c) Composed optimal split α^*_{CP}.

Table A.13: Relative differences between $E[S_0](\alpha^*_{\text{actual}})$ and $E[S_0](\alpha^*_{\text{analytic}})$ for *Erlang-2* service times B_j, $j = 0, 1, 2$, and $\rho_0 = 0.5$.

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.0%	0.1%	0.2%	0.5%	1.3%	1.8%	4.2%	8.7%	12.8%	39.5%
0.1	0.1%	0.0%	0.0%	0.3%	0.5%	1.7%	2.7%	6.3%	11.7%	25.5%
0.2	0.2%	0.0%	0.0%	0.1%	0.2%	1.1%	2.3%	5.8%	10.1%	11.6%
0.3	0.5%	0.3%	0.1%	0.0%	0.0%	0.5%	1.5%	4.3%	7.4%	9.6%
0.4	1.3%	0.5%	0.2%	0.0%	0.0%	0.2%	0.5%	2.3%	5.1%	9.2%
0.5	1.8%	1.7%	1.1%	0.5%	0.2%	0.0%	0.0%	1.6%	2.8%	8.2%
0.6	4.2%	2.7%	2.3%	1.5%	0.5%	0.0%	0.0%	0.0%	0.3%	-
0.7	8.7%	6.3%	5.8%	4.3%	2.3%	1.6%	0.0%	0.0%	-	-
0.8	12.8%	11.7%	10.1%	7.4%	5.1%	2.8%	0.3%	-	-	-
0.9	39.5%	25.5%	11.6%	9.6%	9.2%	8.2%	-	-	-	-

(a) Light traffic optimal split α^*_{LT}.

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.0%	0.5%	1.0%	2.4%	4.1%	3.7%	3.6%	2.2%	1.6%	0.2%
0.1	0.5%	0.0%	0.4%	0.7%	1.5%	2.3%	1.7%	1.7%	1.4%	0.7%
0.2	1.0%	0.4%	0.0%	0.0%	1.1%	1.1%	1.5%	1.5%	1.3%	0.8%
0.3	2.4%	0.7%	0.0%	0.0%	0.0%	0.2%	0.5%	0.8%	1.1%	1.8%
0.4	4.1%	1.5%	1.1%	0.0%	0.0%	0.1%	0.3%	0.5%	0.5%	0.8%
0.5	3.7%	2.3%	1.1%	0.2%	0.1%	0.0%	0.1%	0.2%	0.2%	0.5%
0.6	3.6%	1.7%	1.5%	0.5%	0.3%	0.1%	0.0%	0.0%	0.1%	-
0.7	2.2%	1.7%	1.5%	0.8%	0.5%	0.2%	0.0%	0.0%	-	-
0.8	1.6%	1.4%	1.3%	1.1%	0.5%	0.2%	0.1%	-	-	-
0.9	0.2%	0.7%	0.8%	1.8%	0.8%	0.5%	-	-	-	-

(b) Heavy traffic optimal split α^*_{HT}.

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.0%	0.0%	0.0%	0.2%	0.4%	0.1%	0.1%	0.0%	0.0%	0.0%
0.1	0.0%	0.0%	0.0%	0.0%	0.1%	0.1%	0.1%	0.0%	0.0%	0.0%
0.2	0.0%	0.0%	0.0%	0.0%	0.1%	0.1%	0.1%	0.1%	0.1%	0.0%
0.3	0.2%	0.0%	0.0%	0.0%	0.0%	0.0%	0.1%	0.1%	0.1%	0.0%
0.4	0.4%	0.1%	0.1%	0.0%	0.0%	0.0%	0.0%	0.1%	0.1%	0.3%
0.5	0.1%	0.1%	0.1%	0.0%	0.0%	0.0%	0.0%	0.1%	0.2%	0.4%
0.6	0.1%	0.1%	0.1%	0.1%	0.0%	0.0%	0.0%	0.0%	0.2%	-
0.7	0.0%	0.0%	0.1%	0.1%	0.1%	0.1%	0.0%	0.0%	-	-
0.8	0.0%	0.0%	0.1%	0.1%	0.1%	0.2%	0.2%	-	-	-
0.9	0.0%	0.0%	0.0%	0.0%	0.3%	0.4%	-	-	-	-

(c) Composed optimal split α^*_{CP}.

Table A.14: Relative differences between $E[S_0](\alpha^*_{\text{actual}})$ and $E[S_0](\alpha^*_{\text{analytic}})$ for *deterministic* service times B_j, $j = 0, 1, 2$, and $\rho_0 = 0.5$.

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.0%	0.0%	0.3%	0.3%	1.3%	1.8%	4.2%	8.7%	12.8%	39.4%
0.1	0.0%	0.0%	0.0%	0.2%	0.4%	1.6%	2.7%	6.3%	11.7%	25.4%
0.2	0.3%	0.0%	0.0%	0.0%	0.1%	1.1%	2.6%	5.8%	10.2%	11.6%
0.3	0.3%	0.2%	0.0%	0.0%	0.1%	0.8%	1.8%	4.5%	7.4%	9.6%
0.4	1.3%	0.4%	0.1%	0.1%	0.0%	0.2%	0.4%	2.5%	4.9%	9.2%
0.5	1.8%	1.6%	1.1%	0.8%	0.2%	0.0%	0.2%	1.7%	2.9%	7.9%
0.6	4.2%	2.7%	2.6%	1.8%	0.4%	0.2%	0.0%	0.0%	0.5%	-
0.7	8.7%	6.3%	5.8%	4.5%	2.5%	1.7%	0.0%	0.0%	-	-
0.8	12.8%	11.7%	10.2%	7.4%	4.9%	2.9%	0.5%	-	-	-
0.9	39.4%	25.4%	11.6%	9.6%	9.2%	7.9%	-	-	-	-

(a) Light traffic optimal split α^*_{LT}.

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.0%	0.5%	0.8%	2.1%	4.2%	3.7%	3.6%	2.2%	1.6%	0.1%
0.1	0.5%	0.0%	0.5%	0.6%	1.5%	2.2%	1.7%	1.7%	1.4%	0.7%
0.2	0.8%	0.5%	0.0%	0.3%	0.4%	0.8%	1.4%	1.5%	1.3%	0.9%
0.3	2.1%	0.6%	0.3%	0.0%	0.1%	0.2%	0.7%	0.8%	1.1%	1.8%
0.4	4.2%	1.5%	0.4%	0.1%	0.0%	0.2%	0.4%	0.4%	0.6%	0.8%
0.5	3.7%	2.2%	0.8%	0.2%	0.2%	0.0%	0.0%	0.1%	0.3%	0.4%
0.6	3.6%	1.7%	1.4%	0.7%	0.4%	0.0%	0.0%	0.0%	0.1%	-
0.7	2.2%	1.7%	1.5%	0.8%	0.4%	0.1%	0.0%	0.0%	-	-
0.8	1.6%	1.4%	1.3%	1.1%	0.6%	0.3%	0.1%	-	-	-
0.9	0.1%	0.7%	0.9%	1.8%	0.8%	0.4%	-	-	-	-

(b) Heavy traffic optimal split α^*_{HT}.

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.0%	0.0%	0.0%	0.1%	0.3%	0.1%	0.1%	0.0%	0.0%	0.0%
0.1	0.0%	0.0%	0.0%	0.0%	0.1%	0.1%	0.1%	0.1%	0.1%	0.0%
0.2	0.0%	0.0%	0.0%	0.0%	0.1%	0.1%	0.1%	0.1%	0.1%	0.0%
0.3	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.1%	0.1%	0.1%	0.1%
0.4	0.3%	0.1%	0.1%	0.0%	0.0%	0.0%	0.1%	0.1%	0.1%	0.3%
0.5	0.1%	0.1%	0.1%	0.0%	0.0%	0.0%	0.0%	0.1%	0.2%	0.4%
0.6	0.1%	0.1%	0.1%	0.1%	0.1%	0.0%	0.0%	0.0%	0.2%	-
0.7	0.0%	0.1%	0.1%	0.1%	0.1%	0.1%	0.0%	0.0%	-	-
0.8	0.0%	0.1%	0.1%	0.1%	0.1%	0.2%	0.2%	-	-	-
0.9	0.0%	0.0%	0.0%	0.1%	0.3%	0.4%	-	-	-	-

(c) Composed optimal split α^*_{CP}.

Table A.15: Relative differences between $E[S_0](\alpha^*_{\text{actual}})$ and $E[S_0](\alpha^*_{\text{analytic}})$ for *hyperexponential* service times B_j with $c^2_{B_j} = 2$, $j = 0, 1, 2$, and $\rho_0 = 0.5$.

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.0%	0.1%	0.1%	0.5%	1.3%	1.8%	4.2%	8.7%	12.8%	39.5%
0.1	0.1%	0.0%	0.0%	0.3%	0.6%	1.5%	2.7%	6.3%	11.7%	25.4%
0.2	0.1%	0.0%	0.0%	0.2%	0.4%	0.9%	2.4%	5.8%	10.1%	11.6%
0.3	0.5%	0.3%	0.2%	0.0%	0.1%	0.6%	1.6%	4.2%	7.4%	9.6%
0.4	1.3%	0.6%	0.4%	0.1%	0.0%	0.1%	0.5%	2.5%	5.2%	9.2%
0.5	1.8%	1.5%	0.9%	0.6%	0.1%	0.0%	0.0%	1.6%	2.9%	8.1%
0.6	4.2%	2.7%	2.4%	1.6%	0.5%	0.0%	0.0%	0.0%	0.4%	-
0.7	8.7%	6.3%	5.8%	4.2%	2.5%	1.6%	0.0%	0.0%	-	-
0.8	12.8%	11.7%	10.1%	7.4%	5.2%	2.9%	0.4%	-	-	-
0.9	39.5%	25.4%	11.6%	9.6%	9.2%	8.1%	-	-	-	-

(a) Light traffic optimal split α^*_{LT}.

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.0%	0.5%	0.8%	2.4%	4.1%	3.7%	3.6%	2.2%	1.6%	0.2%
0.1	0.5%	0.0%	0.2%	0.8%	1.8%	2.2%	1.7%	1.7%	1.4%	0.7%
0.2	0.8%	0.2%	0.0%	0.3%	1.0%	1.1%	1.5%	1.5%	1.3%	0.9%
0.3	2.4%	0.8%	0.3%	0.0%	0.2%	0.2%	0.8%	0.8%	1.1%	1.8%
0.4	4.1%	1.8%	1.0%	0.2%	0.0%	0.1%	0.3%	0.5%	0.6%	0.8%
0.5	3.7%	2.2%	1.1%	0.2%	0.1%	0.0%	0.0%	0.3%	0.4%	0.5%
0.6	3.6%	1.7%	1.5%	0.8%	0.3%	0.0%	0.0%	0.2%	0.2%	-
0.7	2.2%	1.7%	1.5%	0.8%	0.5%	0.3%	0.2%	0.0%	-	-
0.8	1.6%	1.4%	1.3%	1.1%	0.6%	0.4%	0.2%	-	-	-
0.9	0.2%	0.7%	0.9%	1.8%	0.8%	0.5%	-	-	-	-

(b) Heavy traffic optimal split α^*_{HT}.

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.0%	0.0%	0.0%	0.1%	0.4%	0.1%	0.1%	0.0%	0.0%	0.0%
0.1	0.0%	0.0%	0.0%	0.0%	0.1%	0.1%	0.1%	0.1%	0.0%	0.0%
0.2	0.0%	0.0%	0.0%	0.0%	0.1%	0.1%	0.1%	0.1%	0.1%	0.0%
0.3	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.1%	0.1%	0.1%	0.0%
0.4	0.4%	0.1%	0.1%	0.0%	0.0%	0.0%	0.0%	0.1%	0.1%	0.3%
0.5	0.1%	0.1%	0.1%	0.0%	0.0%	0.0%	0.0%	0.1%	0.2%	0.4%
0.6	0.1%	0.1%	0.1%	0.1%	0.0%	0.0%	0.0%	0.0%	0.2%	-
0.7	0.0%	0.1%	0.1%	0.1%	0.1%	0.1%	0.0%	0.0%	-	-
0.8	0.0%	0.0%	0.1%	0.1%	0.1%	0.2%	0.2%	-	-	-
0.9	0.0%	0.0%	0.0%	0.0%	0.3%	0.4%	-	-	-	-

(c) Composed optimal split α^*_{CP}.

Table A.16: Relative differences between $E[S_0](\alpha^*_{\text{actual}})$ and $E[S_0](\alpha^*_{\text{analytic}})$ for *hyperexponential* service times B_j with $c^2_{B_j} = 4$, $j = 0, 1, 2$, and $\rho_0 = 0.5$.

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.0%	0.2%	0.3%	0.4%	1.4%	1.8%	4.2%	8.7%	12.8%	39.5%
0.1	0.2%	0.0%	0.0%	0.3%	0.5%	1.6%	2.6%	6.3%	11.7%	25.5%
0.2	0.3%	0.0%	0.0%	0.1%	0.2%	0.8%	2.3%	5.8%	10.2%	11.6%
0.3	0.4%	0.3%	0.1%	0.0%	0.0%	0.6%	1.6%	4.5%	7.4%	9.6%
0.4	1.4%	0.5%	0.2%	0.0%	0.0%	0.4%	0.2%	2.4%	5.2%	9.2%
0.5	1.8%	1.6%	0.8%	0.6%	0.4%	0.0%	0.3%	1.8%	2.8%	8.1%
0.6	4.2%	2.6%	2.3%	1.6%	0.2%	0.3%	0.0%	0.0%	0.2%	-
0.7	8.7%	6.3%	5.8%	4.5%	2.4%	1.8%	0.0%	0.0%	-	-
0.8	12.8%	11.7%	10.2%	7.4%	5.2%	2.8%	0.2%	-	-	-
0.9	39.5%	25.5%	11.6%	9.6%	9.2%	8.1%	-	-	-	-

(a) Light traffic optimal split α^*_{LT}.

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.0%	0.5%	0.8%	2.2%	4.0%	3.7%	3.6%	2.2%	1.6%	0.2%
0.1	0.5%	0.0%	0.3%	0.5%	1.7%	2.2%	1.7%	1.7%	1.4%	0.7%
0.2	0.8%	0.3%	0.0%	0.1%	1.1%	1.4%	1.7%	1.5%	1.3%	0.8%
0.3	2.2%	0.5%	0.1%	0.0%	0.1%	0.3%	0.7%	0.7%	1.1%	1.8%
0.4	4.0%	1.7%	1.1%	0.1%	0.0%	0.1%	0.4%	0.5%	0.6%	0.8%
0.5	3.7%	2.2%	1.4%	0.3%	0.1%	0.0%	0.0%	0.2%	0.4%	0.7%
0.6	3.6%	1.7%	1.7%	0.7%	0.4%	0.0%	0.0%	0.0%	0.3%	-
0.7	2.2%	1.7%	1.5%	0.7%	0.5%	0.2%	0.0%	0.0%	-	-
0.8	1.6%	1.4%	1.3%	1.1%	0.6%	0.4%	0.3%	-	-	-
0.9	0.2%	0.7%	0.8%	1.8%	0.8%	0.7%	-	-	-	-

(b) Heavy traffic optimal split α^*_{HT}.

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.0%	0.0%	0.1%	0.2%	0.4%	0.1%	0.1%	0.0%	0.0%	0.0%
0.1	0.0%	0.0%	0.0%	0.1%	0.1%	0.1%	0.1%	0.1%	0.0%	0.0%
0.2	0.1%	0.0%	0.0%	0.0%	0.1%	0.1%	0.1%	0.1%	0.0%	0.0%
0.3	0.2%	0.1%	0.0%	0.0%	0.0%	0.0%	0.1%	0.1%	0.1%	0.1%
0.4	0.4%	0.1%	0.1%	0.0%	0.0%	0.0%	0.0%	0.1%	0.1%	0.3%
0.5	0.1%	0.1%	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.2%	0.4%
0.6	0.1%	0.1%	0.1%	0.1%	0.0%	0.0%	0.0%	0.0%	0.2%	-
0.7	0.0%	0.1%	0.1%	0.1%	0.1%	0.0%	0.0%	0.0%	-	-
0.8	0.0%	0.0%	0.0%	0.1%	0.1%	0.2%	0.2%	-	-	-
0.9	0.0%	0.0%	0.0%	0.1%	0.3%	0.4%	-	-	-	-

(c) Composed optimal split α^*_{CP}.

Table A.17: Relative differences between $E[S_0](\alpha^*_{\text{actual}})$ and $E[S_0](\alpha^*_{\text{analytic}})$ for *exponential* service times B_j, $j = 0, 1, 2$ with $c^2_{B_j} = 16$, and $\rho_0 = 0.5$.

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.0%	0.0%	0.1%	0.2%	1.4%	2.4%	6.4%	10.8%	9.3%	9.1%
0.1	0.0%	0.0%	0.0%	0.6%	1.5%	2.7%	6.3%	10.5%	7.8%	0.0%
0.2	0.1%	0.0%	0.0%	0.0%	0.5%	0.5%	3.9%	9.5%	6.5%	-
0.3	0.2%	0.6%	0.0%	0.0%	0.2%	0.2%	3.0%	8.6%	-	-
0.4	1.4%	1.5%	0.5%	0.2%	0.0%	0.0%	0.1%	-	-	-
0.5	2.4%	2.7%	0.5%	0.2%	0.0%	0.0%	-	-	-	-
0.6	6.4%	6.3%	3.9%	3.0%	0.1%	-	-	-	-	-
0.7	10.8%	10.5%	9.5%	8.6%	-	-	-	-	-	-
0.8	9.3%	7.8%	6.5%	-	-	-	-	-	-	-
0.9	9.1%	0.0%	-	-	-	-	-	-	-	-

(a) Light traffic optimal split α^*_{LT}.

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.0%	0.0%	0.6%	1.0%	0.5%	0.4%	0.4%	0.3%	0.3%	0.1%
0.1	0.0%	0.0%	0.0%	0.1%	0.6%	0.5%	0.1%	0.1%	0.1%	0.0%
0.2	0.6%	0.0%	0.0%	0.1%	1.0%	0.9%	0.1%	0.0%	0.0%	-
0.3	1.0%	0.1%	0.1%	0.0%	1.2%	0.3%	0.1%	0.0%	-	-
0.4	0.5%	0.6%	1.0%	1.2%	0.0%	0.1%	0.0%	-	-	-
0.5	0.4%	0.5%	0.9%	0.3%	0.1%	0.0%	-	-	-	-
0.6	0.4%	0.1%	0.1%	0.1%	0.0%	-	-	-	-	-
0.7	0.3%	0.1%	0.0%	0.0%	-	-	-	-	-	-
0.8	0.3%	0.1%	0.0%	-	-	-	-	-	-	-
0.9	0.1%	0.0%	-	-	-	-	-	-	-	-

(b) Heavy traffic optimal split α^*_{HT}.

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.0%	0.1%	0.1%	0.2%	0.3%	0.2%	0.1%	0.1%	0.1%	0.1%
0.1	0.1%	0.0%	0.0%	0.1%	0.4%	0.2%	0.2%	0.1%	0.0%	0.0%
0.2	0.1%	0.0%	0.0%	0.0%	0.1%	0.2%	0.0%	0.1%	0.0%	-
0.3	0.2%	0.1%	0.0%	0.0%	0.0%	0.2%	0.1%	0.0%	-	-
0.4	0.3%	0.4%	0.1%	0.0%	0.0%	0.0%	0.0%	-	-	-
0.5	0.2%	0.2%	0.2%	0.2%	0.0%	0.0%	-	-	-	-
0.6	0.1%	0.2%	0.1%	0.1%	0.0%	-	-	-	-	-
0.7	0.1%	0.1%	0.1%	0.0%	-	-	-	-	-	-
0.8	0.1%	0.0%	0.0%	-	-	-	-	-	-	-
0.9	0.1%	0.0%	-	-	-	-	-	-	-	-

(c) Composed optimal split α^*_{CP}.

Table A.18: Relative differences between $E[S_0](\alpha^*_{\text{actual}})$ and $E[S_0](\alpha^*_{\text{analytic}})$ for *exponential* service times B_j, $j = 0, 1, 2$, and $\rho_0 = 0.9$.

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.0%	0.0%	0.2%	0.3%	1.5%	2.4%	6.4%	10.8%	9.3%	9.2%
0.1	0.0%	0.0%	0.1%	0.7%	1.7%	2.8%	6.3%	10.5%	7.8%	0.0%
0.2	0.2%	0.1%	0.0%	0.0%	0.3%	0.4%	3.9%	9.5%	6.5%	-
0.3	0.3%	0.7%	0.0%	0.0%	0.3%	0.3%	3.0%	8.7%	-	-
0.4	1.5%	1.7%	0.3%	0.3%	0.0%	0.0%	0.2%	-	-	-
0.5	2.4%	2.8%	0.4%	0.3%	0.0%	0.0%	-	-	-	-
0.6	6.4%	6.3%	3.9%	3.0%	0.2%	-	-	-	-	-
0.7	10.8%	10.5%	9.5%	8.7%	-	-	-	-	-	-
0.8	9.3%	7.8%	6.5%	-	-	-	-	-	-	-
0.9	9.2%	0.0%	-	-	-	-	-	-	-	-

(a) Light traffic optimal split α^*_{LT}.

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.0%	0.0%	0.6%	1.0%	0.5%	0.5%	0.4%	0.3%	0.3%	0.1%
0.1	0.0%	0.0%	0.0%	0.2%	0.6%	0.6%	0.1%	0.1%	0.1%	0.1%
0.2	0.6%	0.0%	0.0%	0.1%	1.0%	0.9%	0.1%	0.1%	0.1%	-
0.3	1.0%	0.2%	0.1%	0.0%	1.2%	0.3%	0.1%	0.1%	-	-
0.4	0.5%	0.6%	1.0%	1.2%	0.0%	0.1%	0.1%	-	-	-
0.5	0.5%	0.6%	0.9%	0.3%	0.1%	0.0%	-	-	-	-
0.6	0.4%	0.1%	0.1%	0.1%	0.1%	-	-	-	-	-
0.7	0.3%	0.1%	0.1%	0.1%	-	-	-	-	-	-
0.8	0.3%	0.1%	0.1%	-	-	-	-	-	-	-
0.9	0.1%	0.1%	-	-	-	-	-	-	-	-

(b) Heavy traffic optimal split α^*_{HT}.

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.0%	0.1%	0.1%	0.2%	0.3%	0.2%	0.2%	0.1%	0.1%	0.1%
0.1	0.1%	0.0%	0.0%	0.1%	0.5%	0.2%	0.2%	0.1%	0.1%	0.1%
0.2	0.1%	0.0%	0.0%	0.0%	0.2%	0.2%	0.1%	0.1%	0.1%	-
0.3	0.2%	0.1%	0.0%	0.0%	0.1%	0.2%	0.1%	0.0%	-	-
0.4	0.3%	0.5%	0.2%	0.1%	0.0%	0.0%	0.0%	-	-	-
0.5	0.2%	0.2%	0.2%	0.2%	0.0%	0.0%	-	-	-	-
0.6	0.2%	0.2%	0.1%	0.1%	0.0%	-	-	-	-	-
0.7	0.1%	0.1%	0.1%	0.0%	-	-	-	-	-	-
0.8	0.1%	0.1%	0.1%	-	-	-	-	-	-	-
0.9	0.1%	0.1%	-	-	-	-	-	-	-	-

(c) Composed optimal split α^*_{CP}.

Table A.19: Relative differences between $E[S_0](\alpha^*_{\text{actual}})$ and $E[S_0](\alpha^*_{\text{analytic}})$ for *Erlang-2* service times B_j, $j = 0, 1, 2$, and $\rho_0 = 0.9$.

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.0%	0.1%	0.2%	0.3%	1.3%	2.4%	6.4%	10.8%	9.3%	9.2%
0.1	0.1%	0.0%	0.1%	0.6%	1.5%	2.9%	6.3%	10.5%	7.7%	0.1%
0.2	0.2%	0.1%	0.0%	0.0%	0.3%	0.5%	3.6%	9.5%	6.5%	-
0.3	0.3%	0.6%	0.0%	0.0%	0.3%	0.5%	3.2%	8.5%	-	-
0.4	1.3%	1.5%	0.3%	0.3%	0.0%	0.1%	0.2%	-	-	-
0.5	2.4%	2.9%	0.5%	0.5%	0.1%	0.0%	-	-	-	-
0.6	6.4%	6.3%	3.6%	3.2%	0.2%	-	-	-	-	-
0.7	10.8%	10.5%	9.5%	8.5%	-	-	-	-	-	-
0.8	9.3%	7.7%	6.5%	-	-	-	-	-	-	-
0.9	9.2%	0.1%	-	-	-	-	-	-	-	-

(a) Light traffic optimal split α^*_{LT}.

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.0%	0.0%	0.6%	1.1%	0.5%	0.4%	0.4%	0.3%	0.3%	0.1%
0.1	0.0%	0.0%	0.0%	0.1%	0.6%	0.6%	0.2%	0.2%	0.1%	0.1%
0.2	0.6%	0.0%	0.0%	0.1%	1.0%	0.9%	0.1%	0.1%	0.0%	-
0.3	1.1%	0.1%	0.1%	0.0%	1.2%	0.3%	0.1%	0.1%	-	-
0.4	0.5%	0.6%	1.0%	1.2%	0.0%	0.1%	0.0%	-	-	-
0.5	0.4%	0.6%	0.9%	0.3%	0.1%	0.0%	-	-	-	-
0.6	0.4%	0.2%	0.1%	0.1%	0.0%	-	-	-	-	-
0.7	0.3%	0.2%	0.1%	0.1%	-	-	-	-	-	-
0.8	0.3%	0.1%	0.0%	-	-	-	-	-	-	-
0.9	0.1%	0.1%	-	-	-	-	-	-	-	-

(b) Heavy traffic optimal split α^*_{HT}.

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.0%	0.1%	0.1%	0.2%	0.3%	0.2%	0.2%	0.1%	0.1%	0.1%
0.1	0.1%	0.0%	0.1%	0.1%	0.5%	0.2%	0.2%	0.1%	0.1%	0.0%
0.2	0.1%	0.1%	0.0%	0.0%	0.2%	0.2%	0.1%	0.1%	0.1%	-
0.3	0.2%	0.1%	0.0%	0.0%	0.0%	0.2%	0.1%	0.0%	-	-
0.4	0.3%	0.5%	0.2%	0.0%	0.0%	0.0%	0.0%	-	-	-
0.5	0.2%	0.2%	0.2%	0.2%	0.0%	0.0%	-	-	-	-
0.6	0.2%	0.2%	0.1%	0.1%	0.0%	-	-	-	-	-
0.7	0.1%	0.1%	0.1%	0.0%	-	-	-	-	-	-
0.8	0.1%	0.1%	0.1%	-	-	-	-	-	-	-
0.9	0.1%	0.0%	-	-	-	-	-	-	-	-

(c) Composed optimal split α^*_{CP}.

Table A.20: Relative differences between $E[S_0](\alpha^*_{\text{actual}})$ and $E[S_0](\alpha^*_{\text{analytic}})$ for *deterministic* service times B_j, $j = 0, 1, 2$, and $\rho_0 = 0.9$.

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.0%	0.1%	0.2%	0.3%	1.5%	2.4%	6.4%	10.8%	9.3%	9.1%
0.1	0.1%	0.0%	0.0%	0.4%	1.6%	2.6%	6.3%	10.5%	7.8%	0.0%
0.2	0.2%	0.0%	0.0%	0.0%	0.5%	0.5%	4.1%	9.5%	6.5%	-
0.3	0.3%	0.4%	0.0%	0.0%	0.0%	0.4%	3.2%	8.6%	-	-
0.4	1.5%	1.6%	0.5%	0.0%	0.0%	0.2%	0.3%	-	-	-
0.5	2.4%	2.6%	0.5%	0.4%	0.2%	0.0%	-	-	-	-
0.6	6.4%	6.3%	4.1%	3.2%	0.3%	-	-	-	-	-
0.7	10.8%	10.5%	9.5%	8.6%	-	-	-	-	-	-
0.8	9.3%	7.8%	6.5%	-	-	-	-	-	-	-
0.9	9.1%	0.0%	-	-	-	-	-	-	-	-

(a) Light traffic optimal split α^*_{LT}.

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.0%	0.1%	0.6%	1.0%	0.5%	0.4%	0.4%	0.3%	0.3%	0.1%
0.1	0.1%	0.0%	0.0%	0.1%	0.6%	0.6%	0.2%	0.1%	0.1%	0.1%
0.2	0.6%	0.0%	0.0%	0.1%	1.0%	1.0%	0.1%	0.1%	0.1%	-
0.3	1.0%	0.1%	0.1%	0.0%	1.2%	0.3%	0.1%	0.0%	-	-
0.4	0.5%	0.6%	1.0%	1.2%	0.0%	0.1%	0.0%	-	-	-
0.5	0.4%	0.6%	1.0%	0.3%	0.1%	0.0%	-	-	-	-
0.6	0.4%	0.2%	0.1%	0.1%	0.0%	-	-	-	-	-
0.7	0.3%	0.1%	0.1%	0.0%	-	-	-	-	-	-
0.8	0.3%	0.1%	0.1%	-	-	-	-	-	-	-
0.9	0.1%	0.1%	-	-	-	-	-	-	-	-

(b) Heavy traffic optimal split α^*_{HT}.

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.0%	0.1%	0.6%	1.0%	0.5%	0.4%	0.4%	0.3%	0.3%	0.1%
0.1	0.1%	0.0%	0.0%	0.1%	0.6%	0.6%	0.2%	0.1%	0.1%	0.1%
0.2	0.6%	0.0%	0.0%	0.1%	1.0%	1.0%	0.1%	0.1%	0.1%	-
0.3	1.0%	0.1%	0.1%	0.0%	1.2%	0.3%	0.1%	0.0%	-	-
0.4	0.5%	0.6%	1.0%	1.2%	0.0%	0.1%	0.0%	-	-	-
0.5	0.4%	0.6%	1.0%	0.3%	0.1%	0.0%	-	-	-	-
0.6	0.4%	0.2%	0.1%	0.1%	0.0%	-	-	-	-	-
0.7	0.3%	0.1%	0.1%	0.0%	-	-	-	-	-	-
0.8	0.3%	0.1%	0.1%	-	-	-	-	-	-	-
0.9	0.1%	0.1%	-	-	-	-	-	-	-	-

(c) Composed optimal split α^*_{CP}.

Table A.21: Relative differences between $E[S_0](\alpha^*_{\text{actual}})$ and $E[S_0](\alpha^*_{\text{analytic}})$ for *hyperexponential* service times B_j with $c^2_{B_j} = 2$, $j = 0, 1, 2$, and $\rho_0 = 0.9$.

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.0%	0.0%	0.1%	0.2%	1.6%	2.4%	6.4%	10.8%	9.3%	9.1%
0.1	0.0%	0.0%	0.0%	0.5%	1.7%	2.7%	6.3%	10.5%	7.8%	0.0%
0.2	0.1%	0.0%	0.0%	0.1%	0.7%	0.4%	4.1%	9.5%	6.5%	-
0.3	0.2%	0.5%	0.1%	0.0%	0.1%	0.2%	2.9%	8.7%	-	-
0.4	1.6%	1.7%	0.7%	0.1%	0.0%	0.1%	0.1%	-	-	-
0.5	2.4%	2.7%	0.4%	0.2%	0.1%	0.0%	-	-	-	-
0.6	6.4%	6.3%	4.1%	2.9%	0.1%	-	-	-	-	-
0.7	10.8%	10.5%	9.5%	8.7%	-	-	-	-	-	-
0.8	9.3%	7.8%	6.5%	-	-	-	-	-	-	-
0.9	9.1%	0.0%	-	-	-	-	-	-	-	-

(a) Light traffic optimal split α^*_{LT}.

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.0%	0.1%	0.6%	1.1%	0.5%	0.4%	0.4%	0.3%	0.3%	0.1%
0.1	0.1%	0.0%	0.0%	0.1%	0.6%	0.5%	0.2%	0.1%	0.1%	0.1%
0.2	0.6%	0.0%	0.0%	0.1%	1.0%	1.0%	0.1%	0.1%	0.1%	-
0.3	1.1%	0.1%	0.1%	0.0%	1.2%	0.3%	0.1%	0.1%	-	-
0.4	0.5%	0.6%	1.0%	1.2%	0.0%	0.1%	0.1%	-	-	-
0.5	0.4%	0.5%	1.0%	0.3%	0.1%	0.0%	-	-	-	-
0.6	0.4%	0.2%	0.1%	0.1%	0.1%	-	-	-	-	-
0.7	0.3%	0.1%	0.1%	0.1%	-	-	-	-	-	-
0.8	0.3%	0.1%	0.1%	-	-	-	-	-	-	-
0.9	0.1%	0.1%	-	-	-	-	-	-	-	-

(b) Heavy traffic optimal split α^*_{HT}.

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.0%	0.1%	0.1%	0.2%	0.3%	0.2%	0.1%	0.1%	0.1%	0.1%
0.1	0.1%	0.0%	0.1%	0.1%	0.4%	0.2%	0.2%	0.1%	0.1%	0.0%
0.2	0.1%	0.1%	0.0%	0.0%	0.2%	0.2%	0.2%	0.1%	0.1%	-
0.3	0.2%	0.1%	0.0%	0.0%	0.1%	0.2%	0.1%	0.1%	-	-
0.4	0.3%	0.4%	0.2%	0.1%	0.0%	0.0%	0.1%	-	-	-
0.5	0.2%	0.2%	0.2%	0.2%	0.0%	0.0%	-	-	-	-
0.6	0.1%	0.2%	0.2%	0.1%	0.1%	-	-	-	-	-
0.7	0.1%	0.1%	0.1%	0.1%	-	-	-	-	-	-
0.8	0.1%	0.1%	0.1%	-	-	-	-	-	-	-
0.9	0.1%	0.0%	-	-	-	-	-	-	-	-

(c) Composed optimal split α^*_{CP}.

Table A.22: Relative differences between $E[S_0](\alpha^*_{\text{actual}})$ and $E[S_0](\alpha^*_{\text{analytic}})$ for *hyperexponential* service times B_j, $j = 0,1,2$ with $c^2_{B_j} = 4$, and $\rho_0 = 0.9$.

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.0%	0.0%	0.1%	0.1%	1.3%	2.4%	6.4%	10.8%	9.3%	9.1%
0.1	0.0%	0.0%	0.0%	0.8%	1.7%	2.6%	6.3%	10.5%	7.8%	0.0%
0.2	0.1%	0.0%	0.0%	0.0%	0.4%	0.4%	4.1%	9.5%	6.5%	-
0.3	0.1%	0.8%	0.0%	0.0%	0.3%	0.3%	3.1%	8.6%	-	-
0.4	1.3%	1.7%	0.4%	0.3%	0.0%	0.0%	0.1%	-	-	-
0.5	2.4%	2.6%	0.4%	0.3%	0.0%	0.0%	-	-	-	-
0.6	6.4%	6.3%	4.1%	3.1%	0.1%	-	-	-	-	-
0.7	10.8%	10.5%	9.5%	8.6%	-	-	-	-	-	-
0.8	9.3%	7.8%	6.5%	-	-	-	-	-	-	-
0.9	9.1%	0.0%	-	-	-	-	-	-	-	-

(a) Light traffic optimal split α^*_{LT}.

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.0%	0.1%	0.6%	1.1%	0.5%	0.5%	0.4%	0.3%	0.3%	0.1%
0.1	0.1%	0.0%	0.0%	0.2%	0.6%	0.5%	0.2%	0.1%	0.1%	0.1%
0.2	0.6%	0.0%	0.0%	0.1%	1.0%	1.0%	0.1%	0.1%	0.0%	-
0.3	1.1%	0.2%	0.1%	0.0%	1.2%	0.3%	0.1%	0.1%	-	-
0.4	0.5%	0.6%	1.0%	1.2%	0.0%	0.1%	0.1%	-	-	-
0.5	0.5%	0.5%	1.0%	0.3%	0.1%	0.0%	-	-	-	-
0.6	0.4%	0.2%	0.1%	0.1%	0.1%	-	-	-	-	-
0.7	0.3%	0.1%	0.1%	0.1%	-	-	-	-	-	-
0.8	0.3%	0.1%	0.0%	-	-	-	-	-	-	-
0.9	0.1%	0.1%	-	-	-	-	-	-	-	-

(b) Heavy traffic optimal split α^*_{HT}.

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.0%	0.1%	0.1%	0.2%	0.3%	0.2%	0.2%	0.1%	0.1%	0.1%
0.1	0.1%	0.0%	0.1%	0.1%	0.5%	0.2%	0.2%	0.1%	0.1%	0.1%
0.2	0.1%	0.1%	0.0%	0.1%	0.2%	0.2%	0.2%	0.1%	0.1%	-
0.3	0.2%	0.1%	0.1%	0.0%	0.0%	0.2%	0.1%	0.0%	-	-
0.4	0.3%	0.5%	0.2%	0.0%	0.0%	0.0%	0.0%	-	-	-
0.5	0.2%	0.2%	0.2%	0.2%	0.0%	0.0%	-	-	-	-
0.6	0.2%	0.2%	0.2%	0.1%	0.0%	-	-	-	-	-
0.7	0.1%	0.1%	0.1%	0.0%	-	-	-	-	-	-
0.8	0.1%	0.1%	0.1%	-	-	-	-	-	-	-
0.9	0.1%	0.1%	-	-	-	-	-	-	-	-

(c) Composed optimal split α^*_{CP}.

Table A.23: Relative differences between $E[S_0](\alpha^*_{\text{actual}})$ and $E[S_0](\alpha^*_{\text{analytic}})$ for *hyperexponential* service times B_j, $j = 0, 1, 2$ with $c^2_{B_j} = 16$, and $\rho_0 = 0.9$.

A.3 Static Models

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	49.9%	43.6%	39.4%	34.7%	27.4%	20.4%	15.3%	10.2%	5.7%	2.5%
0.1	42.6%	42.7%	39.9%	35.9%	29.8%	24.7%	17.1%	11.8%	8.3%	3.2%
0.2	39.4%	39.9%	40.1%	37.4%	32.7%	26.4%	19.0%	13.3%	8.4%	3.4%
0.3	34.7%	35.9%	37.4%	38.0%	35.3%	29.4%	22.9%	15.9%	10.0%	4.6%
0.4	27.4%	29.8%	32.7%	35.3%	35.3%	32.8%	26.7%	18.6%	10.0%	5.2%
0.5	20.4%	24.7%	26.4%	29.4%	32.8%	33.7%	31.1%	23.8%	14.2%	5.7%
0.6	15.3%	17.1%	19.0%	22.9%	26.7%	31.1%	31.5%	28.5%	19.5%	7.2%
0.7	10.2%	11.8%	13.3%	15.9%	18.6%	23.8%	28.5%	30.7%	26.6%	12.9%
0.8	5.7%	8.3%	8.4%	10.0%	10.0%	14.2%	19.5%	26.6%	28.7%	21.9%
0.9	2.5%	3.2%	3.4%	4.6%	5.2%	5.7%	7.2%	12.9%	21.9%	31.5%

(a) $\rho_0 = 0.1$

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	50.1%	46.3%	43.2%	39.5%	38.0%	33.5%	29.6%	23.2%	18.2%	9.3%
0.1	46.3%	44.4%	42.1%	38.3%	36.6%	33.7%	30.2%	26.2%	18.8%	9.7%
0.2	43.2%	42.1%	41.0%	38.0%	36.4%	34.0%	31.0%	27.8%	24.6%	13.2%
0.3	39.5%	38.3%	38.0%	37.1%	35.3%	34.7%	31.5%	28.1%	26.4%	18.5%
0.4	38.0%	36.6%	36.4%	35.3%	34.2%	33.4%	31.7%	31.0%	28.3%	22.8%
0.5	33.5%	33.7%	34.0%	34.7%	33.4%	33.3%	33.3%	32.9%	28.7%	28.3%
0.6	29.6%	30.2%	31.0%	31.5%	31.7%	33.3%	33.7%	34.6%	35.0%	-
0.7	23.2%	26.2%	27.8%	28.1%	31.0%	32.9%	34.6%	40.0%	-	-
0.8	18.2%	18.8%	24.6%	26.4%	28.3%	28.7%	35.0%	-	-	-
0.9	9.3%	9.7%	13.2%	18.5%	22.8%	28.3%	-	-	-	-

(b) $\rho_0 = 0.5$

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	50.2%	44.6%	42.3%	40.0%	37.6%	33.0%	32.1%	30.8%	27.8%	22.8%
0.1	44.6%	43.7%	40.7%	37.1%	36.1%	33.1%	32.6%	29.2%	30.5%	28.3%
0.2	42.3%	40.7%	38.6%	37.1%	35.9%	34.0%	33.1%	28.9%	36.4%	-
0.3	40.0%	37.1%	37.1%	35.8%	34.2%	34.0%	32.4%	26.1%	-	-
0.4	37.6%	36.1%	35.9%	34.2%	33.1%	32.6%	31.5%	-	-	-
0.5	33.0%	33.1%	34.0%	34.0%	32.6%	28.8%	-	-	-	-
0.6	32.1%	32.6%	33.1%	32.4%	31.5%	-	-	-	-	-
0.7	30.8%	29.2%	28.9%	26.1%	-	-	-	-	-	-
0.8	27.8%	29.1%	36.4%	-	-	-	-	-	-	-
0.9	22.8%	28.3%	-	-	-	-	-	-	-	-

(c) $\rho_0 = 0.9$

Table A.24: Relative differences between $E[S_0]_{\text{select}}(q^*)$ and $E[S_0]_{\text{split}}(\alpha^*)$ for *exponential* service times B_j, $j = 0, 1, 2$.

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	50.0%	45.1%	40.5%	34.4%	28.3%	23.3%	17.2%	11.4%	7.5%	2.7%
0.1	45.1%	45.2%	41.5%	36.0%	30.0%	24.5%	19.1%	12.7%	7.6%	3.5%
0.2	40.5%	41.5%	41.6%	38.8%	33.2%	27.4%	20.4%	14.4%	8.4%	3.8%
0.3	34.4%	36.0%	38.8%	39.9%	36.0%	30.0%	24.1%	16.5%	10.2%	4.6%
0.4	28.3%	30.0%	33.2%	36.0%	36.1%	33.9%	28.1%	19.6%	13.0%	5.0%
0.5	23.3%	24.5%	27.4%	30.0%	33.9%	34.7%	32.1%	24.8%	17.0%	6.5%
0.6	17.2%	19.1%	20.4%	24.1%	28.1%	32.1%	33.8%	29.3%	21.7%	10.1%
0.7	11.4%	12.7%	14.4%	16.5%	19.6%	24.8%	29.3%	32.8%	27.8%	15.4%
0.8	7.5%	7.6%	8.4%	10.2%	13.0%	17.0%	21.7%	27.8%	30.3%	26.9%
0.9	2.7%	3.5%	3.8%	4.6%	5.0%	6.5%	10.1%	15.4%	26.9%	34.2%

(a) $\rho_0 = 0.1$

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	49.8%	46.2%	41.6%	38.3%	35.2%	32.0%	28.5%	22.3%	14.5%	5.5%
0.1	46.2%	44.2%	41.8%	38.4%	35.3%	33.0%	29.7%	25.6%	20.0%	10.8%
0.2	41.6%	41.8%	40.3%	38.4%	36.0%	33.6%	30.8%	27.6%	21.9%	13.8%
0.3	38.3%	38.4%	38.4%	36.9%	35.4%	33.1%	31.4%	29.2%	23.6%	17.2%
0.4	35.2%	35.3%	36.0%	35.4%	34.2%	33.0%	31.9%	30.3%	26.6%	19.9%
0.5	32.0%	33.0%	33.6%	33.1%	33.0%	32.5%	32.0%	31.6%	26.7%	20.4%
0.6	28.5%	29.7%	30.8%	31.4%	31.9%	32.0%	33.4%	33.7%	36.1%	-
0.7	22.3%	25.6%	27.6%	29.2%	30.3%	31.6%	33.7%	34.1%	-	-
0.8	14.5%	20.0%	21.9%	23.6%	26.6%	26.7%	36.1%	-	-	-
0.9	5.5%	10.8%	13.8%	17.2%	19.9%	20.4%	-	-	-	-

(b) $\rho_0 = 0.5$

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	49.6%	44.7%	41.1%	38.3%	35.8%	33.4%	32.2%	25.8%	24.2%	23.1%
0.1	44.7%	44.2%	40.0%	38.9%	35.6%	33.3%	31.7%	26.2%	25.4%	25.2%
0.2	41.1%	40.0%	38.2%	36.7%	35.0%	32.7%	31.0%	28.1%	26.6%	-
0.3	38.3%	38.9%	36.7%	36.4%	35.0%	32.7%	29.7%	29.1%	-	-
0.4	35.8%	35.6%	35.0%	35.0%	32.1%	28.3%	27.7%	-	-	-
0.5	33.4%	33.3%	32.7%	32.7%	28.3%	28.1%	-	-	-	-
0.6	32.2%	31.7%	31.0%	29.7%	27.7%	-	-	-	-	-
0.7	25.8%	26.2%	28.1%	29.1%	-	-	-	-	-	-
0.8	24.2%	25.4%	26.6%	-	-	-	-	-	-	-
0.9	23.1%	25.2%	-	-	-	-	-	-	-	-

(c) $\rho_0 = 0.9$

Table A.25: Relative differences between $E[S_0]_{\text{select}}(q^*)$ and $E[S_0]_{\text{split}}(\alpha^*)$ for *Erlang-2* service times B_j, $j = 0, 1, 2$.

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	50.1%	45.3%	40.0%	33.0%	27.4%	21.5%	15.7%	11.0%	6.3%	2.7%
0.1	45.3%	45.4%	40.9%	35.7%	30.1%	23.7%	17.3%	12.2%	7.4%	3.1%
0.2	40.0%	40.9%	41.3%	38.1%	32.3%	26.7%	20.0%	13.5%	7.5%	3.7%
0.3	33.0%	35.7%	38.1%	38.3%	35.9%	30.1%	23.4%	16.8%	10.1%	4.3%
0.4	27.4%	30.1%	32.3%	35.9%	36.0%	33.3%	26.5%	18.6%	11.3%	4.7%
0.5	21.5%	23.7%	26.7%	30.1%	33.3%	34.6%	31.3%	24.1%	14.0%	7.5%
0.6	15.7%	17.3%	20.0%	23.4%	26.5%	31.3%	31.6%	28.9%	21.0%	8.7%
0.7	11.0%	12.2%	13.5%	16.8%	18.6%	24.1%	28.9%	31.0%	28.0%	15.7%
0.8	6.3%	7.4%	7.5%	10.1%	11.3%	14.0%	21.0%	28.0%	29.8%	25.9%
0.9	2.7%	3.1%	3.7%	4.3%	4.7%	7.5%	8.7%	15.7%	25.9%	34.8%

(a) $\rho_0 = 0.1$

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	49.9%	46.5%	40.9%	38.0%	34.8%	31.4%	27.3%	22.3%	14.7%	5.8%
0.1	46.5%	44.3%	41.3%	37.9%	35.3%	32.1%	29.0%	25.0%	19.4%	8.6%
0.2	40.9%	41.3%	40.0%	37.6%	35.1%	32.7%	30.5%	27.3%	23.1%	13.3%
0.3	38.0%	37.9%	37.6%	37.2%	34.6%	32.9%	31.2%	28.4%	23.9%	19.3%
0.4	34.8%	35.3%	35.1%	34.6%	34.3%	32.2%	31.5%	29.5%	24.9%	22.8%
0.5	31.4%	32.1%	32.7%	32.9%	32.2%	31.6%	31.5%	29.8%	29.3%	28.3%
0.6	27.3%	29.0%	30.5%	31.2%	31.5%	31.5%	31.4%	31.4%	29.7%	-
0.7	22.3%	25.0%	27.3%	28.4%	29.5%	29.8%	31.4%	35.2%	-	-
0.8	14.7%	19.4%	23.1%	23.9%	24.9%	29.3%	29.7%	-	-	-
0.9	5.8%	8.6%	13.3%	19.3%	22.8%	28.3%	-	-	-	-

(b) $\rho_0 = 0.5$

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	49.9%	44.4%	40.0%	39.0%	35.4%	32.1%	29.1%	27.4%	22.2%	20.5%
0.1	44.4%	42.7%	39.9%	38.3%	34.7%	33.8%	30.8%	28.8%	25.6%	22.0%
0.2	40.0%	39.9%	38.2%	37.6%	33.7%	33.7%	30.1%	28.6%	28.5%	-
0.3	39.0%	38.3%	37.6%	35.8%	33.6%	33.6%	29.1%	26.4%	-	-
0.4	35.4%	34.7%	33.7%	33.6%	33.6%	32.1%	27.0%	-	-	-
0.5	32.1%	33.8%	33.7%	33.6%	32.1%	26.1%	-	-	-	-
0.6	29.1%	30.8%	30.1%	29.1%	27.0%	-	-	-	-	-
0.7	27.4%	28.8%	28.6%	26.4%	-	-	-	-	-	-
0.8	22.2%	25.6%	28.5%	-	-	-	-	-	-	-
0.9	20.5%	22.0%	-	-	-	-	-	-	-	-

(c) $\rho_0 = 0.9$

Table A.26: Relative differences between $E[S_0]_{\text{select}}(q^*)$ and $E[S_0]_{\text{split}}(\alpha^*)$ for *deterministic* service times B_j, $j = 0, 1, 2$.

$\rho_1 \backslash \rho_2$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	50.4%	45.1%	40.8%	34.3%	29.7%	23.0%	17.5%	13.2%	6.8%	3.7%
0.1	45.1%	45.2%	43.5%	38.5%	32.5%	25.8%	19.2%	14.0%	7.5%	3.9%
0.2	40.8%	43.5%	44.0%	40.2%	34.8%	27.6%	21.6%	15.6%	9.5%	4.4%
0.3	34.3%	38.5%	40.2%	40.3%	38.0%	32.3%	25.5%	17.9%	12.4%	4.6%
0.4	29.7%	32.5%	34.8%	38.0%	38.7%	35.2%	28.5%	20.8%	12.7%	6.7%
0.5	23.0%	25.8%	27.6%	32.3%	35.2%	36.1%	33.3%	27.4%	16.0%	7.0%
0.6	17.5%	19.2%	21.6%	25.5%	28.5%	33.3%	33.4%	32.0%	21.3%	10.4%
0.7	13.2%	14.0%	15.6%	17.9%	20.8%	27.4%	32.0%	34.6%	29.7%	17.2%
0.8	6.8%	7.5%	9.5%	12.4%	12.7%	16.0%	21.3%	29.7%	31.0%	26.7%
0.9	3.7%	3.9%	4.4%	4.6%	6.7%	7.0%	10.4%	17.2%	26.7%	40.0%

(a) $\rho_0 = 0.1$

$\rho_1 \backslash \rho_2$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	49.2%	46.6%	41.7%	38.6%	35.7%	33.0%	28.1%	22.7%	18.4%	4.6%
0.1	46.6%	44.3%	41.8%	39.6%	37.9%	34.3%	30.2%	25.3%	18.5%	8.9%
0.2	41.7%	41.8%	41.3%	39.2%	37.6%	34.9%	31.0%	27.7%	25.1%	14.7%
0.3	38.6%	39.6%	39.2%	38.6%	36.8%	35.7%	32.4%	30.3%	26.6%	19.0%
0.4	35.7%	37.9%	37.6%	36.8%	36.0%	35.6%	33.4%	31.1%	26.7%	22.3%
0.5	33.0%	34.3%	34.9%	35.7%	35.6%	33.4%	32.4%	31.6%	31.2%	31.1%
0.6	28.1%	30.2%	31.0%	32.4%	33.4%	32.4%	32.4%	32.4%	31.3%	-
0.7	22.7%	25.3%	27.7%	30.3%	31.1%	31.6%	32.4%	35.1%	-	-
0.8	18.4%	18.5%	25.1%	26.6%	26.7%	31.2%	31.3%	-	-	-
0.9	4.6%	8.9%	14.7%	19.0%	22.3%	31.1%	-	-	-	-

(b) $\rho_0 = 0.5$

$\rho_1 \backslash \rho_2$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	49.7%	45.5%	40.8%	39.3%	36.6%	34.2%	30.7%	28.6%	25.4%	24.8%
0.1	45.5%	44.6%	40.4%	38.2%	35.7%	33.7%	31.2%	30.6%	27.9%	26.5%
0.2	40.8%	40.4%	39.5%	37.5%	35.4%	33.3%	32.7%	30.6%	28.7%	-
0.3	39.3%	38.2%	37.5%	37.3%	34.9%	32.9%	32.9%	30.5%	-	-
0.4	36.6%	35.7%	35.4%	34.9%	34.6%	32.8%	29.9%	-	-	-
0.5	34.2%	33.7%	33.3%	32.9%	32.8%	31.8%	-	-	-	-
0.6	30.7%	31.2%	32.7%	32.9%	29.9%	-	-	-	-	-
0.7	28.6%	30.6%	30.6%	30.5%	-	-	-	-	-	-
0.8	25.4%	27.9%	28.7%	-	-	-	-	-	-	-
0.9	24.8%	26.5%	-	-	-	-	-	-	-	-

(c) $\rho_0 = 0.9$

Table A.27: Relative differences between $E[S_0]_{\text{select}}(q^*)$ and $E[S_0]_{\text{split}}(\alpha^*)$ for *hyperexponential* service times B_j with $c^2_{B_j} = 2$, $j = 0, 1, 2$

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	50.5%	46.0%	42.1%	35.9%	30.6%	24.2%	18.1%	12.2%	9.0%	3.3%
0.1	46.0%	46.1%	42.7%	37.8%	32.6%	25.0%	20.3%	15.8%	11.4%	5.0%
0.2	42.1%	42.7%	42.8%	40.2%	36.5%	30.9%	21.8%	17.0%	12.2%	5.7%
0.3	35.9%	37.8%	40.2%	41.2%	38.3%	32.5%	28.0%	19.1%	11.5%	6.0%
0.4	30.6%	32.6%	36.5%	38.3%	38.5%	34.5%	30.4%	21.6%	13.4%	7.8%
0.5	24.2%	25.0%	30.9%	32.5%	34.5%	36.9%	34.1%	29.7%	18.4%	13.2%
0.6	18.1%	20.3%	21.8%	28.0%	30.4%	34.1%	34.3%	33.5%	24.2%	14.1%
0.7	12.2%	15.8%	17.0%	19.1%	21.6%	29.7%	33.5%	33.8%	33.0%	23.3%
0.8	9.0%	11.4%	12.2%	11.5%	13.4%	18.4%	24.2%	33.0%	36.8%	33.8%
0.9	3.3%	5.0%	5.7%	6.0%	7.8%	13.2%	14.1%	23.3%	33.8%	40.4%

(a) $\rho_0 = 0.1$

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	49.7%	46.7%	43.4%	40.0%	38.0%	33.4%	29.7%	24.1%	17.1%	8.4%
0.1	46.7%	44.8%	42.5%	39.0%	37.8%	34.0%	30.3%	27.1%	20.1%	11.9%
0.2	43.4%	42.5%	42.4%	38.8%	36.4%	34.3%	31.6%	28.8%	23.7%	17.9%
0.3	40.0%	39.0%	38.8%	37.7%	36.2%	35.2%	33.2%	30.4%	28.0%	21.3%
0.4	38.0%	37.8%	36.4%	36.2%	35.6%	35.8%	34.0%	32.4%	29.0%	28.2%
0.5	33.4%	34.0%	34.3%	35.2%	35.8%	35.2%	34.1%	33.5%	32.6%	32.5%
0.6	29.7%	30.3%	31.6%	33.2%	34.0%	34.1%	35.0%	33.9%	33.1%	-
0.7	24.1%	27.1%	28.8%	30.4%	32.4%	33.5%	33.9%	36.0%	-	-
0.8	17.1%	20.1%	23.7%	28.0%	29.0%	32.6%	33.1%	-	-	-
0.9	8.4%	11.9%	17.9%	21.3%	28.2%	32.5%	-	-	-	-

(b) $\rho_0 = 0.5$

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	49.3%	45.2%	41.0%	40.7%	36.8%	36.6%	30.8%	29.6%	26.4%	19.1%
0.1	45.2%	43.4%	40.8%	39.6%	37.2%	34.8%	31.4%	30.5%	28.4%	22.4%
0.2	41.0%	40.8%	38.8%	39.4%	36.7%	34.4%	34.1%	31.4%	29.7%	-
0.3	40.7%	39.6%	39.4%	36.7%	34.5%	32.3%	31.4%	30.4%	-	-
0.4	36.8%	37.2%	36.7%	34.5%	33.7%	32.1%	31.3%	-	-	-
0.5	36.6%	34.8%	34.4%	32.3%	32.1%	31.9%	-	-	-	-
0.6	30.8%	31.4%	34.1%	31.4%	31.3%	-	-	-	-	-
0.7	29.6%	30.5%	31.4%	30.4%	-	-	-	-	-	-
0.8	26.4%	28.4%	29.7%	-	-	-	-	-	-	-
0.9	19.1%	22.4%	-	-	-	-	-	-	-	-

(c) $\rho_0 = 0.9$

Table A.28: Relative differences between $E[S_0]_{\text{select}}(q^*)$ and $E[S_0]_{\text{split}}(\alpha^*)$ for *hyperexponential* service times B_j with $c^2_{B_j} = 4$, $j = 0, 1, 2$.

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	50.6%	46.8%	43.4%	37.5%	31.7%	25.5%	18.5%	11.0%	11.2%	2.9%
0.1	46.8%	47.0%	41.8%	37.2%	32.9%	24.2%	21.4%	17.7%	15.4%	6.1%
0.2	43.4%	41.8%	41.4%	40.2%	38.2%	34.1%	21.9%	18.6%	14.8%	7.1%
0.3	37.5%	37.2%	40.2%	41.7%	38.6%	32.8%	30.6%	20.5%	10.4%	7.4%
0.4	31.7%	32.9%	38.2%	38.6%	38.1%	34.0%	32.5%	22.5%	14.1%	8.9%
0.5	25.5%	24.2%	34.1%	32.8%	34.0%	37.6%	34.9%	31.7%	20.7%	19.5%
0.6	18.5%	21.4%	21.9%	30.6%	32.5%	34.9%	35.6%	34.8%	27.0%	18.0%
0.7	11.0%	17.7%	18.6%	20.5%	22.5%	31.7%	34.8%	33.2%	36.1%	29.4%
0.8	11.2%	15.4%	14.8%	10.4%	14.1%	20.7%	27.0%	36.1%	42.8%	40.7%
0.9	2.9%	6.1%	7.1%	7.4%	8.9%	19.5%	18.0%	29.4%	40.7%	40.6%

(a) $\rho_0 = 0.1$

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	50.1%	46.8%	45.2%	41.6%	40.6%	33.6%	31.2%	25.3%	15.7%	12.4%
0.1	46.8%	45.3%	42.9%	38.5%	37.6%	33.8%	30.4%	28.8%	21.6%	14.9%
0.2	45.2%	42.9%	43.5%	38.6%	35.0%	33.7%	32.4%	29.9%	22.3%	21.0%
0.3	41.6%	38.5%	38.6%	36.6%	35.6%	34.6%	33.7%	30.4%	29.4%	23.8%
0.4	40.6%	37.6%	35.0%	35.6%	35.5%	36.0%	34.7%	33.5%	31.3%	34.1%
0.5	33.6%	33.8%	33.7%	34.6%	36.0%	36.8%	35.5%	35.1%	34.2%	34.1%
0.6	31.2%	30.4%	32.4%	33.7%	34.7%	35.5%	37.5%	35.3%	34.6%	-
0.7	25.3%	28.8%	29.9%	30.4%	33.5%	35.1%	35.3%	36.9%	-	-
0.8	15.7%	21.6%	22.3%	29.4%	31.3%	34.2%	34.6%	-	-	-
0.9	12.4%	14.9%	21.0%	23.8%	34.1%	34.1%	-	-	-	-

(b) $\rho_0 = 0.5$

ρ_1 \ ρ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	48.8%	44.8%	41.2%	42.3%	36.8%	39.1%	30.9%	30.7%	27.6%	13.5%
0.1	44.8%	42.1%	40.9%	40.9%	38.4%	36.1%	31.7%	30.1%	28.9%	18.4%
0.2	41.2%	40.9%	37.9%	41.3%	38.1%	35.7%	35.6%	32.0%	30.6%	-
0.3	42.3%	40.9%	41.3%	36.1%	34.1%	31.7%	30.2%	30.4%	-	-
0.4	36.8%	38.4%	38.1%	34.1%	32.9%	31.4%	32.8%	-	-	-
0.5	39.1%	36.1%	35.7%	31.7%	31.4%	32.3%	-	-	-	-
0.6	30.9%	31.7%	35.6%	30.2%	32.8%	-	-	-	-	-
0.7	30.7%	30.1%	32.0%	30.4%	-	-	-	-	-	-
0.8	27.6%	28.9%	30.6%	-	-	-	-	-	-	-
0.9	13.5%	18.4%	-	-	-	-	-	-	-	-

(c) $\rho_0 = 0.9$

Table A.29: Relative differences between $E[S_0]_{\text{select}}(q^*)$ and $E[S_0]_{\text{split}}(\alpha^*)$ for *hyperexponential* service times B_j with $c^2_{B_j} = 16$, $j = 0, 1, 2$.

Bibliography

[1] J.L. van den Berg. Sojourn times in feedback and processor sharing queues. 1990.

[2] J.L. van den Berg; R. Litjens; J. Laverman. HSDPA flow level performance: the impact of key system and traffic aspects. *Proceedings of the 7th ACM international symposium on modeling, analysis and simulation of wireless and mobile systems*, 2007.

[3] S. Bhulai; G. Koole. Stochastic optimization. 2007.

[4] S.C. Borst. User-level performance of channel-aware scheduling algorithms in wireless data networks. *IEEE/ACM transactions on networking*, Vol. 13(No. 3), 2005.

[5] S.C. Borst; M. Jonckheere. Flow-level stability of channel-aware scheduling algorithms. 2006.

[6] O.J. Boxma. Stochastic performance modelling. 2002.

[7] S.K. Cheung. Processor-sharing queues and resource sharing in wireless LANs. 2007.

[8] E.G. Coffman Jr; R.R. Muntz; H. Trotter. Waiting time distributions for processor-sharing systems. *Journal of the Association for Computing Machinery*, Vol. 17:123–130, 1970.

[9] J.W. Cohen. The multiple phase service network with generalized processor sharing. *Acta Informatica*, Vol. 12:245–284, 1979.

[10] I. Forkel; H. Klenner; A. Kemper. High speed downlink packet access (HSDPA) - enhanced data rates for UMTS evolution. *Computer Networks*, pages 325–340, 2005.

[11] A. Haider; R. Harris; H. Sirisena. Implementation and simulation of HSDPA functionality. 2005.

[12] R. Litjens. HSDPA flow level performance and the impact of terminal mobility. *IEEE Communications Society*, Vol. 3:1657–1663, 2005.

[13] R. Litjens; J.L. van den Berg. Performance analysis of adaptive scheduling in integrated services UMTS networks. pages 3–7, 2002.

[14] S. Parvall; E. Dahlman; P. Frenger; P. Beming; M. Persson. The evolution of WCDMA towards higher speed downlink packet data access. 2001.

[15] H.C. Zhao; J.Q. Wu. Implementation and simulation of HSDPA functionality. 2005.