Integrating Inflation-Linked Instruments in the Asset Liability Management Framework

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Internship report

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Preface

The last phase of the study Business Mathematics and Informatics at the Free University Amsterdam includes a six-month internship. My internship took place at the Group Asset Liability Management (GALM) department within ABN AMRO bank. This thesis is the end result of my internship.

The experience that I gained during my internship is valuable. First of all, my gratitude goes to all the people within the GALM department for their support and help during my internship. I would like to thank Jan Remmerswaal in particular for giving me the opportunity to do my internship within the GALM department. My gratitude goes certainly to Benno Rummel who is my main supervisor at the GALM department. I am grateful for his guidance, the objective comments and the encouragement he provided me during my research and writing this thesis. I would also like to thank Sandjai Bhulai who is my supervisor at the Free University. I am grateful for his critical comments on my thesis. His constant support since the beginning until the end of my internship meant a lot to me.

Further, I also want to thank Alessandro Drago from BU Brazil and the people from the ALM modeling department for their advices and good ideas which contributed a lot to the modeling part of this research. I would like to thank Floris Kleemans from the Economic Department in Amsterdam for providing me an introduction of macro-economic situation in Brazil and the data that I needed for the analysis within my research.

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Contents

Management Summary

There are two main inflation indexes in Brazil; the IGP-M index and the IPCA index. Bonds are sold on both indexes in Brazilian economic markets; NTNB bonds which are linked to IPCA index and NTNC bonds which are linked to IGP-M index. Since the principal of the inflation-linked bonds is adjusted by a change in inflation over a period, the cash flows would increase in the situation of increasing inflation. When a sudden upward shock in interest rate occurs, which is accompanied with an increase in inflation, investing in inflation-linked bonds would provide protection against the negative effects of increasing interest rates.

The development of both inflation indexes was analyzed and models were obtained that explain the development of the inflation. It is observed that the movements of the inflation indexes based on the models have the same direction as the long end of the yield curve. However, the IGP-M index is more correlated with the long-term interest rates than the IPCA index. This coincides with observed historical movements. The historical data show that the IGP-M index has been more volatile than the IPCA index.

Given this fact, the NTNC bonds, bonds whose coupons and principal are kept in line with IGP-M index, would provide more protection against the negative effects of increasing interest rates than the NTNB bonds whose coupons and principal are kept in line with IPCA index (since the IPCA index is much less correlated with the long-term interest rates). However, the opposite is also true for decreasing interest rates. In this kind of situation, investing in other than NTNC bonds would be a cheaper alternative.

From this research, simple models for explaining both inflation indexes have been obtained in the form of Autoregressive Distributed Lags models. These models can be integrated in the ALM framework with little effort. These models were obtained given a small set of macro-economic variables that were theoretically analyzed and empirically proven for their ability to explain the two inflation indexes in Brazil. Below please find the preferred models for explaining the inflation indexes.

$$\Delta IGPMyoy_t = 0.580\ \Delta IGPMyoy_{t-1} + 0.071(Slope_{t-1} - Slope_{t-13}) + 0.011\ (\Delta FX_{t-1} - \Delta FX_{t-13} + \Delta FX_{t-2} - \Delta FX_{t-14}),$$

$$\Delta IPCAyoy_t = 0.440\ \Delta IPCAyoy_{t-1} - 0.327\ \Delta IPCAyoy_{t-2} + 0.167\ (\Delta Selic_{t-1} - \Delta Selic_{t-13}) + 0.056\ (EMBI_{t-4} - EMBI_{t-16}) + 0.009\ (\Delta FX_{t-2} - \Delta FX_{t-14}).$$

It is proposed to start using these models in the ALM framework to explain the development of both inflation indexes. The current models that are proposed by BU Brazil should be replaced. Below please find the current models for explaining the inflation indexes.

$$\Delta IGPMyoy_t = 0.960\ \Delta OneY_{t-1} + 0.930\ \Delta IGPMyoy_{t-1} - 0.330\ \Delta IGPMyoy_{t-2},$$

$$\Delta IPCAyoy_t = 0.830\ \Delta OneY_{t-1} + 0.570\ \Delta IPCAyoy_{t-1} - 0.220\ \Delta IPCAyoy_{t-2}.$$

1 Introduction

This internship project is for Group Asset Liability Management (GALM) within ABN AMRO bank. The research of this project will be focusing on Inflation-linked bonds specific for Banco Real, an ABN AMRO subsidiary in Brazil.

One of the responsibilities of GALM is managing the interest income on the balance sheets with respect to the interest rate movements. The strategic goal within interest income management is to optimize/stabilize the development of the interest earnings (earnings perspective) and the market value of equity (market value perspective). For instance, the position of the balance sheet should be taken into consideration to achieve the stability of the net interest income.

The most likely economic scenario for Brazil predicts a gradually decreasing interest rate to a significantly lower level. With such a scenario it is advantageous to lock in the current interest rate by investing in long-term securities such as bonds. In other words, investing in the long-term bonds would give an opportunity to secure the interest rate during the lifetime of the bonds. It is known that there is an inverse relationship between the interest rate and the market value of the bond; as the interest rate decreases, the market value of the bond will increase. Investing in long term bonds corresponds to increasing the duration of the balance sheet. However, the risk still exists that a crisis, a sudden upward shock in the interest rate that occurred in the past, might occur again in the future. Like many other emerging market economies, Brazil has suffered a series of major external financial shocks since the mid 90's. As long-term bonds are subject to a greater interest rate risk than shorter term bonds, holding a long term position during a crisis situation would have an adverse effect.

The primary risk of instruments such as bonds is the change in the market price and the earnings due to changes in the interest rate. The real interest rate risk and the inflation risk are some of the main sources of interest rate risk. The changes in interest rate can be explained by the changes in either the real interest rate or changes in the inflation.

When an increase in the interest rate is accompanied with an increasing inflation, investing in inflation-linked bonds would be a good deal for investors. It is because the principal amount and future coupon payments of inflation-linked bonds are adjusted to keep in line with the inflation realized over the lifetime of the bond. Therefore, inflation-linked bonds would offset some of the negative effects of the increasing interest rates. On the other hand, when an increase in the interest rate is not accompanied with an increasing inflation, the protection provided by the inflation-linked bonds would not work. In this kind of situation, maybe investing in instruments other than inflation-linked bonds would be a cheaper alternative. In some of the crisis situation occurred in the past, the sudden upward in interest rates were accompanied by an increase in inflation.

The main objective of this project is integrating inflation-linked bonds in the ALM metrics specific for the situation in Brazil. To make this possible, the relationship between the inflation, the nominal interest rate, and maybe other leading macro-economic indicators has to be studied. Based on this relationship, one could analyze whether the inflation-linked bonds indeed fit to the ALM perspectives; earnings and market value perspectives.

In the first chapter, a brief economic overview of Brazil is given. Further, an overview of Group Asset Liability Management and its perspectives, perspectives within interest income management in particular, is given. The characteristics of inflation-linked bonds are described in Chapter 3. This chapter includes an illustration of the impact of the interest rate and inflation movements on a simple balance sheet in which the inflation-linked bond is positioned from an interest income management point of view. The considerations of integrating inflation-linked bonds in the ALM framework are given in Chapter 4. The analysis of possible macro-economic indicators for predicting inflation is given in Chapter 5. Based on the theoretical analysis in Chapter 5 and the statistical analysis in Chapter 6 as a background motivation for the research, various Autoregressive Distributed Lags (ADL) models have been estimated during the analysis. Several models are presented and evaluated in Chapter 7 for each inflation index. The focus is on the forecast errors resulting from the model specification. The question how to forecast the explanatory variables is out of the scope of this project.

At last, the impact analysis is done in Chapter 8 given the stress scenarios for the yield curve and other macro-economic variables and the corresponding inflation developments based on the models. The stress scenarios used for the impact analysis are proposed based on the principal component analysis (PCA). The aim of the impact analysis is to analyze what would happen with the market value risk and the earnings risk when the inflation-linked bonds are integrated in the ALM framework, and compare these results with the case when the inflation-linked bonds are not integrated. The impact analysis will be done by using a simple balance sheet in which inflation-linked bonds are positioned at the asset side.

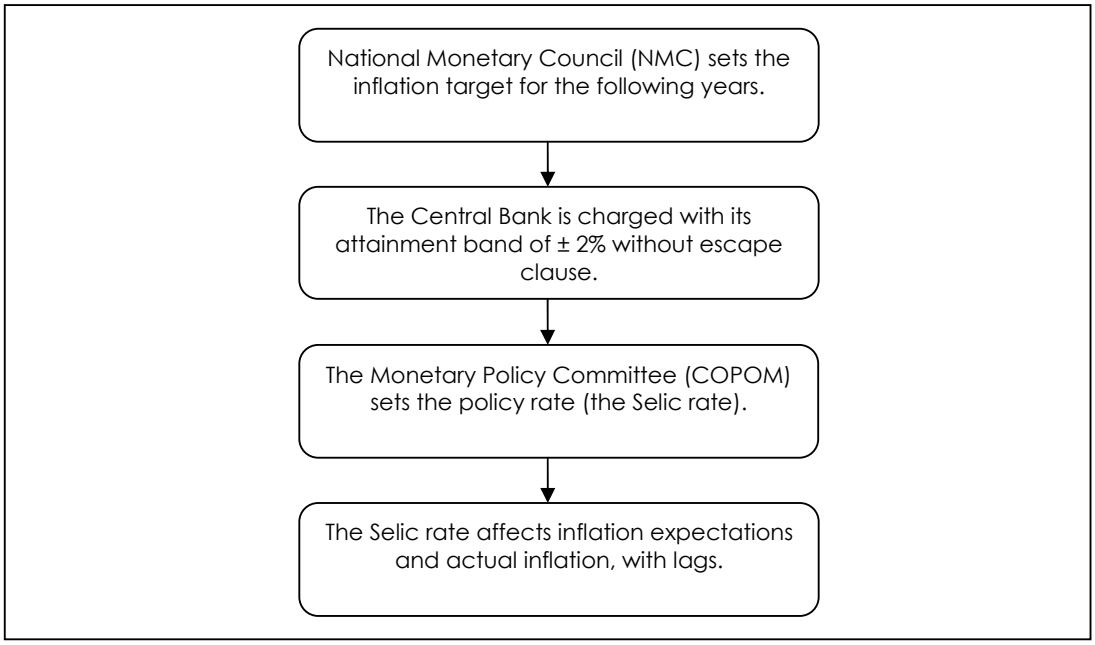
2 Brazil's economic overview

2.1 Inflation targeting regime, monetary policy

The economic development in Brazil has been hampered by high inflation and foreign debt. In the 80's, Brazil had the highest inflation level in the world. Further, Brazil experienced a series of major economy shocks since the mid 90's. Up to 1998, the Real was pegged to the US dollar. Starting in January 1999, it was no longer the case. Soon after the transition from the peg exchange rate to the floating exchange rate regime, Brazil adopted the inflation targeting framework for the conduct of the monetary policy. The policy is about keeping inflation at a targeted level. A global institutional structure of inflation targeting regime framework is illustrated in Figure 2-1.

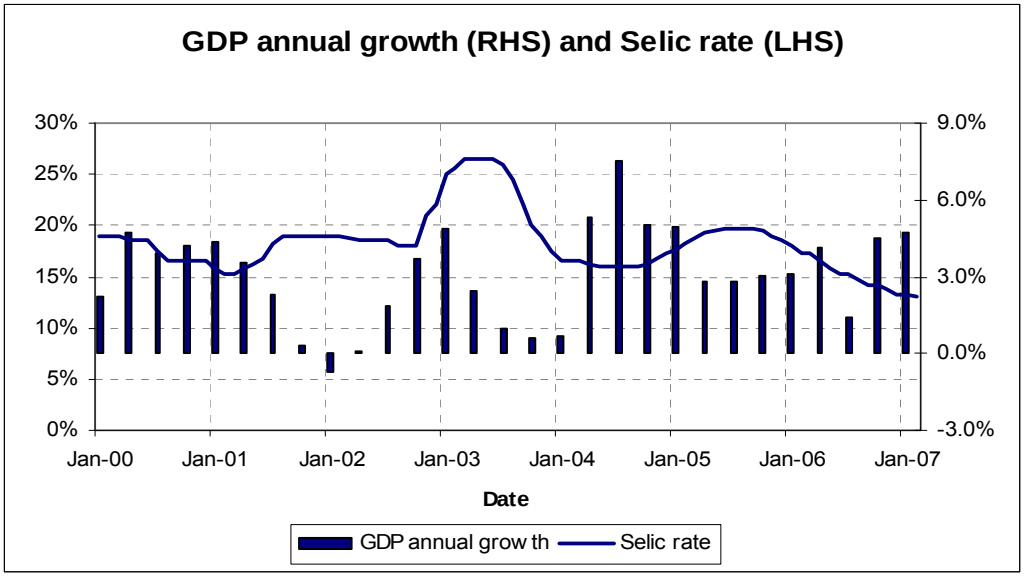
In the inflation targeting regime framework, inflation targets for the following two years are set in the middle of each year by National Monetary Council (NMC). The Central Bank has one instrument, the monetary policy, to ensure price stability. The policy has been based on some basic principles implemented by the Monetary Policy Committee (COPOM). In Brazil, the COPOM's monetary policy decisions have the achievement of the inflation targets through a periodic adjustment to the policy rate, the Selic rate. It is a short-term interest rate in which the Central Bank charges other banks for overnight government bonds. It is observed that the monetary policy has important effects on the economy. The increase in the Selic rate leads to a decline in the output. The quarterly year-on-year GDP growth and the monthly Selic rate are given in Figure 2-2. Further, it affects the inflation expectations and actual inflation, with certain lags. The effects on inflation will be analyzed in the further chapter.

Figure 2-1: Institutional structure of Inflation Targeting Regime Framework.



Source: Paulo Vieira da Cunha (2007)

Figure 2-2: The quarterly y-o-y GDP growth and the monthly Selic rate.



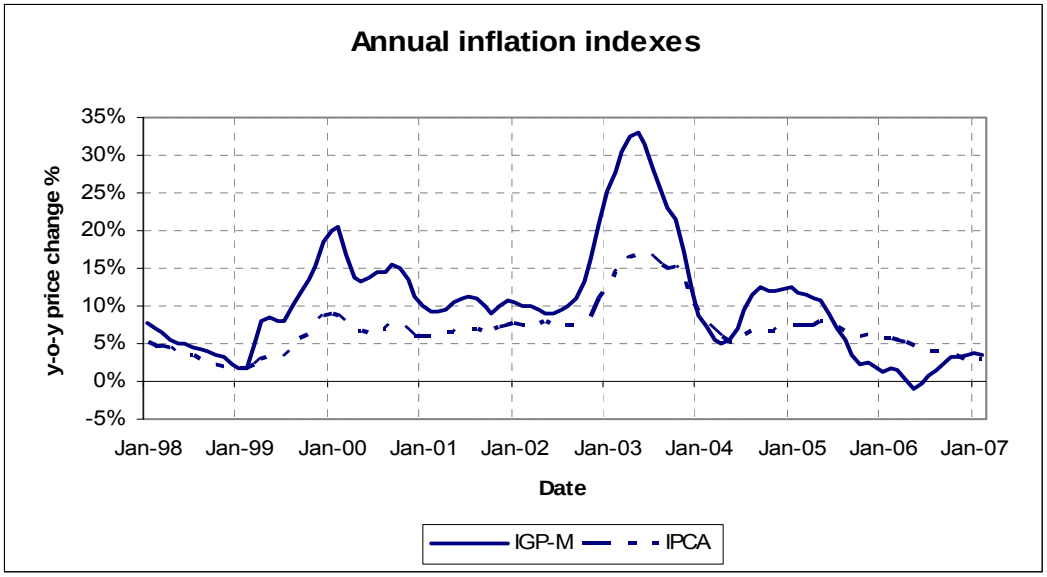
Source: Bloomberg (BZGDYOY% Index, BZSTSETA Index)

2.1.1 Inflation index in Brazil

The main inflation indexes in Brazil are the IPCA index and the IGP-M index. The IPCA is the nationwide consumer's price index. It is used by the Central bank for monitoring the objectives established in the inflation targeting regime. The IGP-M is the market general price index. It is composed of three indexes: the wholesale price index, the consumer price index and the construction cost national index that represent 60%, 30%, and 10%, respectively, of the IGP-M[1]. As depicted in Figure 2-3 and Figure 2-4 below, the impacts of the depreciation of the Real were reflected in both main price indexes. However, the IGP-M index has been more volatile than the IPCA index because the wholesale prices track exchange rate movements much more closely. The bonds linked to the IPCA and IGP-M indexes are called the NTNB and the NTNC, respectively.

[1] http://64.151.125.190/gafisa2006/web/conteudo_en.asp?idioma=1&tipo=91&conta=44. Last update: July 19, 2006.

Figure 2-3: Annual inflation indexes.

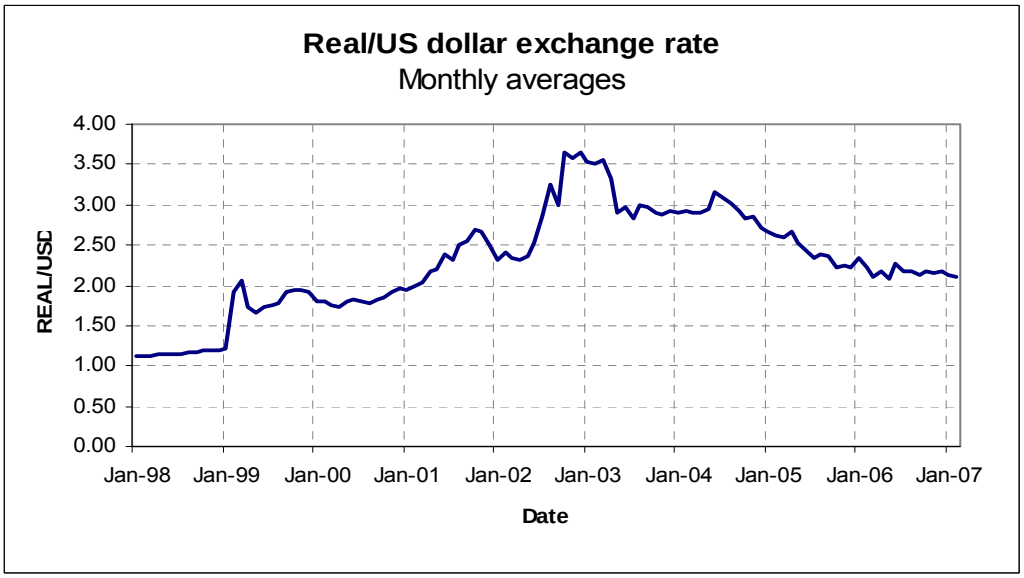


Source: Bloomberg (BZPIIPCY Index, IBREGPMY Index)

2.2 Crisis scenarios

Figure 2-4 shows that in January 1999, the nominal exchange rate jumped from 1.21 BRL/USD to 1.98 BRL/USD. Further, there were depreciation periods in the year 2001, which is gradual, and quite severely in the second half of 2002.

Figure 2-4: The monthly exchange rate.



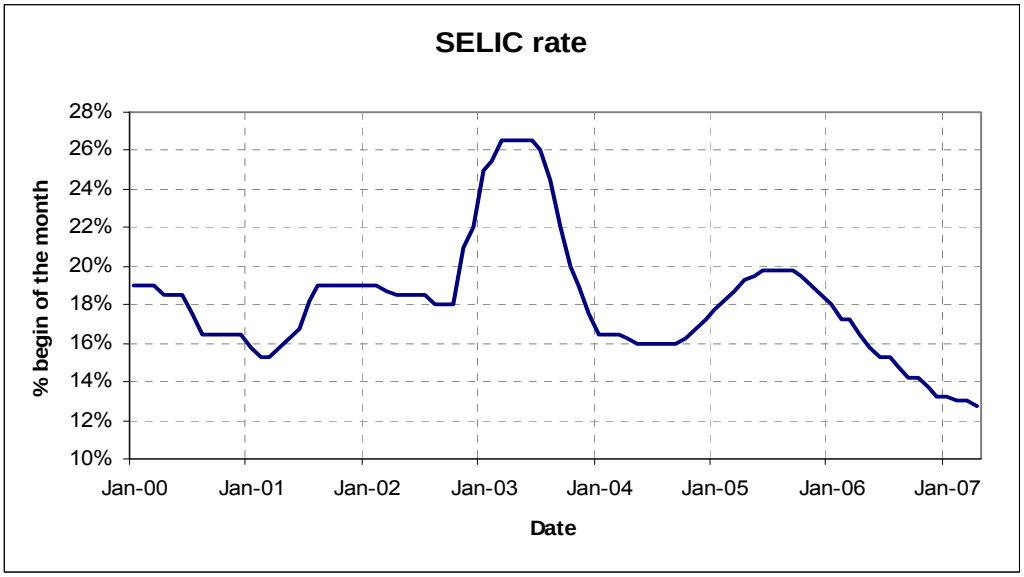
Source: Bloomberg

2.2.1 Foreign and domestic crisis in 2001

The strong pressure on the exchange rate in 2001 can be explained by a sequence of domestic and international events. On the domestic side there were the energy crisis and the political disarray inside the government coalition. On the international side, it became clear that the US economy entered a recession and that the Argentina crisis worsened considerably. The aftermath of the September 11 terrorist attacks in the United States brought contagion to Brazil.

Figure 2-5 shows that the interest rate was increased several times. The Selic rate increased from 15.75% in the beginning of year 2001 to 19% in the end of 2001. The yield curve steepened drastically (this will be further analyzed in Section 6.2.3). Until September 2001, the exchange rate depreciated continually.

Figure 2-5: Selic rate.



Source: Bloomberg (BZSTSETA Index)

2.2.2 Confidence crisis in 2002

In 2002, there was another story behind the wave of depreciation. Although the economy was growing in 2000 (see Figure 2-2), the economy slowed down after that due to the confidence crisis. The GDP growth was even negative in the beginning of 2002. This is because the year 2002 was a presidential election year in Brazil and the leftist candidate (currently President) Lula became the front runner in the public opinion surveys. There was a concern, in particular, about the inability of the new government to pay the large public debt. This unfortunate combination of an increase of global risk aversion in that period with fears of a Brazilian default on the debt was the cause of country risk hikes after April 2002. This confidence crisis led to a weakening of the currency and high inflation.

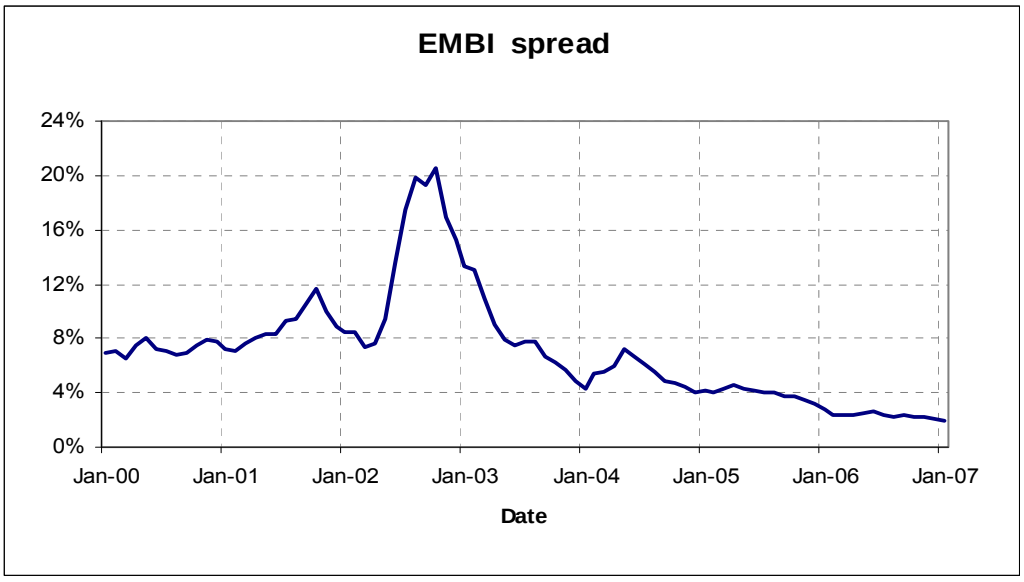
The inflation target regime, monetary policy, and the exchange rate are responsible for economic stability. It is observed in the crisis situations that these three economic factors are related to each other. The exchange rate depreciation is likely accompanied with a higher inflation rate which can be explained by the impact on tradable goods. The effect of exchange rate changes on inflation will be further described in Section 6.1. In order to control inflation, the monetary policy increases the Selic rate to reduce demand.

2.3 Country risk in Brazil

2.3.1 Emerging Market Bond index (EMBI) spread

The EMBI spread is the difference between the yield on a dollar-denominated bond issued by the Brazilian government and a corresponding one issued by the US Treasury. This variable gives information about the market's assessment of the probability that Brazil might default on its debt obligations. The Brazilian EMBI spread was around 700 basis points in February 2002 and reached a peak in September 2002. After the election in October, the spread has gradually fallen. The EMBI spread is considered a good indicator for these crises because it increased significantly during these episodes.

Figure 2-6: EMBI (Emerging Market Bond index) spread, monthly average.

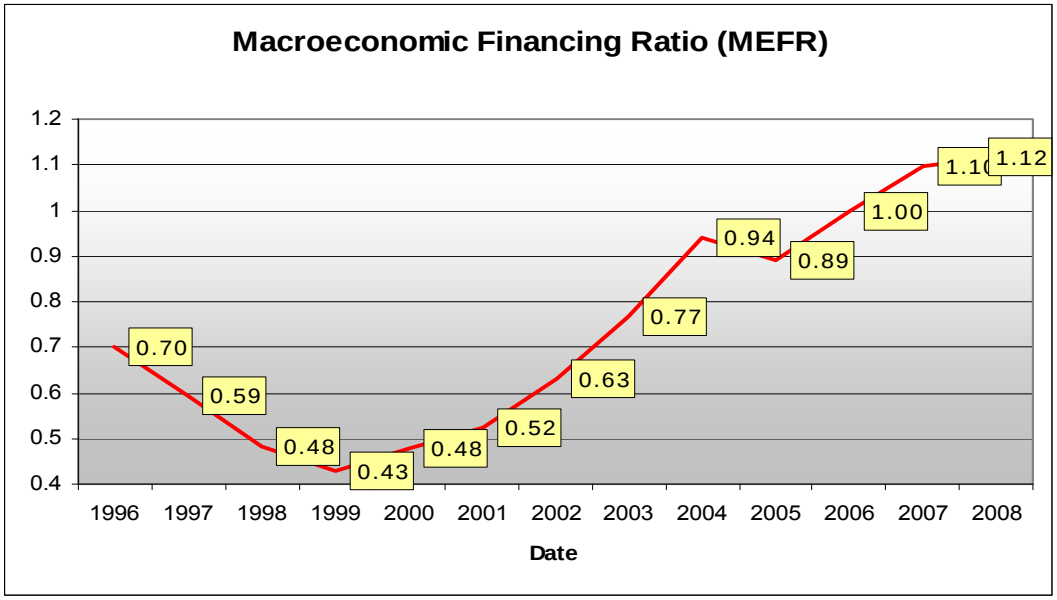


Source: Datastream, JPMPBRA(SSPRD)

2.3.2 Macro-economic Financing ratio

Another indicator for country risk in Brazil is the Macro-economic Financing Ratio (MEFR). It is the ratio of the total means and the total spending. The spending is supposed to be financed by the means. The means include the foreign reserves exclusive gold, the net foreign assets commercial banks, the undisbursed credit commitments at BIS banks, and exports of goods, services and net transfers. The total spending includes the imports of goods and services, the principal repayments on medium and long term debt, and the short term debt. A low MEFR indicates the necessity of extra financing to be able pay to the spending and, at the same time, this means a high credit risk of a country. The MEFR in the period 1999 reached the lowest point of 0.43. In the current situation, the MEFR is around 1.

Figure 2-7: Macro-economic Financing Ratio (MEFR).



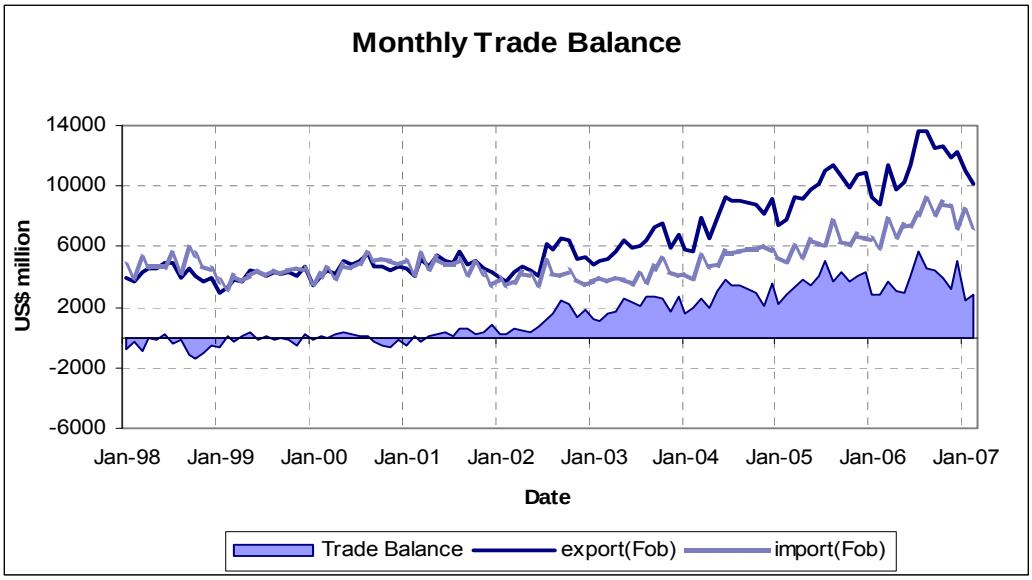
Source: Economic Department, ABN AMRO in the Netherlands.

2.4 Current situation

There are trends observed in the Brazilian economic environment; there is a reduction in the debt-to-GDP ratio (see Figure 2-10), an increasing trend in the GDP growth (see Figure 2-2), growing exports (see Figure 2-8), currency strengthening and a stable low inflation. The economic environment in Brazil is improving relative to the previous years. As the reaction of the Central Bank, the Selic rate was lowered in the period of low inflation.

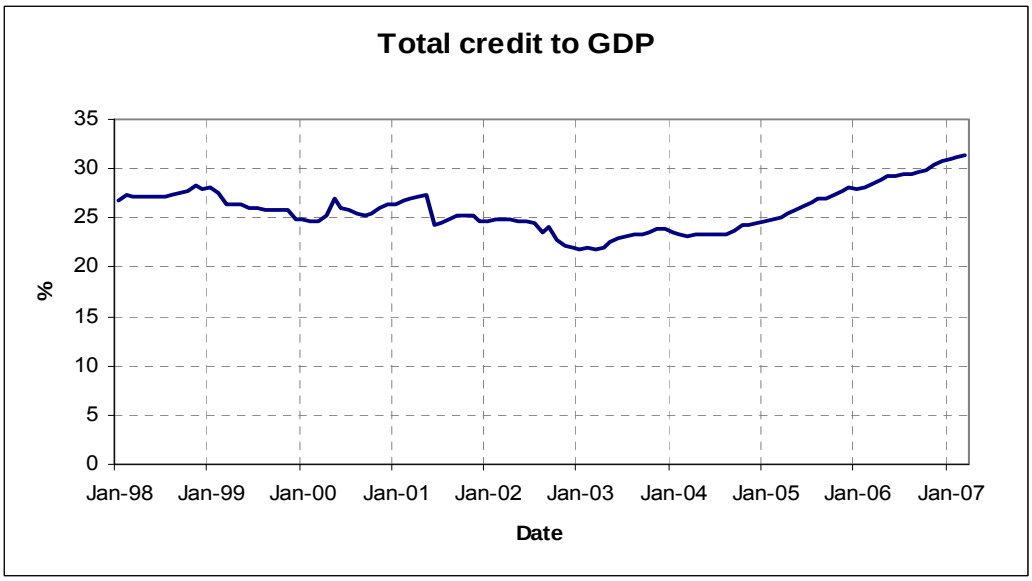
The credit-to-GDP is growing (see Figure 2-9). However, compared with other emerging market economies, the credit-to-GDP ratio in Brazil is still low. Therefore, the real interest rate remains high to raise the investment attractiveness and credibility in Brazil. The real interest rate in Brazil is still very high compared to the real interest rates in other countries. For the comparisons of the Brazilian real interest rate one has to take into consideration the historical problems such as a high inflation and an inappropriate fiscal management. However, Brazil's capacity to pay the foreign debt gives the positive assessments of the Brazilian country risk and the foreign exchange reserve remains high to increase the credit rating of the nation.

Figure 2-8: Monthly trade balance.



Source: MDIC/Secex (code 2946, 3034)

Figure 2-9: Total credit-to-GDP.



Source: BCB-DEPEC (code 11400)

2.5 Possible scenarios

Based on the current situation, there are several scenarios according to the Economics department of ABN AMRO bank in the Netherlands.

2.5.1 High growth

The most likely scenario in Brazil is that the inflation remains stable and the interest rates will decrease to significantly lower levels. It is partly due to that Brazil is regaining the credibility. Therefore, there is a higher growth and more capital inflows that lead to more currency strengthening.

2.5.2 Stable growth

To raise the investment attractiveness in Brazil, the real interest rate is set to a high level. By setting the interest rate to a high level, the inflation and economic growth are limited. A high interest rate corresponds to a high cost for borrowing and this leads to less consumption. The stable economic growth at a low level can also be interpreted as consequence of high taxes.

2.5.3 Low growth

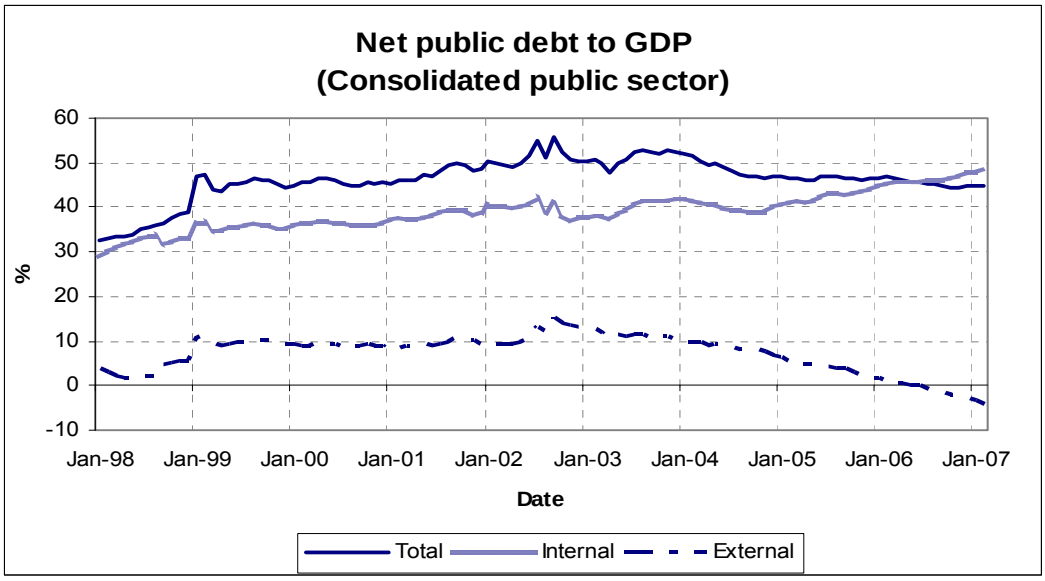
When government expenditures exceed government revenues, a budgetary deficit results; and vice versa for a surplus. A government's public debt is defined as the sum of all the budgetary surpluses and deficits it has accumulated over a specific period of time.

Despite registering a decline, Brazil's government debt remains high, largely domestic (see Figure 2-10). The negative external debt that corresponds to a surplus situation can be explained by a strengthening of the Real relative to the US dollar that lowers the value of the external debt while the government finds it hard to meet the internal debt.

There is a lax fiscal stance. Government expenditures continue to grow fast, and that would increase the debt. The surplus would be used to pay the debt. Higher expenditures would also lead to higher taxes for the private sector, further crowding out the private sector, following the negative growth prospects.

The increasing internal debt and lower earnings would cause both a rise in interest rates and the weakening of the currency, followed by a high expected inflation. These rising interest rates would lead to a lower growth (vicious circle).

Figure 2-10: Net public debt-to-GDP.



Source: BCB-DEPEC (code 4513, 4524, 4535)

3 Asset and Liability Management

3.1 Introduction

Risk management is a core competency for banks. The objectives of risk management of the bank are to analyze and assess risks in an early stage, to propose measures for reducing risks, and to set and manage the risk limits. Risk management takes place within the Group Risk Committee (GRC) and the Group Asset and Liability Committee (GALCO) within the ABN AMRO bank.

The GRC is responsible for the control of credit risks, country risks, market value risks in trading portfolios, and operational risks. The GALCO is responsible for managing the earnings (interest rate risk management) and the capital (capital management) position in the non-trading portfolio with respect to interest rates and currency movements (FX risk management), for liquidity management, and for managing the longer-term debt issuance. Within this report, the focus will be on the area of the responsibility of GALCO.

The Group Asset Liability Management (GALM) department advises the GALCO about the consolidated balance sheet position and positioning of all Business Units of ABN AMRO. The GALCO is responsible for the overall risk limits for market risks regarding the decision making on asset and liability management transactions.

The major aspects of market risks are interest income, liquidity risk, capital, and FX risk management.

3.1.1 Interest rate risk management

Interest rate risk management focuses on the income optimization with respect to possible economic scenarios constrained by risk in stress scenarios. The strategic goal within interest income management is to optimize/stabilize the development of the interest earnings and the market value of equity. It is done through understanding the dynamics of the balance sheet and the drivers of the earnings, ensuring alignment with business objectives and incentives, strategic positioning of the balance sheet, and monitoring and managing exposures.

Considerations within interest rate risk management are for example; which transactions should be taken into consideration to achieve stability of the net interest income? What is the mismatch position at the moment? How attractive are savings at the moment with respect to other financial possibilities?

3.1.2 Liquidity management

Liquidity risk arises in any bank's general funding of its activities. For example, a bank may be unable to fund its portfolio of assets at appropriate maturities and rates, or may find itself unable to liquidate a position in a timely manner at a reasonable price. Liquidity management focuses on the process of achieving a balance between the maximization of liquidity premium earnings and the need to have sufficient liquidity available. This is achieved through the analysis and active management of assets and liabilities.

3.1.3 Capital management

Capital management focuses on managing the balance between demand and supply for capital, and reducing capital costs constrained by capital requirements.

3.1.4 FX risk management

FX risk management focuses on actively monitoring and hedging open currency positions of capital and income with respect to currency movements.

3.2 Interest Rate Risk

One of the market risk variables is the interest rate. The interest rate risk is generally defined as the adverse impact of interest rate movements on the earnings or the market value of a portfolio. Because this project is attached to the interest rate risk, a brief introduction to the interest rate risk will be given in this section.

Two main perspectives for assessing a bank's interest rate risk exposure are the earnings perspective and the market value perspective.

3.2.1 Earnings perspective

As the name denotes, the earnings perspective focuses on the impact of interest rate movements on a bank's earnings. ALM within ABN AMRO defines scenarios with a time horizon of a maximum of two years. The basis metric for earnings is the Net Interest Income (NII), which is the difference between the total interest received (on assets) and the total interest paid (on liabilities). Future developments such as new production are included in the NII calculation.

ALM within ABN AMRO analyzes two developments; the interest rate development (this is called the scenario) and the balance sheet position/volume and margin development (this is called the strategy). There are basically six interest rate scenarios defined within ALM at ABN AMRO, the most likely scenario, the base case scenario, and four stress scenarios. These scenarios are analyzed based on the most likely strategy.

The NII on the base case scenario is simply the NII when there are no interest rate changes. The stress scenarios are the interest rate scenarios when there are changes in the interest rate movement in the sense of the changes in the yield curve's level (ramp up and ramp down) or shape (clockwise rotation and counter clockwise rotation), see Table 3-1 as an example for the interest rate scenarios for the interest balance sheet of Banco Real[2]. The earnings risk scenarios reflect a gradual move of the interest rate over the course of one year. The NII on the stress scenarios will be compared to the base case scenario. A higher variance in these results corresponds to the higher interest rate risk within the balance sheet.

Table 3-1: Earnings risk scenarios.

Scenario	Magnitude (b.p.)	Rate scenario
Most likely		forecasted scenario

[2] The scenarios given in Table 3.1 and Table 3.2 are used to calculate the earnings risk and the market value risk on the balance sheets of Banco Real (reported in Standard ALCO Report – part 1, Interest income and interest rate risk, as of December 2006).

Base case		no changes in interest rates
Ramp up	+1100	gradual increase in the first year
Ramp down	−800	gradual decrease in the first year
Clockwise rotation	+550/−550	gradual increase and decrease for short and long term, respectively
Counter clockwise	− 550/+550	gradual decrease and increase for short and long term, respectively

This earnings risk shows the percentage change to the NII under the predefined stress scenarios and a time horizon of 12, and 24 months. The predefined stress scenarios reflect a gradual move of the interest rate over the course of one year.

3.2.2 Market value perspective

The market value perspective focuses on the potential impact of interest rate movements on the market value of the assets, the liabilities, and the off-balance sheet instruments of the bank. With help of the market value perspective, the impact on the value of future cash flows is included based on the balance sheet's structure. Therefore, it gives insight into the potential long-term effects of the interest rate movements. However, no value is attached to future developments such as new productions.

The market value of equity (MVE) is the main metric for the market value perspective. It is the difference between the net present value of all cash flows from assets and the net present value of all cash flows from liabilities plus the net present value of the off-balance sheet instruments. The cash flows are valued using the market rates, which are considered as alternative investment rates for the asset positions, and alternative borrowing costs for the liability positions.

This market value of equity is compared to the market value of equity in the situation of increasing and decreasing rates as a measure of risk. For example, the possible interest rate shock scenarios for the interest balance sheet of Banco Real are given in Table 3-2. There is a sudden shock instead of a gradual movement. The intention is to manage the balance sheet in order to limit these risks.

Table 3-2: Market value risk scenarios.

Scenario	Magnitude (b.p.)	Rate scenario
Base case		No changes in interest rate
Rates rise	+320	Parallel shock
Rates fall	−230	Parallel shock
Clockwise rotation	+160/−160	Shock
Counter clockwise	−160/+160	Shock

The market value risk shows the sensitivity of that market value of equity to changes in interest rates, specified by PV25, the effective duration, and several predefined scenarios. The PV25 is defined as the absolute sensitivity of the market value of equity to changes in rates using a shock of 25 basis points. The effective duration is defined as the relative sensitivity of the market value of equity to changes in rates using a shock of 100 basis points. The effective duration gives the indication of the average duration of the interest balance sheet.

3.3 Sources of interest rate risk

As intermediaries of financial institutions, banks encounter interest rate risks in several ways. Several sources of interest rate risk are described below.

3.3.1 Repricing Risk

The repricing risk arises if there are differences in the maturities (for fixed rate) and repricing (for floating rate) of bank assets, liabilities, and off-balance sheet positions. Repricing is any occasion on which interest rates are to be reset. This can expose the bank's earnings and market value to uncertain interest rate movements in the sense of changes in the yield curve level.

As an illustration, consider a balance sheet Table 3-3 consisting of both asset and liability sides. The asset side includes a five-year fixed rate consumer loans and the liability side includes a one-year saving deposits with a book value of EUR 1000. This can be interpreted as follows; a bank funds a long-term fixed rate loans with a short-term deposit. The net interest income is equal to 1000(5% - 3%) = EUR 20.

Table 3-3: An example of a balance sheet.

Asset	**Balance sheet (starting point)**		**Liability**
	Book Value		Book Value
5% loans (5 year)	1000	3% deposits (1 year)	1000

When the interest rates increase with 1% after 1 year, the earnings would be negatively affected. The NII would be equal to 1000(5%-4%) = EUR 10. The decline in the bank's earnings is caused by the fixed rate loans over its lifetime and increasing rate on the deposits that leads to higher expenses on the liability side. This illustrates the liability sensitivity which means that the liabilities reprice earlier. On the other hand, when the interest rate decreases with 1% the earnings would be higher since the rate on the deposits is decreasing that leads to lower expenses on the liability side. The NII would be 1000(5%-2%) = EUR 30.

Assume that the interest rate is 5% and 3% for long-term and short-term rates, respectively at the starting point, which makes the market values equal to the book values at the starting point. When the interest rates suddenly increase with 1%, the market value of equity would also be negatively affected. This is caused by the greater impact on the market value of the loans compared with the impact on the deposits, given the longer maturity of the loans. On the other hand, when the interest rates suddenly decrease with 1%, the market value of equity would be positively affected in this case. This is illustrated in Table 3-4 and Table 3-5.

Table 3-4: An illustration of how the MVE changes with a sudden increase in the interest rates.

Asset	**Balance sheet (interest rate +1%)**			**Liability**		
	Book Value	MV		Book Value	MV
5% loans (5 year)	1000	957.88	4% deposits (1 year)	1000	990.38

MVE = 957.88 –990.38 = -32.51

Table 3-5: An illustration of how the MVE changes with a sudden decrease in the interest rates.

Asset	**Balance sheet (interest rate -1%)**			**Liability**	
	Book Value	MV		Book Value	MV
5% loans (5 year)	1000	1044.52	2% deposits (1 year)	1000	1009.80
MVE =1044.52 –1009.80 =34.72					

3.3.2 Yield Curve Risk

The yield curve risk is one form of repricing risk. The yield curve risk arises when unanticipated changes in the slope and shape of the yield curve have adverse effects on the earnings and the underlying market value.

3.3.3 Basis Risk

The basis risk can occur when two offsetting positions with the same maturities but different indexes are taken. In other words, it defines the possibility of loss from imperfectly matched risk offsetting positions in two related but not identical markets. When the interest rates change, these differences can give increasing unexpected changes to the market value and the earnings spread between assets, liabilities, and off-balance sheet instruments of similar maturities or repricing frequencies. For example, a bank funds a one-year loan that reprices monthly based on the one-month U.S. Treasury Bill rate with a one-year deposit that reprices monthly based on the one-month LIBOR. This strategy exposes the bank to the risk that the spread between the two index rates may change unexpectedly.

4 Inflation-linked bonds

The overall increase in prices is called inflation. Inflation is considered as a major economic problem because increasing prices lead to less purchasing power. Inflation-linked bonds are designed to protect the purchasing power of an investor's savings by indexing the coupons and the principal payment to consumer prices.

This chapter is meant to give a clear overview of how inflation-linked bonds work (i.e., how inflation-linked bonds differ from the conventional bonds, i.e. how the cash flows of inflation-linked bonds are calculated). Further, the impact of the interest rates and inflation movements on the balance sheet (in which inflation-linked bonds are positioned) from an interest rate risk management point of view is given.

4.1 How do inflation-linked bonds differ from conventional bonds?

4.1.1 Conventional bonds

The conventional fixed rate bonds promise fixed payments of interest and the principal which are not adjusted to inflation. The purchasing power of future payments is not known because of the unknown future inflation. The purchasing power of the bond's payment is defined as the basket of goods and services it can buy. Therefore, both the purchaser and the issuer of the conventional bond face inflation risk, the risk of unanticipated changes in purchasing power of the principal and interest payments promised by the bond.

As an illustration, consider a one-year EUR 1,000 bond with a 5% annual coupon payment. At maturity, this bond will pay EUR 1,050 to the purchaser. The purchasing power of the payment depends on what happens to the prices. Suppose the **expected inflation** is 3% over the year. This would mean that while the purchaser gets EUR 1,050 at the end of the year, the price of something that costs EUR 1,000 at the beginning of the year would cost EUR 1,030 at the end of the year. Thus, the expected extra purchasing power at the end of the year is EUR 20; this corresponds to a real rate of return of 1.94% (= (1,050 – 1,030) / 1,030).

Suppose that the inflation turns out to be higher than the expected inflation, let us say 5%. This would lead to zero extra purchasing power at the end of the year because the price of something that costs EUR 1,000 at the beginning of the year increases to EUR 1,050. The higher inflation rate eliminates the expected extra purchasing power. On the contrary, if the inflation turns out to be lower than the expected inflation, for instance 1%, the extra purchasing power will rise.

4.1.2 Inflation-linked bonds

With an inflation-linked bond, the fixed rate is pre-specified in advance, and the principal amount and future coupon payments will be adjusted to keep in line with an index, for instance a consumer price index, which is a measure of inflation over the lifetime of the bond. Hence, inflation-linked bonds would offset some of the negative effects of increasing interest rates. The coupon rates for conventional bonds are generally higher than the coupon rates for inflation-linked bonds. This can be seen as the premium paid for the inflation protection.

Consider a one-year EUR 1,000 inflation-linked bond that pays a fixed annual coupon of 1.94% (which was the expected real rate of return of the conventional bond) on

the principal. But in this case, the principal will be adjusted each year to keep in line with the consumer price index. Suppose the expected inflation is again 3% over the year, the principal will be adjusted to EUR 1,030 and the coupon will be calculated based on this new principal amount. At maturity, the investor will receive a total amount of EUR 1,050 (coupon amount of EUR 20). In this case, the real rate of return is 1.94% (= (1,050 – 1,030) / 1,030).

If the inflation turns out to be 5%, the principal will be adjusted to EUR 1,050 and the coupon will be 1.94% on this new principal amount, which is EUR 20.37. At maturity, the investor will receive a total amount of EUR 1,070.37. An inflation of 5% means that the price of something that costs EUR 1,000 at the beginning of the year increases to EUR 1,050. The extra purchasing power at the end of the year is EUR 20.37 (=1,070.37-1,050) that corresponds to the real rate of return of 1.94%.

The above examples illustrate that inflation-linked bonds protect the purchasing power of an investor's savings by indexing the coupon and the principal payment to consumer prices. It makes sure that the purchasing power of the pre-specified fixed interest payment will not be eroded by inflation.

There is a situation when inflation-linked bonds will underperform relative to conventional bonds. In case of the mentioned investments, when the inflation turns out to be 1%, the real rate of return for the inflation-linked bond remains at 1.94% while the rate of return for the conventional bond will rise to 3.96%[3].

In case of the mentioned investments, a one-year inflation-linked bond with a 1.94% fixed coupon rate and a one-year conventional bond with a 5% fixed coupon rate, the real rate of returns of both investments are the same when the inflation turns out to be 3%. This is usually called the break-even inflation. If the inflation is higher than the break-even inflation, inflation-linked bonds will outperform relative to the conventional bond. On the contrary, when the inflation is lower than the break-even inflation, the inflation-linked bonds will underperform relative to the conventional bond.

4.2 A simple balance sheet using inflation-linked bonds

The primary risk of instruments such as bonds is the change in market price due to changes in the interest rate. There are several sources of interest rate risks. In highly inflationary economies, the real interest rate risk and the inflation risk are some of the main sources of interest rate risk. The changes in the interest rate can be explained by changes in either the real interest rate or changes in the inflation.

An increase in inflation will cause less purchasing power of future payments of a conventional bond. On the other hand, as described in Section 4.1.2, an increase in the inflation leads to an increase in the return of the inflation-linked bond and thus offsets some of the negative effects of the increasing interest rate. To see what the impact of the interest rate and inflation movements is on the balance sheet from an ALM point of view, a simple balance sheet is introduced in which inflation-linked bonds are positioned at the asset side.

[3] The calculation of the real rate of return for the conventional bond is as follows.

$$\frac{(1{,}000 \times 1.05) - (1{,}000 \times 1.01)}{1{,}000 \times 1.01} = 3.96\%$$

Consider a two-year EUR 1,000 inflation-linked bond with a 10% fixed coupon rate on the asset side and a one-year EUR 1,000 conventional bond with an 8% fixed coupon rate on the liability side. As a new production at the end of year one, a one-year EUR 1,000 conventional bond with a 10% fixed coupon rate is positioned on the liability side. This new production is included when analyzing the earnings risk, but it will not be taken into account when analyzing the market value risk.

Assume that the nominal interest rate at the settlement date is 10% and that the yield curve will remain **flat**. Also assume that there is a linear relationship between the inflation and the **one-year** nominal interest rates. The relation is defined as follows.

Equation 4-1

$$\text{Inflation} = \beta \times \text{interest rate} + c$$

The coupon is compounded on a monthly basis and paid out on a semi-annual basis. The compounded coupon refers to the fact that whenever the coupon payment is calculated, it is based not only on the indexed principal but also on any unpaid coupons that have been added to the indexed principal. The indexed principal is repaid at maturity. The effect of indexation of the principal is taken into the NII on a monthly basis. See Appendix B for illustration.

4.2.1 Cumulative Earnings risk

The main metric of the earnings risk is the net interest income. This earnings risk shows the percentage change to the NII under the predefined stress scenarios and a time horizon of 12, and 24 months. As explained in Section 3.2.1, the earnings risk is defined as the relative change of the NII on the stress scenarios compared to the NII on the base case scenario.

The relative change of the NII is obtained by calculating the difference between the cumulative NII on the stress scenario compared to the cumulative NII on the base case scenario and that divided by the cumulative NII on the base case scenario. The predefined stress scenarios reflect a gradual move of the interest rate over the course of one year. The stress scenarios are defined in terms of the interest rate and the inflation.

The interest rate scenarios used to generate the earnings risk for the simple balance sheet are

- Ramp up, a gradual increase in the interest rate in the first year with 200 b.p.
- Ramp down, a gradual decrease in the interest rate in the first year with 200 b.p.

Assume that the current inflation is 5%. The parameters that describe the relation between the inflation and the interest rate are summarized in the table below.

Table 4-1: The parameters that describe the relation between the inflation and the interest rate.

Scenario	c	β
1	15.00%	-1
2	12.50%	-0.75
3	10.00%	-0.5
4	7.50%	-0.25
5	5.00%	0
6	2.50%	0.25

7	0.00%	0.5
8	-2.50%	0.75
9	-5.00%	1

The earnings risks on the different inflation scenarios are depicted in Figure 4-1 for the ramp up scenarios and in Figure 4-2 for the ramp down scenarios. It is observed that the NII remains the same when there is a gradual move of the interest rate at which the inflation remains the same, in other words, when there is no relation between the inflation and the interest rate (β =0). As the relation of the inflation and the interest rate becomes stronger, that corresponds to the greater absolute value of β , the relative change is greater. This can be explained by the fact that the coupons are calculated on the indexed principal and the effect of indexation of the principal is taken into the NII on a monthly basis. A higher volatility in these results corresponds to the higher interest rate risk within the balance sheet. In the ramp up scenarios, a positive β corresponds to a positive relative change in the NII and a negative β corresponds to a negative relative change in the NII. In the case of ramp down scenarios, it is exactly the other way around.

Figure 4-1: Earnings risk in ramp up scenarios (200 b.p.).

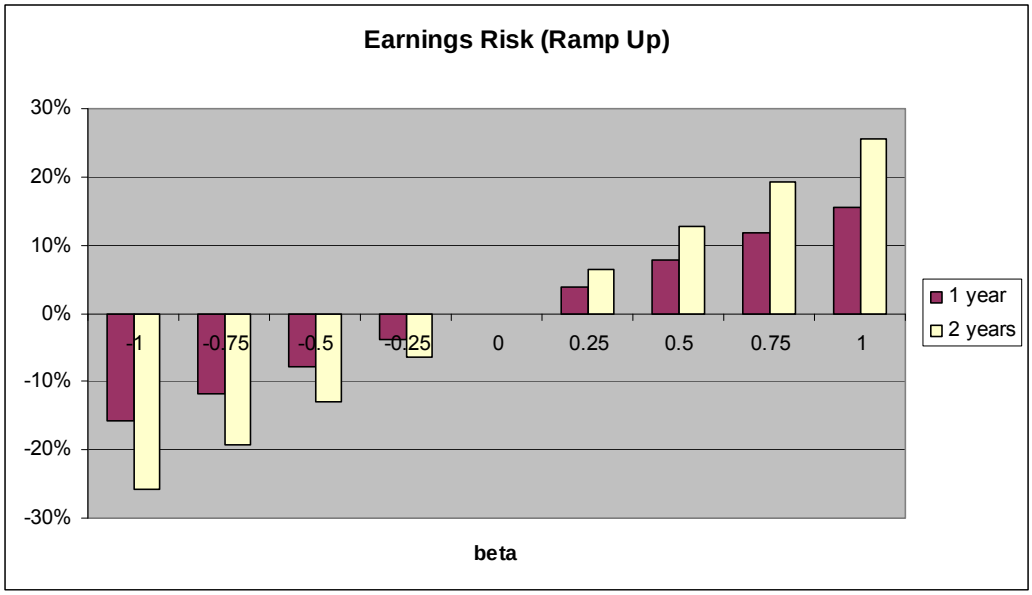
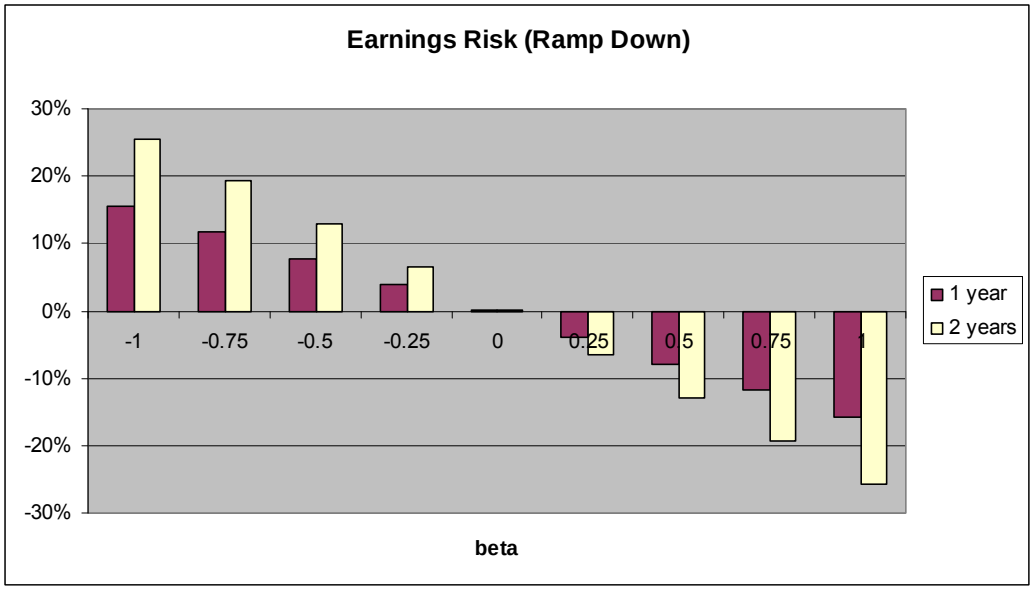


Figure 4-2: Earnings risk in ramp down scenarios (200 b.p.).



4.2.2 Market value risk

The market value of equity (MVE) is the main metric for the market value perspective. In the case of the simple balance sheet, the market value of equity is the difference between the net present value of all cash flows from assets and the net present value of all cash flows from liabilities.

The market value risk shows the sensitivity of the market value of equity to changes in interest rates. This is calculated by comparing the market value of equity on the base case situation to the market value of equity in the situation of an increasing and decreasing rate as a measure of risk.

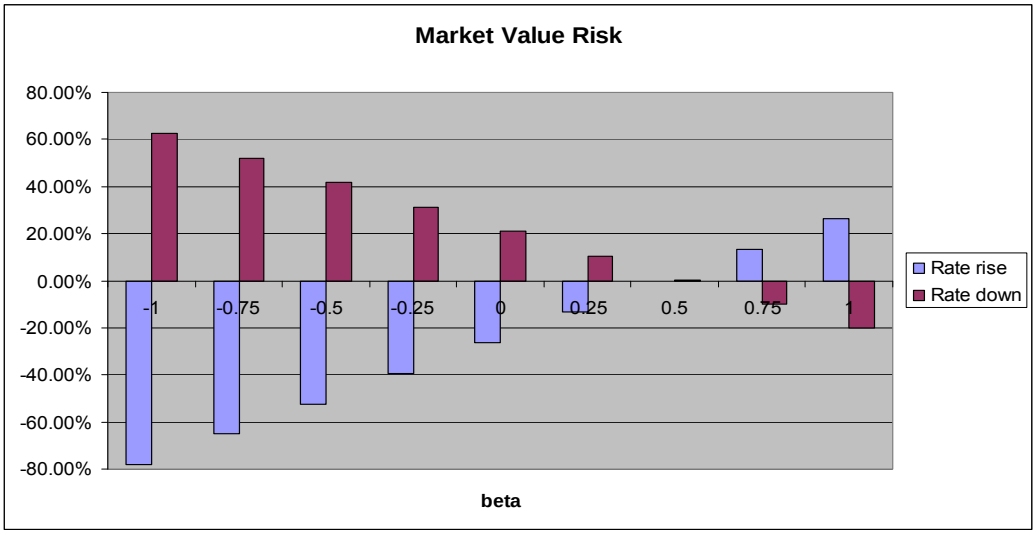
The interest rate scenarios used to generate the market value risk are

- Rate rise, a shock increase in the interest rate with 320 b.p.
- Rate down, a shock decrease in the interest rate with 230 b.p.

The inflation scenarios used to generate the market value risk are the same as the inflation scenarios used to generate the earnings risk (see Table 4-1).

The results of the market value risk are depicted in Figure 4-3. Note that changes in the interest rate and the inflation affect the market value of equity because of two changes; changes in the cash flows and the discount factor. The inflation movement has impact on the cash flows since the coupons are calculated on the indexed principal while the interest rate movement has impact on the discount factors. When there is a positive relationship between the inflation and the interest rate, the cash flows are greater and the discount factors become less as the interest rate increases. Given certain strength of correlation, the different impacts on the cash flows and the discount factors can stabilize the market value. This is the case in the simple balance sheet when β =0.5; the market value risk is -0.16% in the rate rise scenario. The greater cash flows are offset by a smaller discount factor that leads to a small change of the market value.

Figure 4-3: Market value risk.



4.2.3 Conclusion

Based on this simple example, it is observed that a change in the interest rate and the inflation may impact the earnings risk and the market value risk in different ways. It is observed that in the situation of an increasing interest rate, there is a positive impact on the market value and the earnings when there is a high positive correlation between the inflation and the nominal interest rate. In the situation where the interest rate decreases, there is positive impact on the market value and the earnings when the inflation and the nominal interest rate are negatively correlated.

There are several assumptions made regarding the inflation development. It is assumed that there is a linear relationship between inflation and the nominal interest rate, and the one-year interest rate is chosen to be linked with the inflation. In the further chapters, the relationship between the inflation and the interest rate will be studied.

5 Consideration of integrating the inflation-linked bonds in the ALM framework

As mentioned in Chapter 2, the most likely scenario for Brazil predicts a gradually decreasing interest rate to a significantly lower level and the inflation will remain stable. With such a scenario it is advantageous to lock in the current interest rate by investing in long-term securities such as bonds. In other words, investing in the long-term bonds would give an opportunity to secure the interest rate during the lifetime of the bonds. It is known that there is an inverse relationship between interest rate and the market value of the bond; as the interest rate decreases, the market value of the bond will increase. Investing in long-term securities corresponds to increasing the duration of the balance sheet. However, the risk still exists that a crisis, a sudden upward shock in the interest rates that occurred in the past, might occur again in the future. Like many other emerging market economies, Brazil has suffered a series of major external financial shocks since the mid 90's. Holding a long-term position during a crisis situation would have an adverse effect and investing in long term bonds are subject to the greater interest rate risk than investing in short-term bonds. In the previous crisis situation in Brazil, the sudden upward shock in the interest rates was accompanied by an increase in the inflation.

The primary risks of instruments such as bonds are changes in the market value and the earnings due to changes in the interest rate. As described above, there are several sources of interest rate risks. In highly inflationary economies, the real interest rate risk and the inflation risk are some of the main sources of interest rate risks. The changes in the interest rate can be caused by the changes in either the real interest rate or changes in the (expected) inflation. With an inflation-linked bond, the fixed rate is pre-specified in advance, and the principal amount and future coupon payments are adjusted to keep in line with the inflation realized over the lifetime of the bond. In other words, an increase in the inflation leads to an increase in the coupon and principal payments. Therefore, inflation-linked bonds would offset some of the negative effects of the increasing interest rates when it is accompanied with an increasing inflation. Investing in inflation-linked bonds might then be a good investment since it provides a protection against the possible upward shock in the interest rate and inflation[4] (see Chapter 4).

On the other hand, there are situations when investing in instruments other than inflation-linked bonds would be a cheaper alternative. Consider a situation at which the most likely scenario applies and the inflation develops as expected or is lower than expected (there is no crisis situation). In this situation, the real rate of returns of the conventional bond would be higher than the real rate of returns of the inflation-linked bonds.

Another situation that might also occur is a crisis situation with an interest rate shock but with a stable inflation. Investing in inflation-linked bonds in this kind of situation, the coupon and the principal payments would not be increased since the inflation remains stable. Therefore, inflation-linked bonds would provide no protection in this kind of situation.

[4] Investing in inflation-linked bonds is one of few alternatives to be able to invest over a longer term.

6 Leading indicators for explaining inflation

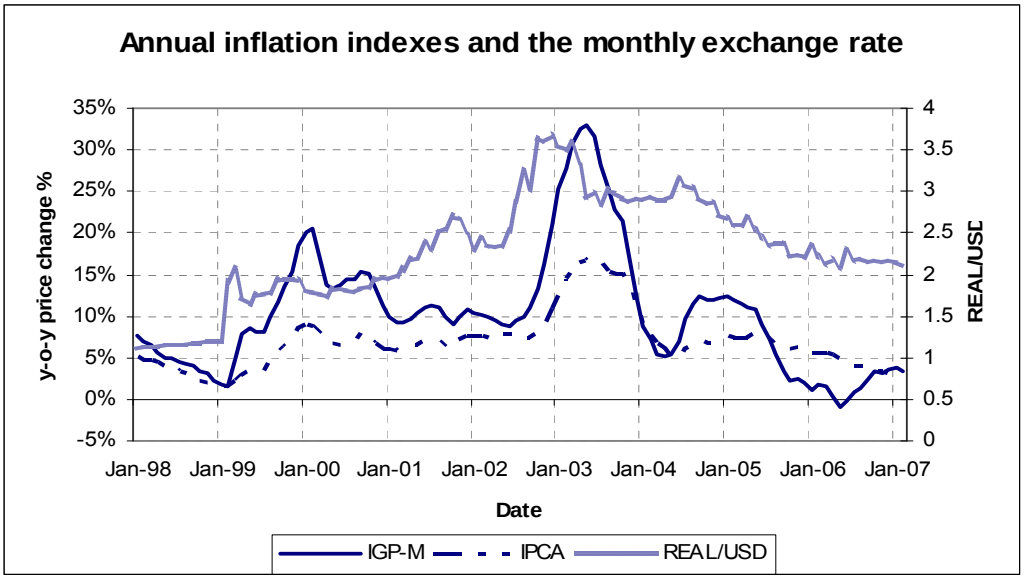
The main objective of this project is integrating inflation-linked bonds in the ALM metrics. To make this possible, the relationship between the inflation, the nominal interest rate, and maybe other possible macro-economic variables has to be studied. Macro-economics is the field of economics that studies the behavior of the aggregate economy. In this field, all variables are related to each other. This section includes the analysis of the possible leading indicators to explain the inflation development in Brazil, especially in the crisis situation, with respect to macro-economic theories and the business environment in Brazil.

The data that are used for further analysis are also introduced in this section. The data are time series data, which is a sequence of data points, measured at successive time. In this study, the monthly data are used to look at possible relationships between inflation and the interest rates. Certain data sets are published in a daily interval exclusive weekends and holidays; therefore the average within a month is taken in order to obtain monthly data. Given that the floating exchange rates regime in Brazil is conducted since January 1999, it does not seem reasonable to use the historical data before that. Therefore, the data period is from January 2000 until April 2007.

6.1 Exchange rates

As mentioned in Chapter 2, the crisis periods are noticed by the depreciation of the Brazilian currency Real with respect to the US dollar. This weakening of the Real was accompanied by a high inflation. The annual inflation indexes and the nominal exchange rate are given in Figure 6-1.

Figure 6-1: Annual inflation indexes and the monthly exchange rate.



There exists a dual causality between the exchange rate and the inflation. This means that the exchange rate would affect the inflation and the inflation would affect the

exchange rate in return. Before getting into that, two types of exchange rates and the relation between them are described below.

Exchange rates are distinguished between two types; the nominal and the real exchange rates. The nominal exchange rate is the relative price of the currency of two countries. When the nominal exchange rate between the Brazilian Real and the US dollar is 2.10, this means that one dollar can be exchanged for 2.10 Reais. The real exchange rate is the relative price of the goods of two countries. In other words, it gives information about the rate at which the goods of Brazil can be traded for the goods of the US.

As an illustration, consider a single good X produced in Brazil and the US. Suppose good X costs P_f dollars in the US and a similar good X costs P Reais in Brazil. To compare the prices of these two goods, the prices would be converted into the same currency. If a dollar is worth FX Reais (this is the nominal exchange rate between the Real and the US dollar), then the good X in the US would cost $FX \times P_f$ Reais. The real exchange rate is the ratio of the price of the American good X and the price of the Brazilian good X. This gives information about the purchasing power of the currency. Thus, the relation between the real exchange rate (FX_r) and the nominal exchange rate (FX) can be expressed by the following equation

$$FX_r = \frac{FX \times P_f}{P}, \qquad \textbf{(6-1)}$$

or

$$FX = \frac{FX_r \times P}{P_f}, \qquad \textbf{(6-2)}$$

where FX is the nominal exchange rate (the number of Reais per US dollar), FX_r is the real exchange rate, P the domestic price index (the Brazilian price index) and P_f the foreign price index (the US price index). The relation between these exchange rates can be restated in terms of rate changes,

$$\frac{\Delta FX}{FX} = \frac{\Delta FX_r}{FX_r} + \frac{\Delta P}{P} - \frac{\Delta P_f}{P_f}. \qquad \textbf{(6-3)}$$

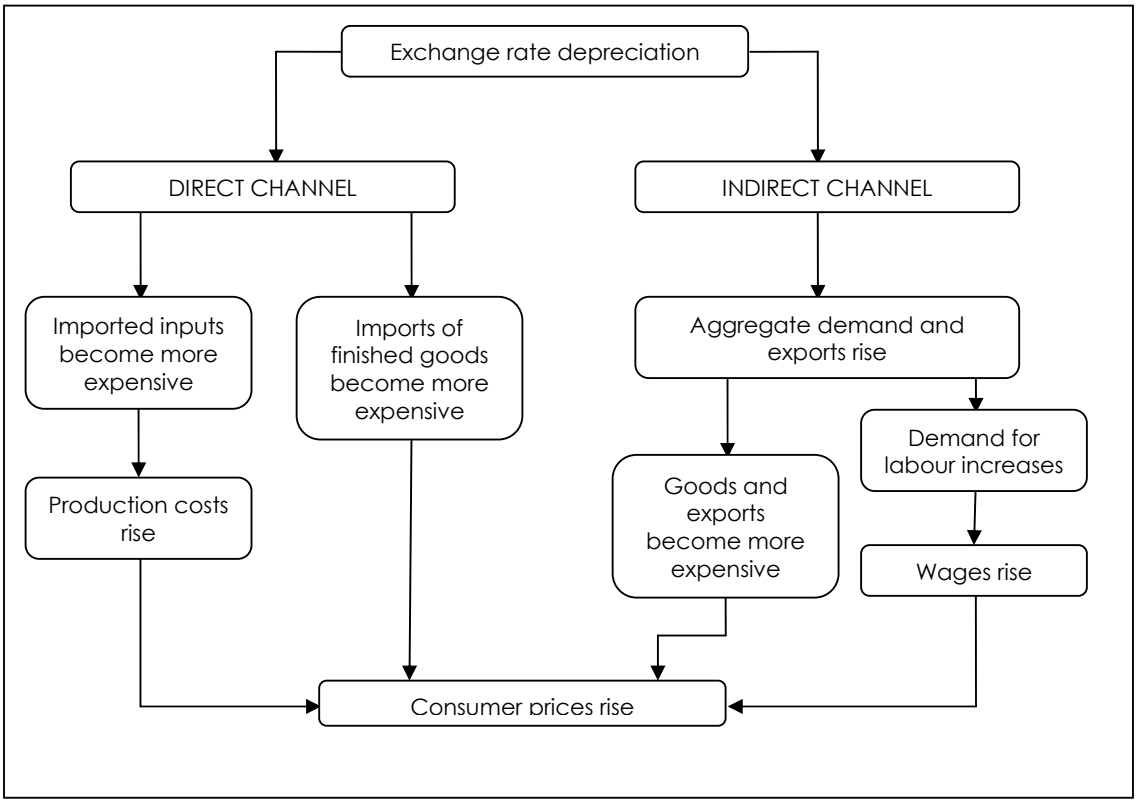
The percentage change in P and P_f are the inflation rates in Brazil and the US, respectively. This equation states that the percentage change in the nominal exchange rate can be related to the percentage change in the real exchange rate and the inflation rates in both countries.

According to theoretical considerations, there are two channels through which the exchange rate can affect the inflation. The first channel is the direct channel through the price of imports. If the nominal exchange rate in terms of Brazilian currency per unit of US dollar depreciates, which corresponds to more Reais to be exchanged for one US dollar, then the Real price of the imported goods will increase. The change in nominal exchange rate affects not only prices of imported goods, but also the price of domestic goods which are either under competitive pressure from imported goods or whose inputs are imported which leads to the higher production costs.

The second channel is the indirect channel through the competitiveness of international markets. When the domestic currency (Real) depreciates, then the domestic goods become relatively cheaper for foreign buyers which lead to an increase in exports and aggregate demand. This induces an increase in the domestic price level. This also induces an increase in the demand for labor which leads to an

increase in wages. The increase in wage increases the overall price level. The mechanism of direct and indirect exchange rate effects is displayed in Figure 6-2.

Figure 6-2: The mechanism of direct and indirect exchange rate effects on inflation.



On the other hand, the effect of inflation on the nominal exchange rate can be explained by using the theory called "The Law of one Price" that states that the same goods cannot sell for different prices in different locations at the same time. The law of one price applied to the international marketplace is called the purchasing power parity (PPP). The purchasing power parity hypothesis states that the nominal exchange rate will adjust for differences in the domestic and foreign inflation rates, which implies a constant real exchange rate (see Equation **(6-3)**). A more rapid inflation in Brazil relative to the US would cause weakening of the Real.

6.2 Interest rates

6.2.1 Why interest rate changes?

Interest rates are important variables for macro-economics to understand because they link the economy of the present and the economy in the future through their effects on savings and investments. To understand why the interest rates change, it is necessary to distinguish them.

Interest is the fee paid on money for the borrower. As for the lender, it is a compensation for lending the money. There are two definitions of interest rates; the **nominal** and the **real** interest rate. The nominal interest rate includes compensation for the lender's lost value due to inflation, whereas the real interest rate excludes inflation. The **Fisher effect** describes the link between the nominal interest rates and the expected inflation. It states that the real interest rate is the nominal interest rate minus the expected inflation rate.

Interest rates are determined by various markets. This depends on the length of the maturities of the securities. The short-term interest rates are determined by the money market. In the case of Brazil, the short-term interest rates except the Selic rate are determined by the supply and demand between banks. The Selic rate is set by the Central bank and it influences the market for other short term securities.

The long-term interest rates are determined by the capital market. They tend to move in anticipation of changes in the economy and inflation.

In the case of the real interest rate, it depends on the demand (investments) and the supply (savings) from the private sector. It varies to make sure that the total private investments and the total savings are in equilibrium. In a period when many people are saving money, in the sense that there is more supply than demand, any investment will get special rate discounts to be very competitive, keeping the real rates low. On the contrary, when there is more demand than supply, the real interest rate would be higher.

6.2.2 Dual causality between the inflation and the interest rate

There exists a dual causality relationship between inflation and interest rates. This means that the inflation would affect the interest rate and the interest rate would affect the inflation in return. The effect of the inflation on interest rates can be explained by the monetary policy. As mentioned earlier, the Selic rate is the indicator of the Brazil's Central bank to control inflation. If the inflation starts to rise, the Central bank may raise the Selic rate in order to dampen the inflationary pressure and the whole term structure will be influenced. The Selic rate would affect the inflation expectations that would influence the long-term rates. With certain lags, it would affect the effective inflation. On the other hand, if the inflation is low the Central Bank may decrease the Selic rate to stimulate economic growth.

The effect of the interest rate on inflation can be explained by the law of supply and demand. A high interest rate corresponds to a high cost for borrowing. On the other hand, a high interest rate means that savings offer high returns. This situation would induce people to have more savings than borrowing which would lead to less consumption and demand. Less demand would decrease the price level, which corresponds to a decreasing inflation.

6.2.3 Term structure of the interest rates

This section gives an interpretation of the term structure of the interest rates. The term structure of the interest rates, also known as the yield curve, is the relation between the interest rate and the time to maturity of the securities. It describes the relationship between the short-term, the medium-term, and the long-term rates at a given point in time. Based on the expectation theory, the yield curve is a measure of the market expectations about the future interest rates given the current market conditions.

Various macro-economic variables affect the shape of the yield curve, including the expectation of economic growth and the inflation. There are several shapes of the yield curve; **normal**, **steep**, **flat**, and **inverted**.

A **normal** yield curve is upward-sloping, of which the short end of the curve is lower than the long end of the curve. This reflects the risk premium associated with the credit risk and the temporary loss of liquidity. The former is referred to as the credit premium and the latter is referred to as the liquidity premium.

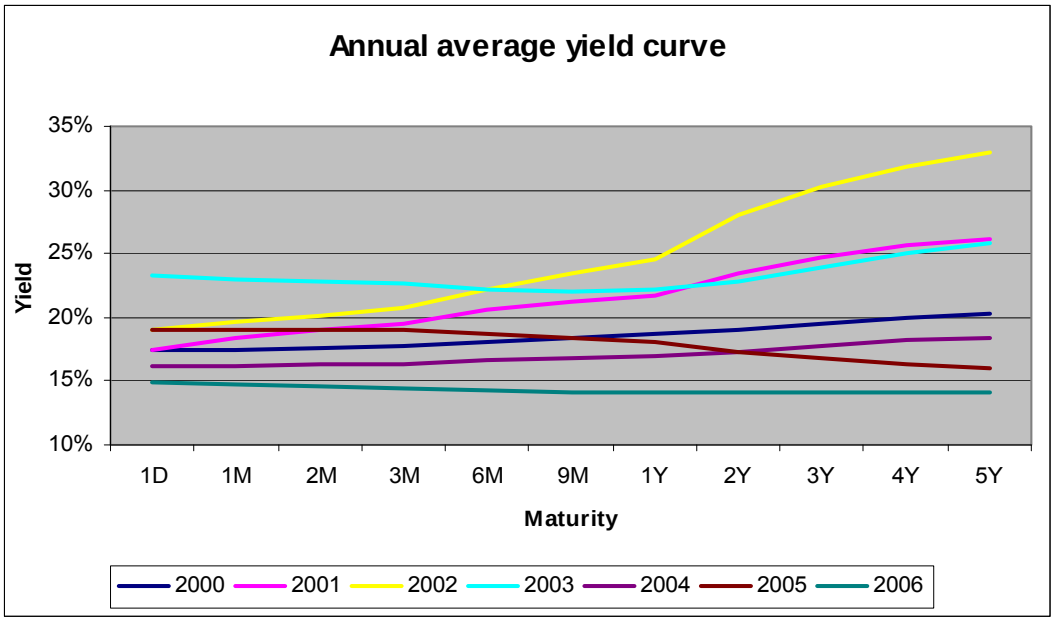
A **steep** yield curve is also upward-sloping, of which the long end of the curve is much higher then the short end of the curve. This strong positive slope reflects the expectations for the economy to grow in the future and this growth is associated with an expectation that the inflation will rise in the future. This expectation of higher inflation leads to expectations that the Central bank will tighten the monetary policy by raising short-term interest rates in the future to slow down the economic growth and thus dampen the inflationary pressure.

A **flat** yield curve is a yield curve in which the rate level from different maturities is more or less the same. A flat curve reflects the market's expectation that the interest rate may decrease, typically due to an economic slowdown. This concern offsets the risk premiums.

An **inverted** yield curve occurs when the short-term rates are higher than the long-term rates. This reflects the market's expectations that the decline in the economy will be extreme enough such that it cannot be offset by the risk premiums. The expectation of an economic decline is associated with the expectation of a lower inflation and future short interest rate (see more explanation at Section 6.2.2). This expectation of a lower inflation leads to expectations that the Central bank will loosen its monetary policy by reducing the short-term interest rates in the future.

The annual average yield curves from the year 2000 until the year 2006 in Brazil are plotted below (see Figure 6-3). The shape of the yield curves reflects the economic situation for each year. Because of the market liquidity in Brazil, the longest term yield related in the yield curve is a five-year yield to maturity. As noted earlier, the shape of the yield curve is influenced partly by the market's economic outlook based on the current situation. As it is observed, the shape of the yield curve of the year 2000 is normal compared to the shape of the yield curve of the years 2001and 2002. This can be explained by the crisis situation in 2001 and 2002 (see Chapter 2). For the year 2003, a sort of a flat curve is depicted. This can be explained by the fact that the inflation rate has been lowered and that led to an expectation of a lower interest rate. The shape of the yield curve depicted for the year 2004 is upward-sloping, but much less than the case of the years 2001and 2002. In case of the years 2005 and 2006, an inverted yield curve has been observed. This corresponds to the market's expectation of a lower inflation.

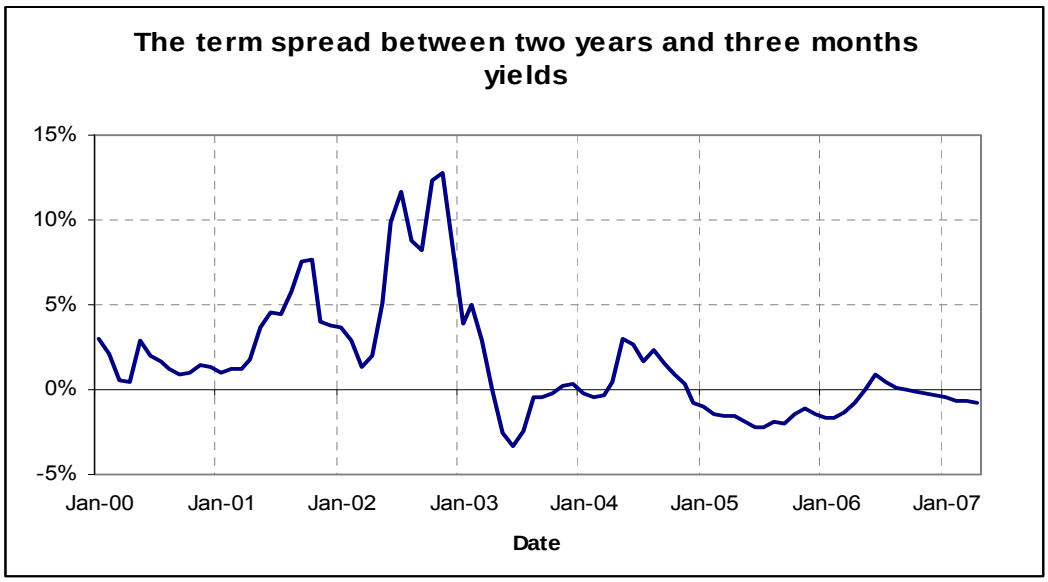
Figure 6-3: Annual average yield curve.



Source: ABN AMRO, BU Brazil.

As explained and observed above, the shape of the yield curve can be a good leading indicator for explaining the inflation development. *Cuaresma, Gnan and Ritzberger-Grünwald (2005)* **[2]** attempted to evaluate empirically the predictive power of the yield spread for euro area output and inflation. The term spread, i.e., the difference between the long-term (two year) and the short-term (three month) yield to maturities is given in Figure 6-4. As depicted, there is great positive difference between the long-term and the short-term yield to maturities around the crisis situation in the year 2001and 2002.

Figure 6-4: Annual inflation indexes and the difference between two-year and three-month yield to maturities.



Source: ABN AMRO, BU Brazil.

6.3 Emerging market bond index spread.

As mentioned in Section 2.3.1, the EMBI spread is the difference between the yield on a dollar-denominated bond issued by the Brazilian government and a corresponding one issued by the US Treasury. This variable is considered as a good indicator for country risk in Brazil.

Many macro-economic variables in Brazil are highly correlated with the EMBI spread, most importantly the exchange rates. A high credit risk of a country leads to the sudden stop of capital flows and to a depreciation of the currency needed to generate the trade surplus (a positive balance of trade, i.e., exports exceed imports) required to offset the decrease in capital flows. In turn, the exchange rate shock leads to a change in tradable good prices which corresponds to a high inflation (see Section 6.1). At the same time, a shock in the exchange rates would lead to a change in market expectations of the inflation. This induces the Central bank to increase the Selic rate.

The interest rates at longer maturities are also affected by the EMBI spread. This can be explained as follows. Indirectly, it is because the Selic rate moves the term structure. Directly, the domestic financial instruments of longer maturities are not immune from credit risk. A high EMBI spread indicates a high credit risk, which leads to a higher credit risk premium included in the interest rate. A steep yield curve would have been observed in this kind of situation.

7 Data analysis

Having introduced some possible indicators for explaining the inflation indexes in Brazil; the next step is a deeper analysis of the time series properties. With the results from this section, preliminary analysis in the subsequent section can be undertaken.

7.1 Stationarity

The concept of stationarity is essential in time series analysis because it can strongly influence the result of the analysis and it can be spurious. A stochastic process y_t is weakly stationary if the following conditions are valid.

$$\begin{aligned} E(y_t) &= \mu < \infty, \\ Var(y_t) &= \sigma^2 < \infty, \\ \operatorname{cov}(y_t, y_{t-k}) &= E\{(y_t - \mu)(y_{t-k} - \mu)\} = \gamma_k, k = 1,2,3,\ldots \end{aligned} \quad \textbf{(7-1)}$$

Weak stationary implies that the mean, variance and covariance of a process exist and are time-invariant. One example of a stationary process is a process that is characterized by the property that values at different time instants are uncorrelated, in the sense that $\operatorname{cov}(y_t, y_{t-k}) = 0$ for all $k \neq 0$. This process is called ***white noise***.

Many macro-economic variables are non-stationary, presenting trending or seasonal behavior. There are different types of non-stationary processes; e.g., the deterministic and the stochastic linear trend. One can consider a deterministic linear trend process as follows,

$$y_t = a + bt + \varepsilon_t, \quad \textbf{(7-2)}$$

where ε_t is a white noise term, that can be transformed into a stationary process by subtracting the trend $a + bt$. Another type of non stationarity is the stochastic linear trend as follows,

$$y_t = y_{t-1} + \varepsilon_t. \quad \textbf{(7-3)}$$

This is also called a ***random walk***. The presence of a deterministic trend in the explanatory variables does not raise any problem. But many economic time series are non-stationary due to the presence of a stochastic trend, or so called ***unit roots***. When non-stationary time series are used in a regression model one may obtain significant relationships from unrelated variables. This is called ***spurious regression***. The spurious regression can be signaled by a high goodness of fit of the regression, but with a presence of autocorrelation in the residuals from the regression analysis. For example, consider a regression model

$$y_t = \beta_0 + \beta_1 x_t + \varepsilon_t, \quad \textbf{(7-4)}$$

where y_t and x_t are unrelated random walks,

$$\begin{aligned} \Delta y_t &= y_t - y_{t-1} = \varepsilon_t, \\ \Delta x_t &= x_t - x_{t-1} = \varepsilon_t. \end{aligned} \quad \textbf{(7-5)}$$

Neither variable affects one another, so it would be expected that β_0, β_1 would tend significantly to zero and the goodness of fit R^2 would also tend to zero. But the null hypothesis of no relationship is not rejected along with very high R^2 and the presence

of autocorrelation in the residuals from the regression model. This can be explained by the existence of the random walk x_t in y_t which is also a random walk.

A stationary series is integrated of order 0. A series which becomes stationary after first differencing is said to be integrated of order one, denoted I(1).

7.2 Unit root test

Checking the stationarity of time series can be done by looking at the time series plot. Based on the plot, it can be observed whether the mean and variance are constant, and whether the shock in the time series has a temporary effect. A plot of its ***autocorrelation function*** (acf) will be also useful. The autocorrelation of a stationary time series should diminish rapidly as the lag length increases.

A formal unit root test can also be done to test whether the time series are stationary. One of the tests that can be used is the ***Dickey Fuller*** test. As an illustration, consider the following AR(1) model for time series $\{y_t\}$,

$$y_t = \gamma_1 y_{t-1} + \varepsilon_t , \qquad \textbf{(7-6)}$$

where ε_t is a white noise process, which is identically, independently distributed with mean zero and a constant variance of σ^2. The null hypothesis is $\gamma_1 = 1$, which corresponds to a presence of a unit root and the alternative hypothesis is $\gamma_1 < 1$. For practical reasons the testing regression is given by

$$\Delta y_t = (\gamma_1 - 1) y_{t-1} + \varepsilon_t = \beta y_{t-1} + \varepsilon_t . \qquad \textbf{(7-7)}$$

The latter equation can be obtained by subtracting y_{t-1} from both sides. This Dickey Fuller test uses the t-test. The null hypothesis in the Dickey Fuller test is then β=0. If the appropriate order of the AR model is more than one, the term $\{\Delta y_{t-1}\}$ should be added to the regression model. Consider an auto regression of order ***p***, AR(p), for time series $\{y_t\}$,

$$y_t = \gamma_1 y_{t-1} + \gamma_2 y_{t-2} + \ldots + \gamma_p y_{t-p} + \varepsilon_t . \qquad \textbf{(7-8)}$$

Then the regression model can be written as follows (see Appendix I),

$$\Delta y_t = \beta y_{t-1} + \alpha_1 \Delta y_{t-1} + \alpha_2 \Delta y_{t-2} + \ldots + \alpha_{p-1} \Delta y_{t-p+1} + \varepsilon_t , \qquad \textbf{(7-9)}$$

where

$$\beta = \left(\sum_{i=1}^{p} \gamma_i \right) - 1 , \qquad \textbf{(7-10)}$$

and

$$\alpha_j = -\sum_{k=j+1}^{p} \gamma_k . \qquad \textbf{(7-11)}$$

By adding the difference terms into the model, the t-test of this model is referred as the ***Augmented Dickey Fuller*** (adf) test. The test of unit root can be carried out in the same way as for the Dickey Fuller test. If β = 0, than there is a unit root.

The test was conducted for level and first difference data. In the table below, the Augmented Dickey Fuller (adf) test statistics have been computed for the examined

time series. The second column gives the optimal length of the lag order for the differenced terms. The third column gives the t- statistics and the fourth column gives the corresponding critical value at 5% significance level. The last column gives the summary of the previous columns; the null hypothesis that there is a unit root in the time series would be rejected if the corresponding t-statistic is smaller than the critical value. The appropriate length of the lag is determined by the ***Schwarz criterion***. Based on the observation, no time series show a clear long-term trend direction. It is decided to include only an intercept but to exclude a deterministic time trend from a regression model in the adf test.

Based on the adf test, most of the level data indeed have a unit root. However, the slope of the yield curve, the Selic rate, and both inflation indexes in terms of variation per month seem to be stationary according the adf test at a significance level of 5%. Further, the unit root test is applied for the first difference data. The null hypotheses of the presence of the unit root in all first difference data are rejected.

Table 7-1: ADF test statistics for level data.

	Level Data			
	Lag (SIC)	**t-statistics**	**5% CV**	**Unit root**
IGPMyoy	3	-2.775804	-2.8972	Yes
IPCAyoy	1	-2.396426	-2.8963	Yes
IGPMmom	1	-3.604264	-2.8963	No
IPCAmom	0	-4.149926	-2.8959	No
Slope	4	-2.989148	-2.8976	No
Selic	2	-3.299843	-2.8967	No
FX	0	-1.568732	-2.8959	Yes
EMBI	1	-1.935941	-2.8963	Yes
OneY	1	-1.613258	-2.8963	Yes
TwoY	1	-1.796258	-2.8963	Yes

Table 7-2: ADF test statistics for first difference data.

	1st Difference			
	Lag (SIC)	**t-statistics**	**5% CV**	**Unit root**
IGPMyoy	2	-2.95672	-2.8972	No
IPCAyoy	1	-4.172876	-2.8967	No
IGPMmom	1	-7.648452	-2.8967	No
IPCAmom	1	-8.910151	-2.8967	No
Slope	1	-8.113201	-2.8967	No
Selic	2	-3.477736	-2.8972	No
FX	1	-5.003812	-2.8967	No
EMBI	0	-5.025955	-2.8963	No
OneY	0	-7.027076	-2.8963	No
TwoY	0	-7.020739	-2.8963	No

8 Modeling

The usage of these models within the ALM framework is to explain the inflation development given the stress scenarios for the interest rates. Based on the usage of these models within the ALM framework, only one direction of causality between the inflation and the interest rate is of interest for this project, which is the effect of the interest rates on the inflation. Therefore, a single equation model will be estimated.

There were two types of models estimated for explaining the inflation indexes IGP-M and IPCA in the previous analysis. The first model was an Autoregressive Distributed Lag (ADL) model which is estimated based on the Ordinary Least Square (OLS) technique, and the second model was a Vector Error Correction (VEC) model in order to find the presence of a long-run relationship. In this chapter, further analysis will be done by building the ADL models with other explanatory variables. Based on the ADL models, Error Correction models can be derived to find the long-run effects as well as the short-run effects. This derivation will be described in Appendix C.

8.1 Autoregressive distributed lag models for IGP-M index

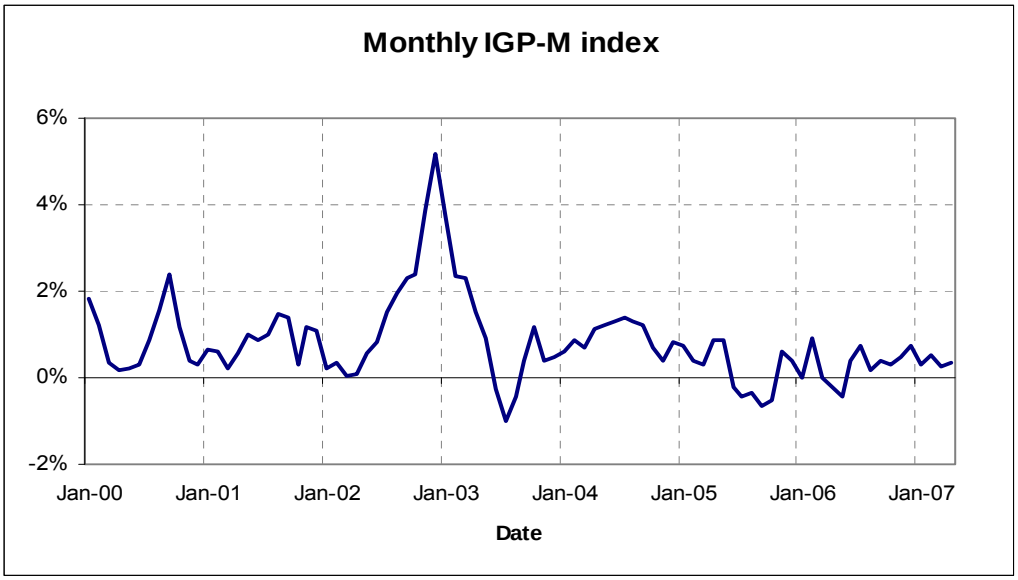
As mentioned in the previous section, most of the variables are non-stationary. Therefore, it is necessary to guard against the spurious relationship. One of the solutions for this is to difference the non-stationary variables to achieve stationarity and use them in that transformed form together with the other stationary variables. One disadvantage of this prescription is ignoring the long-run relations embodied in the level variables. Another way to build a model is to include the variable time lags which minimize the possibility of estimating spurious relations while retaining long-run information.

As observed in the time series data, there is a possibility of a linear relationship and variable time lags in the effects of inflation and interest rate variables. Besides including the explanatory variables at the current time value for explaining the inflation, it would be useful to add the lagged values of the explanatory variables in the regression model. This model is called a **distributed-lag** model. If the model also includes lagged values of the dependent variable, the inflation variable, among its explanatory variables, the model is called an **autoregressive distributed lag** model.

Based on the theoretical analysis as a background motivation for the research, various models have been estimated during the analysis. This modeling is estimated by using the 'general to specific' methodology; the unrestricted equation will be estimated using the Ordinary Least Square (OLS) technique and the lagged variables whose coefficients are statistically significant to zero, with significance level of 5%, will progressively dropped.

The time series data of both inflation indexes are available on monthly basis as well as on annual basis. The estimates of the models are based on the monthly inflation indexes. This choice is considered due to econometric reasons. As analyzed in the Section 7.2, both annual inflation time series have unit roots, while the monthly inflation times series are stationary. Making a regression model based on non-stationary variables can lead to spurious regression. The monthly IGP-M index that is used for regression is given in Figure 8-1.

Figure 8-1: The monthly IGP-M index.



For the IGP-M inflation index, three models will be presented in this section. The focus is on empirically assessing the out-of-sample forecasting abilities of the chosen explanatory variables for each inflation index. The presented models will then be compared to the benchmark model that is a simple autoregressive (AR) process, which is a natural benchmark of comparison when the predictive ability of other explanatory variables have to be evaluated.

8.1.1 The models for the IGP-M index

The sample used for estimating the models is from period January 2000 until April 2007. The estimates of the models for the monthly IGP-M index are given in the Table 8-1.

Table 8-1: Model estimation for IGP-M inflation index.

IGPM_ADL1				**IGPM_ADL2**			
Expl. Var	**Coefficient**	**t-Stat.**	**p-value**	**Expl. Var**	**Coefficient**	**t-Stat.**	**p-value**
(Intercept)	-0.007	-2.273	0.026	(Intercept)	0.002	3.070	0.003
$IGPMmom_{t-1}$	0.940	8.929	0.000	$IGPMmom_{t-1}$	0.723	7.163	0.000
$IGPMmom_{t-2}$	-0.330	-3.061	0.003	$IGPMmom_{t-2}$	-0.205	-2.314	0.023
$OneY_{t-1}$	0.050	3.023	0.003	$Slope_{t-1}$	0.114	6.039	0.000
R^2	0.720	Adj. R^2	0.706	R^2	0.785	Adj. R^2	0.774
IGPM_ADL3				**IGPM_Benchmark**			
Expl. Var	**Coefficient**	**t-Stat.**	**p-value**	**Expl. Var**	**Coefficient**	**t-Stat.**	**p-value**
$IGPMmom_{t-1}$	0.546	7.929	0.000	(Intercept)	0.002	2.330	0.022
$Slope_{t-1}$	0.069	2.981	0.004	$IGPMmom_{t-1}$	1.017	9.455	0.000
FX_{t-1}	0.013	3.533	0.001	$IGPMmom_{t-2}$	-0.247	-2.292	0.025
FX_{t-3}	-0.012	-3.320	0.001				
R^2	0.802	Adj. R^2	0.795	R^2	0.687	Adj. R^2	0.675

The coefficients included in the models are significantly different from zero (all the p-values are smaller than 5%). The first model, **IGPM_ADL1**, is the model that is estimated

from previous research. The model includes the first two lags of the dependent variable, an intercept, and the first lag of the one-year interest rate.

The second model (**IGPM_ADL2**) uses the first lag of the term spread, i.e., the difference of the two-year and the three-month interest rates, instead of the one-year interest rate. As explained in Section 6.2.3 based on the expectation theory, the shape of the yield curve reflects the expectation of the economy, which is associated with the expectation of the inflation and the effective inflation in the future. The model IGPM_ADL2 is better in explaining the monthly IGP-M than the model IGPM_ADL1 based on their goodness of fits. Note that the monthly inflation index is a percentage change of the inflation level in one month period. Thus, the relation explained by the model IGPM_ADL2 can be interpreted as follows; a change in the term spread of the previous month gives a positive change in the IGP-M inflation level. Every 1% (0.01 units) increase in the term spread gives effect on 0.114 units increase in the monthly IGP-M index, holding other explanatory variables fixed.

The third model, **IGPM_ADL3**, includes the time lags of exchange rates as explanatory variables. In economic term, the exchange rates variable has a great impact on the IGP-M index movement. A shock in the exchange rates would raise the prices on the tradable goods and raise the wholesale price index which is the most part of IGP-M index. Based on the statistical test, the one month and three months lags of the exchange rates give significant information in explaining the IGP-M index. The coefficients for the time lags of exchange rates are almost similar in absolute terms. If the Wald test (see Appendix F) is applied, the null hypothesis that these coefficients are similar in absolute terms would not be rejected at a significance level of 5%.

Note that the terms

$$0.013\ FX_{t-1} - 0.012\ FX_{t-3} \qquad \textbf{(8-1)}$$

can be formulated as

$$0.013\ (FX_{t-1} - FX_{t-2}) + 0.012\ (FX_{t-2} - FX_{t-3}) = 0.013\ \Delta FX_{t-1} + 0.012\ \Delta FX_{t-2}, \qquad \textbf{(8-2)}$$

for which the coefficient of FX_{t-2} is significantly equal to zero. Thus, the relation explained by the IGPM_ADL3 can be interpreted as follows; a change in the term spread of the previous month and the movements in the exchange rate in the previous month and two months before have a positive effect to a change in the inflation level. Thus, IGPM_ADL3 can be re-estimated with the sum of the 1st and 2nd lags of the first difference of the exchange rate, ($\Delta FX_{t-1} + \Delta FX_{t-2}$), as explanatory variable. The result of the new estimation is given in Table 8-2.

Table 8-2: The adjusted estimation model IGPM_ADL3 for IGP-M index.

IGPM_ADL3			
Expl. Var	**Coefficient**	**t-Stat.**	**p-value**
(Intercept)	0.002	3.293	0.002
$IGPMmom_{t-1}$	0.580	8.785	0.000
$Slope_{t-1}$	0.071	3.033	0.003
$\Delta FX_{t-1} + \Delta FX_{t-2}$	0.011	3.186	0.002
R^2	0.796	Adj. R^2	0.789

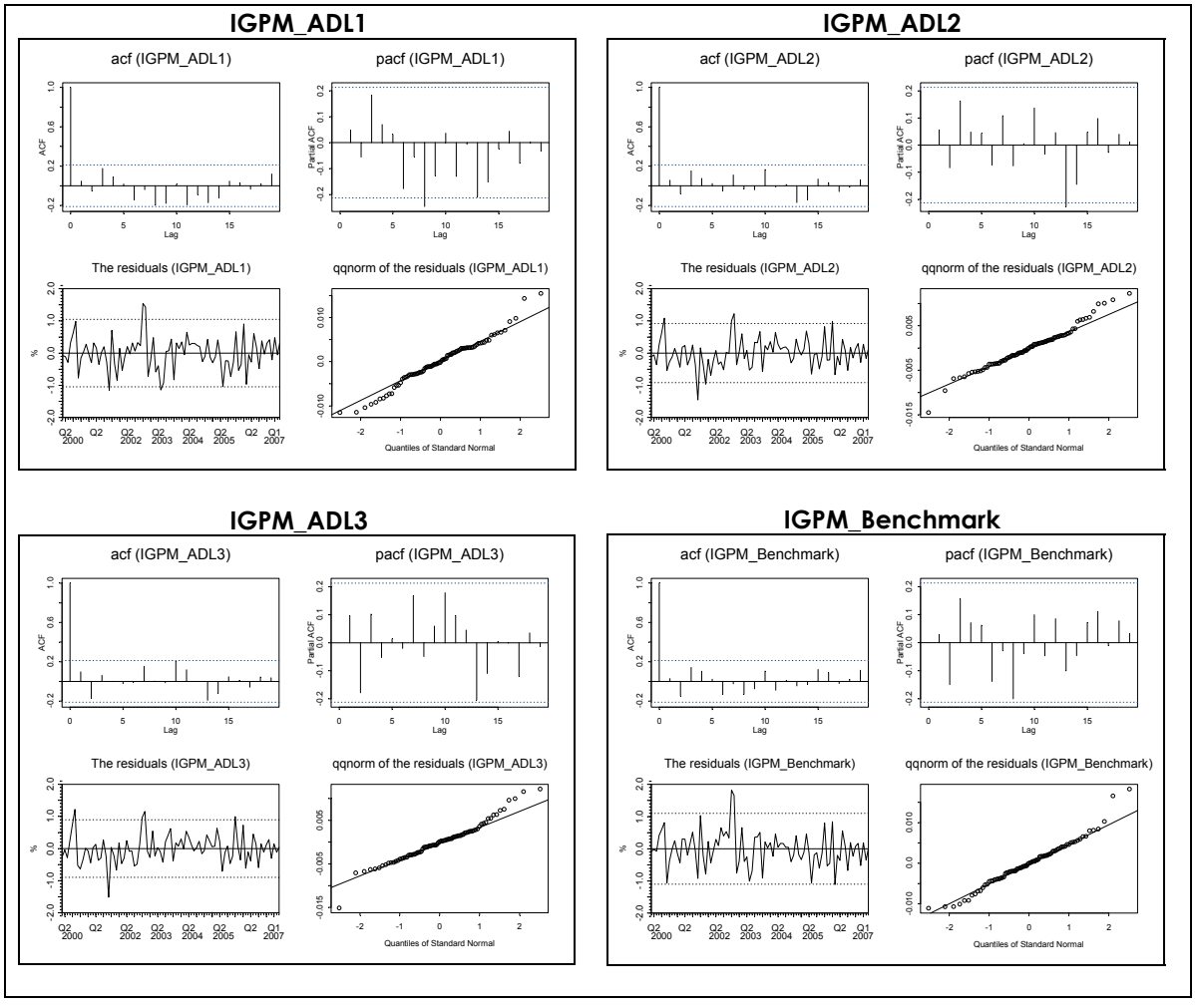
By adding the exchange rate time lags, the two-month lag of the IGP-M index becomes less obvious. This implies that the lags of the exchange rates give more fundamental information for the IGP-M index. The effect of the slope of the yield curve and the one-month lag of the IGP-M also become less, i.e., from 0.114 to 0.071 and from 0.730 to 0.580, respectively. This can be explained by the fact that the

exchange rates variable time lags absorb some of the slope and autoregressive effects. The model **IGPM_ADL3** gives a slightly higher adjusted R^2 than **IGPM_ADL2**.

Several plots are given below to assess the residuals obtained from each model. In Figure 8-2, the correlogram (the autocorrelation function and the partial autocorrelation function), the residuals itself, and the normal quantile plot of the residuals are given. The autocorrelation at lag *k* measures the correlation of the residuals time series with itself lagged *k* months. The partial autocorrelation at lag *k* is the autocorrelation at lag *k* after first removing the autocorrelation with an AR(*k-1*) model, in order to look for additional correlation that is not caused by the correlation at shorter lags.

The normal quantile plot is a scatterplot of the percentiles of the residuals versus the percentiles of random data that are normally distributed. If the resulting points fall closely along a straight line, the assumption that the residuals are normally distributed can be accepted. As observed in Figure 8-2, the autocorrelations of the residuals from all models are all within the 95% confidence interval which can be interpreted as zero autocorrelations. However, some partial autocorrelations are significantly non-zero. The assumption that the residuals are normally distributed can also be accepted based on the normal quantile plots. The normal quantile plot suggests that the residuals closely follow the normal distribution, except for large (absolute) values.

Figure 8-2: Information about the residuals of the models for IGP-M index.



The correlations for the coefficients of the models are given in Appendix J. In case of IGPM_ADL2 and IGPM_ADML3, the correlation coefficients are below 0.8 based on the data sample. This implies no case of multicollinearity, which means that the explanatory variables have an either exact or approximately exact linear relationship.

For the in-sample forecasts the parameters of the model are estimated using the full sample, which is the data period from January 2000 until April 2007, and kept constant throughout the forecasting exercise. The data that is used for estimating the model is called in-sample data. The fitted values, the results from the in-sample forecasts, based on each model are given in Figure 8-3 up to Figure 8-6, plotted with the effective value of the monthly IGP-M index. It is hard to make a comparison between the models based on these plots. However, it is remarkable that the reaction of IGPM_ADL2 and IGPM_ADL3 on the crisis period around the fourth quarter of 2002 (the index in this period is circled in the graphs) is faster compared to the other models, which results a better fitting of the crisis period.

Figure 8-3: Fitted vs. effective IGP-M monthly index (IGPM_ADL1).

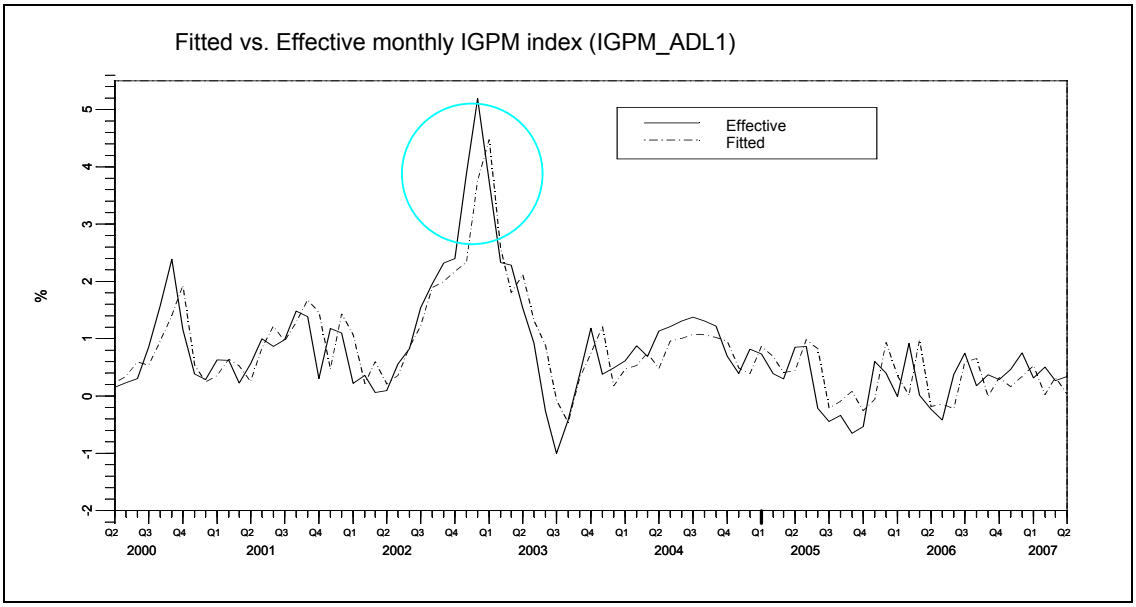


Figure 8-4: Fitted vs. effective IGP-M monthly index (IGPM_ADL2).

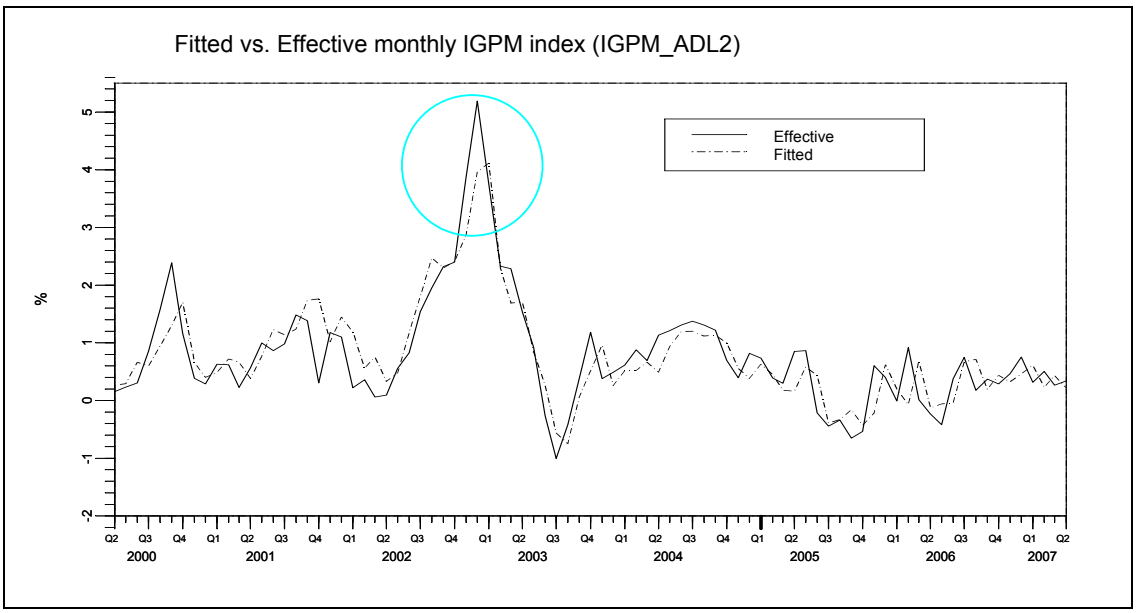


Figure 8-5: Fitted vs. effective IGP-M monthly index (IGPM_ADL3).

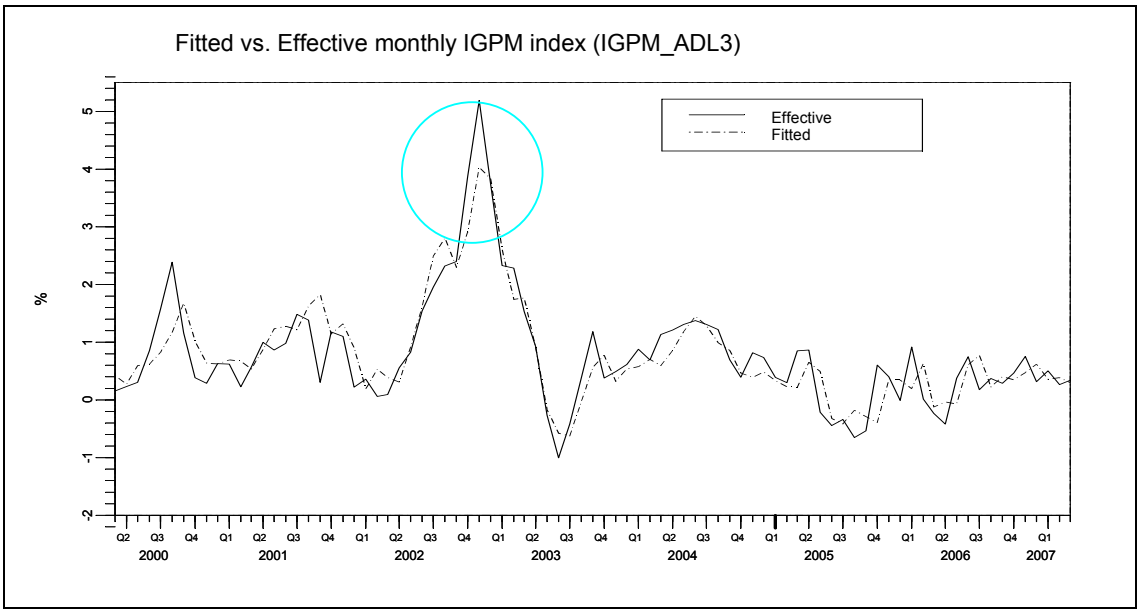
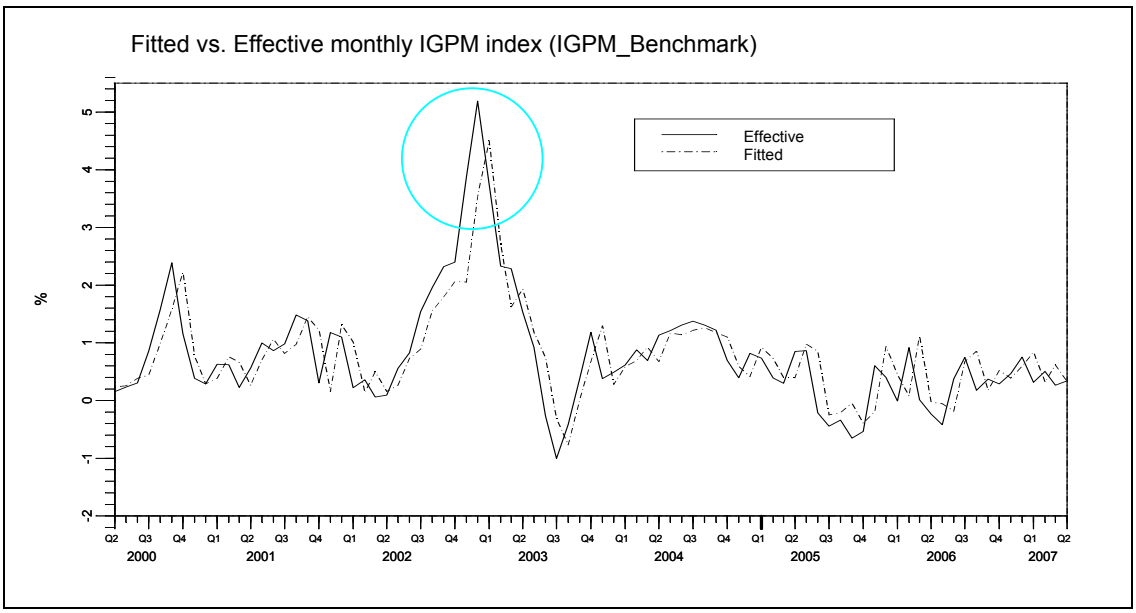


Figure 8-6: Fitted vs. effective IGP-M monthly index (IGPM_Benchmark).



8.1.2 Out-of-sample forecasting comparison: the IGP-M index.

The h steps-ahead forecast is done based on a method called 'the chain rule of forecasting'. This method exploits the fact that the *h* steps-ahead forecast can be obtained given the *i* steps-ahead forecasts, where i={1,…,h-1}. For an illustration, consider the 3 steps-ahead forecasts based on model IGPM_ADL1. The model is estimated by using the variable data up to period T. Let $\mathbf{IGPMmom_{T+1|T}}$ denote a 1 step-ahead forecast of $\mathbf{IGPMmom_{T+1}}$ based on the monthly IGP-M data up to time T and the observed one-year interest rate variable. First, the 1 step-ahead forecast is obtained by

$$IGPMmom_{T+1|T} = \beta_0 + \beta_1\, IGPMmom_T + \beta_2\, IGPMmom_{T-1} + \beta_3\, OneY_T. \qquad \textbf{(8-3)}$$

Given **(8-3)**, the 2 steps-ahead forecasts can be obtained by

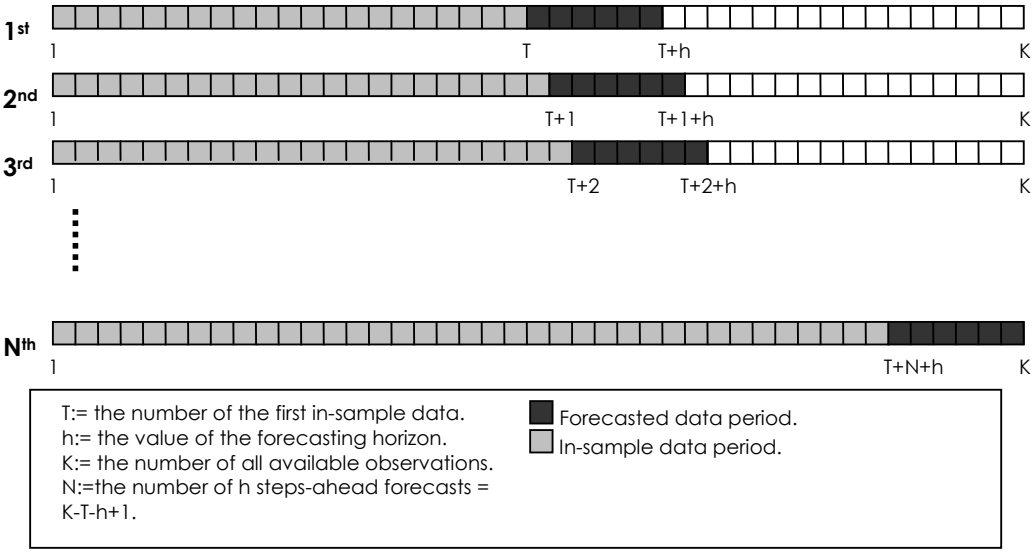
$$IGPMmom_{T+2|T} = \beta_0 + \beta_1\, IGPMmom_{T+1|T} + \beta_2\, IGPMmom_T + \beta_3\, OneY_{T+1}. \qquad \textbf{(8-4)}$$

At last, given the values obtained by **(8-3)** and **(8-4)**, the 3 steps-ahead forecast is computed as

$$IGPMmom_{T+3|T} = \beta_0 + \beta_1\, IGPMmom_{T+2|T} + \beta_2\, IGPMmom_{T+1|T} + \beta_3\, OneY_{T+2}. \qquad \textbf{(8-5)}$$

The forecasting procedure is carried out as follows. For a given value of the forecasting horizon, *h*, the monthly IGP-M index is estimated by using the variables data up to period *T*. With the estimated model, *h* steps-ahead out-of-sample forecast is generated as illustrated above. The next step, the observations of period *T+1* are included to the estimation sample, the model is re-estimated. Based on the latter model, another *h steps-ahead* forecast is computed. This is repeated until forecasts are obtained for all available observations of the IGP-M index since time *T+h*. Figure 8-7 illustrates the forecasting procedure. The blocks represent the available data periods.

Figure 8-7: An illustration for forecasting procedure: the rolling regressions.



The forecasting ability of the models will be compared in terms of the root mean square forecasting error (RMSE). The h steps-ahead RMSE is given by

$$RMSE(h) = \sqrt{\frac{1}{N}\sum_{i=T}^{T+N}\left(IGPMmom_{i+h|i} - IGPMmom_{i+h}\right)^2}\ , \qquad \textbf{(8-6)}$$

where N is the number of out-of-sample forecasts carried out. The RMSE of the different models are given in the Table 8-3.

Table 8-3 reports the results of the forecasting horizons from six months to two years. In all cases, the first estimated model is based on data period January 2000 until May 2004 that makes T equal to 50 adjusted data points which includes the crisis period in 2002, and the forecasts were computed up to April 2007, the last available observation. Based on the results, all three models outperform the benchmark model in forecasting. In Figure 8-8, the forecasted values are plotted with the effective monthly IGP-M index. It is clearly observed that the forecasting ability of the benchmark model is very poor; the forecasted values are constantly above the effective values. This can be expected since the benchmark model predicts the inflation only based on its own past values while the other models predict the inflation based on its own past values and additional information from other observable data.

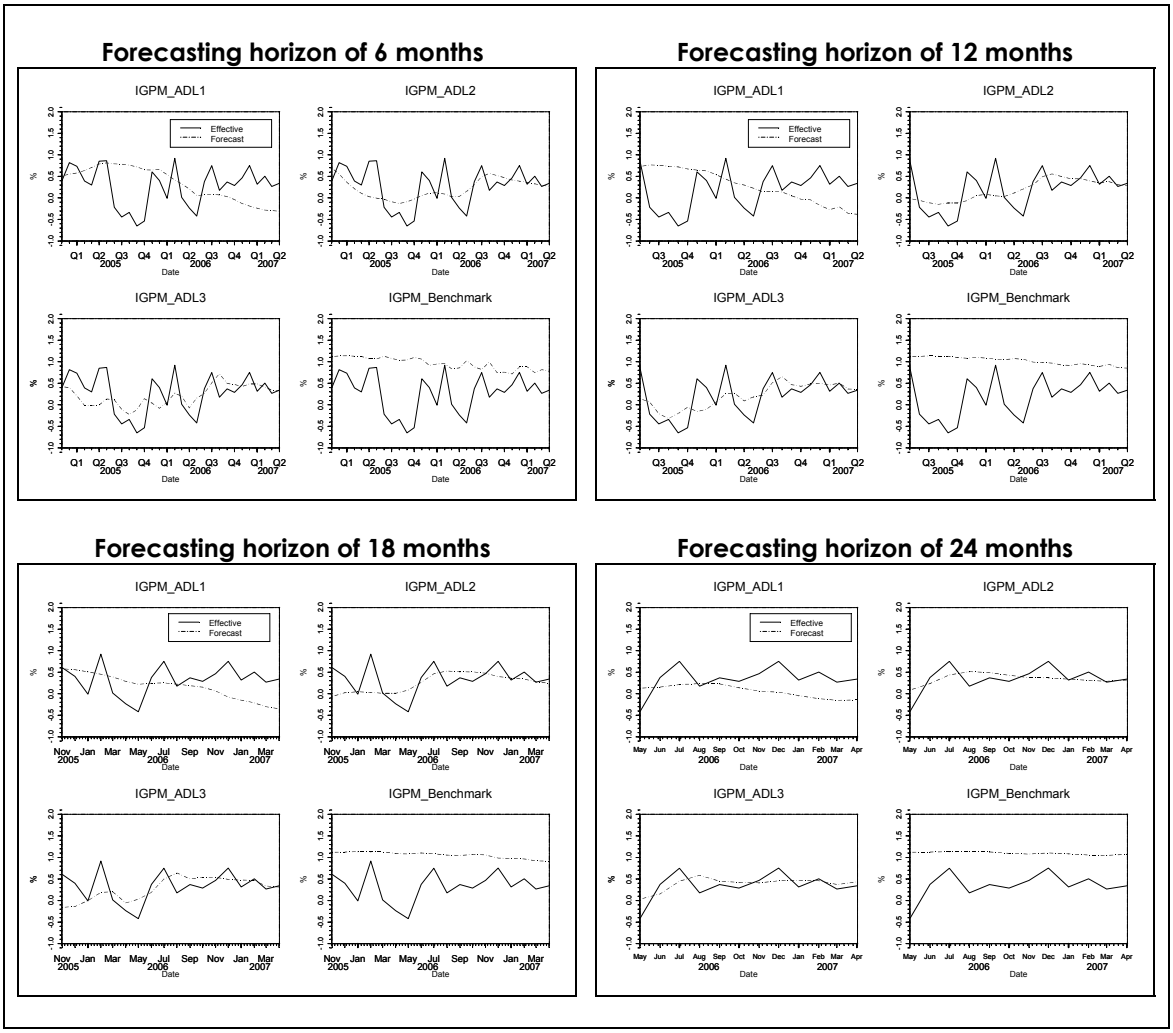
Table 8-3: Forecasting comparison: RMSE of the models for IGP-M index.

	RMSE			
	6 months	**12 months**	**18 months**	**24 months**
IGPM_ADL1	6.37E-03	7.01E-03	4.74E-03	4.37E-03
IGPM_ADL2	3.88E-03	3.90E-03	3.46E-03	2.43E-03
IGPM_ADL3	4.03E-03	3.73E-03	3.47E-03	2.36E-03
IGPM_Benchmark	8.40E-03	9.65E-03	8.05E-03	8.03E-03

Among the presented three models, IGPM_ADL2 and IGPM_ADL3 significantly outperform IGPM_ADL1, with reductions of the RMSE around 40% for forecasting horizons of 6, 12, and 24 months and reductions of the RMSE around 10% for a forecasting horizon of 18 months. These results can be interpreted as evidence that

the slope of the yield curve and the exchange rate have significant ability in forecasting the IGP-M index. Based on RMSE, the difference of the IGPM_ADL2 and the IGPM_ADL3 is very small.

Figure 8-8: The forecasted values based on the models are plotted with the effective monthly IGP-M index, for the forecasting horizons of 6, 12, 18, and 24 months.



8.1.3 Coefficient's stability

As observed in the previous sections, the models IGPM_ADL2 and IGPM_ADL3 perform better compared to the other models in explaining the IGP-M index in-sample as well as in out-of-sample forecasting with the forecasting horizons of up to two years. This implies that the coefficients in the models IGPM_ADL2 and IGPM_ADL3 are more stable than in the other models.

The 'in-sample' forecast was done based on the model that was estimated by using the data from January 2000 until April 2007, while the first out-of-sample forecast was done based on the model that was estimated by the data up to February 2004. The models for the 'in-sample' and 'out-of-sample' forecasting were then estimated by using the data including the crisis period in 2002.

The coefficients of the each model are plotted against the date of the last observation included for estimating the models in Figure 8-9 up to Figure 8-12. The first estimate uses the sample data up to May 2004.

Figure 8-9: The coefficients of the models IGPM_ADL1 plotted against the date of the last observation included in the 'in-sample' data.

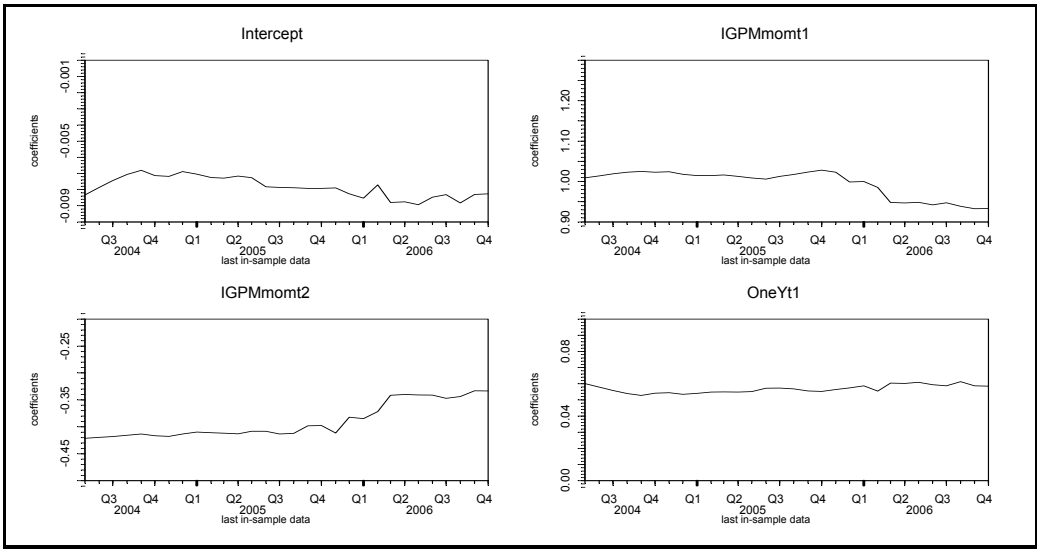


Figure 8-10: The coefficients of the models IGPM_ADL2 plotted against the date of the last observation included in the 'in-sample' data.

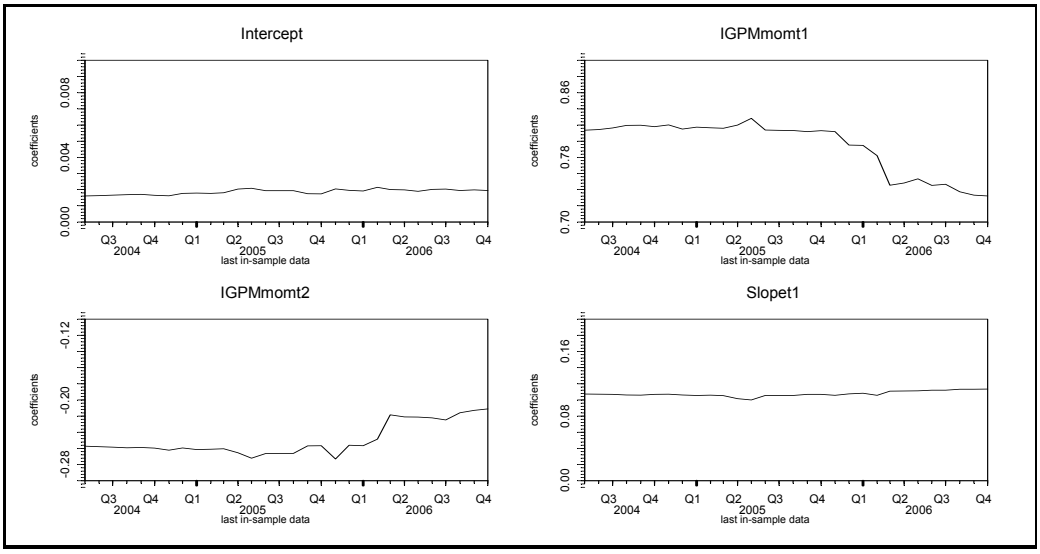


Figure 8-11: The coefficients of the models IGPM_ADL3 plotted against the date of the last observation included in the 'in-sample' data.

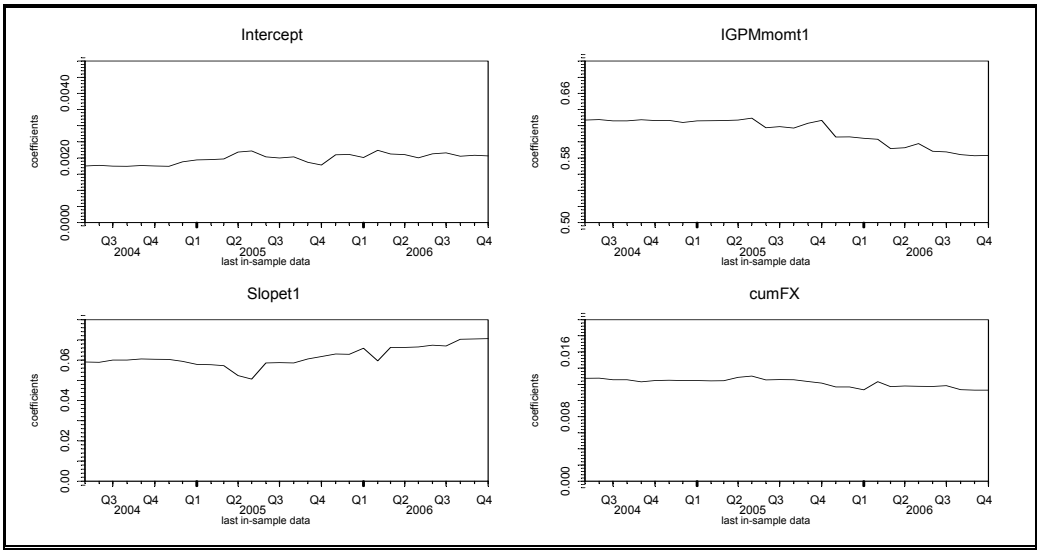
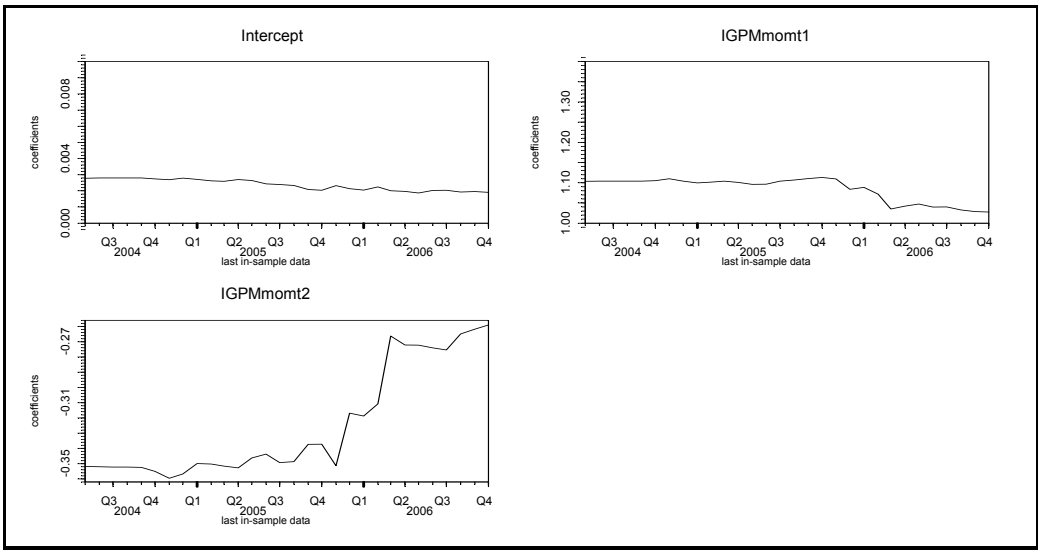


Figure 8-12: The coefficients of the models IGPM_Benchmark plotted against the date of the last observation included in the 'in-sample' data.



However, it is hard to make a comparison of the coefficient's stability between the models due to the different range of graphs. In order to compare the coefficient's stability more easily, the standard deviation and the mean of the coefficient's estimate are given in Table 8-4. Similar as in the case of the 'out-of-sample' forecasting procedure, the first estimate is based on a data sample period up to May 2004.

Based on this summary, a poor forecasting ability of the IGPM_ADL1 and the IGPM_Benchmark may be caused partly by the variation of the coefficient of $IGPMmom_{t-1}$. The standard deviations of the coefficient of $IGPMmom_{t-1}$ in model IGPM_ADL1 and IGPM_Benchmark are quite large compared with the other variables. Moreover, the mean of this coefficient is also high in both models which

make the effect of its deviation greater. Based on the standard deviation of the coefficients in model IGPM_ADL2 and IGPM_ADL3, model IGPM_ADL3 would be preferred.

Table 8-4: The mean and standard deviation of the coefficient estimations.

IGPM_ADL1			IGPM_ADL2		
Coefficients	**St. Dev.**	**Mean**	**Coefficients**	**St. Dev.**	**Mean**
(Intercept)	6.33E-04	-0.008	(Intercept)	1.57E-04	0.002
$IGPMmom_{t-1}$	3.38E-02	0.995	$IGPMmom_{t-1}$	3.29E-02	0.794
$IGPMmom_{t-2}$	3.24E-02	-0.390	$IGPMmom_{t-2}$	1.98E-02	-0.250
$OneY_{t-1}$	2.43E-03	0.057	$Slope_{t-1}$	3.28E-03	0.108
IGPM_ADL3			**IGPM_Benchmark**		
Coefficients	**St. Dev.**	**Mean**	**Coefficients**	**St. Dev.**	**Mean**
(Intercept)	1.60E-04	0.002	(Intercept)	3.45E-04	0.002
$IGPMmom_{t-1}$	1.66E-02	0.613	$IGPMmom_{t-1}$	3.03E-02	1.084
$Slope_{t-1}$	4.93E-03	0.062	$IGPMmom_{t-2}$	3.66E-02	-0.325
$\Delta FX_{t-1} + \Delta FX_{t-2}$	5.12E-04	0.012			

8.1.4 Conversion of the models into annual terms: the IGP-M index.

The payments of inflation-linked products are calculated based on the annual inflation index. Since the estimates of the models are based on the monthly inflation index, the models have to be converted into models in terms of annual inflation index.

Given the monthly IGP-M index, the annual IGP-M index is computed as follows,

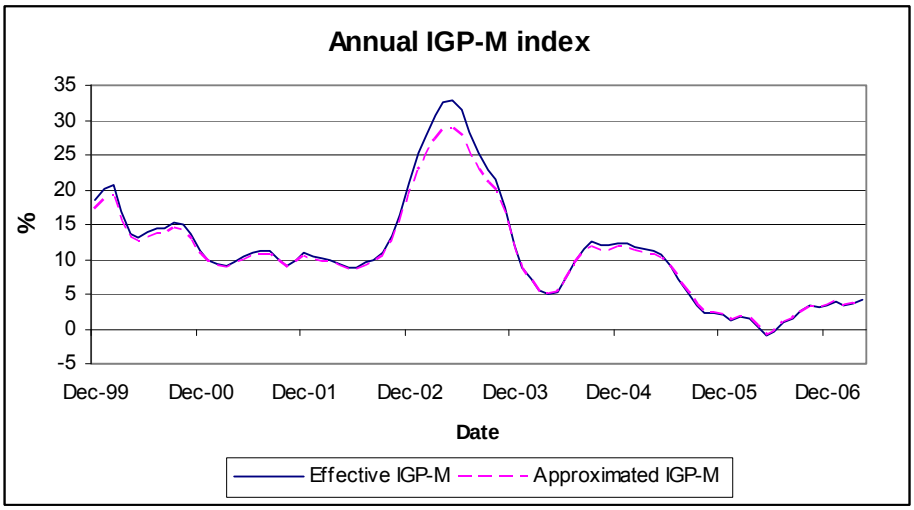
$$IGPMyoy_t = \left(\prod_{i=t-11}^{t} IGPMmom_i + 1\right) - 1, \qquad \textbf{(8-7)}$$

where $\mathbf{IGPMyoy_t}$ is the annual IGP-M index at time t and $\mathbf{IGPMmom_t}$ is the monthly IGP-M index at time t. For the sake of simplification of the conversion, the annual IGP-M index can be approximated by

$$IGPMyoy_t = \sum_{i=t-11}^{t} IGPMmom_i. \qquad \textbf{(8-8)}$$

The approximation of the annual IGP-M index using **(8-8)** is plotted with the effective annual IGP-M index in Figure 8-13. The approximated values are very close to the real values, except for the high value of inflation. Fortunately, the deviation from the effective index can still be accepted.

Figure 8-13: The approximated and effective annual IGP-M index.



Given the estimated parameters (see Table 8-1), the IGPM_ADL2 in terms of the monthly inflation index is given by

$$IGPMmom_t = 0.002 + 0.723\ IGPMmom_{t-1} - 0.205\ IGPMmom_{t-2} + 0.114\ Slope_{t-1}. \quad \textbf{(8-9)}$$

The conversion of the IGPM_ADL2 into a model with annual terms is given by

$$IGPMyoy_t = \sum_{i=t-11}^{t} (0.002 + 0.723\ IGPMmom_{i-1} - 0.205 IGPMmom_{i-2} + 0.114 Slope_{i-1})$$

$$= 12x0.002 + 0.723\ IGPMyoy_{t-1} - 0.205\ IGPMyoy_{t-2} + 0.114 \sum_{i=t-11}^{t} Slope_{i-1}. \quad \textbf{(8-10)}$$

A change in the annual IGP-M index based on model IGPM_ADL2 is then

$$\Delta IGPMyoy_t = IGPMyoy_t - IGPMyoy_{t-1} = \sum_{i=t-11}^{t} IGPMmom_i - \sum_{j=t-12}^{t-1} IGPMmom_j =$$

$$0.723\ \Delta IGPMyoy_{t-1} - 0.205\ \Delta IGPMyoy_{t-2} + 0.114\left(\sum_{i=t-11}^{t} Slope_{i-1} - \sum_{j=t-12}^{t-1} Slope_{j-1}\right) =$$

$$0.723\ \Delta IGPMyoy_{t-1} - 0.205\ \Delta IGPMyoy_{t-2} + 0.114\ (Slope_{t-1} - Slope_{t-13}). \quad \textbf{(8-11)}$$

With the same procedure, a change in the annual IGP-M index based on model IGPM_ADL3 can be obtained. The IGPM_ADL3 in terms of the monthly inflation index is given by

$$IGPMmom_t = 0.002 + 0.580\ IGPMmom_{t-1} + 0.071\ Slope_{t-1} + 0.011\ (\Delta FX_{t-1} + \Delta FX_{t-2}). \quad \textbf{(8-12)}$$

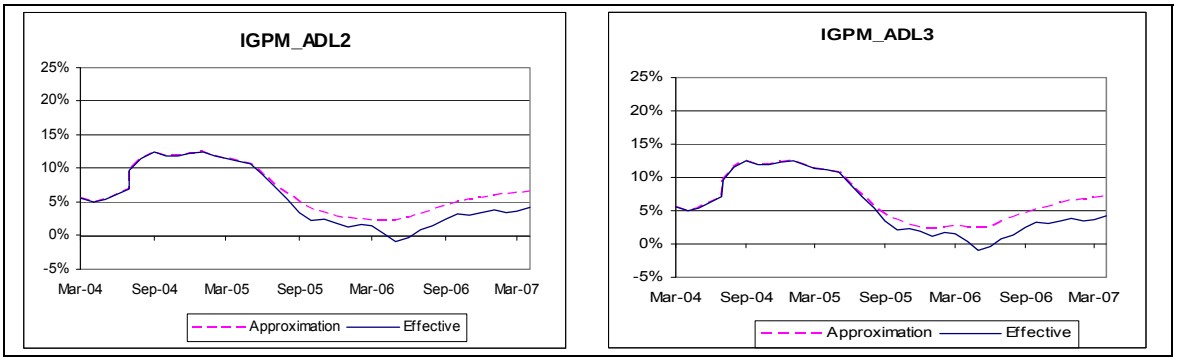
A change in the annual IGP-M index based on the model IGPM_ADL3 is then

$$\Delta IGPMyoy_t = 0.580\ \Delta IGPMyoy_{t-1} + 0.071(Slope_{t-1} - Slope_{t-13}) +$$
$$0.011(\Delta FX_{t-1} - \Delta FX_{t-13} + \Delta FX_{t-2} - \Delta FX_{t-14}) \quad \textbf{(8-13)}$$

In Figure 8-14, the annual IGP-M approximations using equation **(8-11)**, obtained from model IGPM_ADL2, and equation **(8-13)**, obtained from model IGPM_ADL3, are plotted with the effective annual IGP-M index. The approximations are obtained by

using the annual IGP-M data up to June 2005 and observable data for the exchange rate and yield curve slope variables.

Figure 8-14: Annual IGP-M approximation from data period July 2005 until April 2007.



8.2 Autoregressive distributed lag models for the IPCA index

8.2.1 The models for IPCA index.

Similar as in the case of the IGP-M index, the sample used for estimating the models for the IPCA index is from period January 2000 until April 2007. The regression is based on the monthly IPCA index given in Figure 8-15. The estimates of the models are given in the Table 8-5 below.

Figure 8-15: The monthly IPCA index.

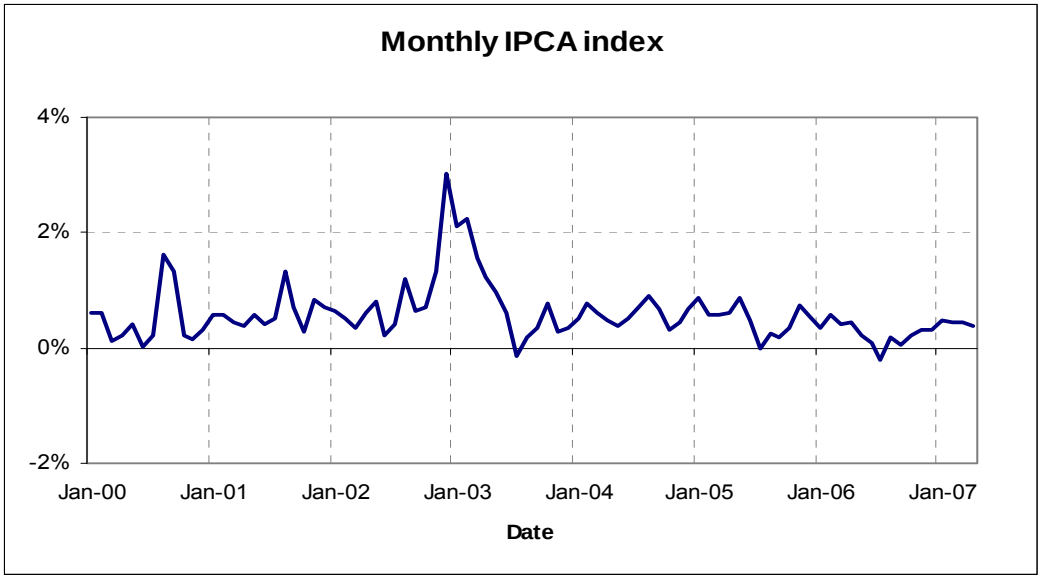


Table 8-5: Model estimation for IPCA inflation index.

IPCA_ADL1				IPCA_ADL2			
Expl. Var	**Coefficient**	**t-Stat.**	**p-value**	**Expl. Var**	**Coefficient**	**t-Stat.**	**p-value**
(Intercept)	-0.004	-2.374	0.020	$IPCAmom_{t-1}$	0.426	4.327	0.000
$IPCAmom_{t-1}$	0.465	5.045	0.000	$IPCAmom_{t-2}$	-0.398	-4.018	0.000
$OneY_{t-1}$	0.038	3.714	0.000	$Selic_{t-1}$	0.215	3.871	0.000
				$Selic_{t-2}$	-0.208	-3.927	0.000
				$EMBI_{t-4}$	0.063	4.782	0.000
R^2	0.508	Adj. R^2	0.496	R-squared	0.653	Adj. R^2	0.636
IPCA_ADL3				**IPCA_Benchmark**			
Expl. Var	**Coefficient**	**t-Stat.**	**p-value**	**Expl. Var**	**Coefficient**	**t-Stat.**	**p-value**
$IPCAmom_{t-1}$	0.407	4.455	0.000	(Intercept)	0.002	3.185	0.002
$IPCAmom_{t-2}$	-0.342	-3.772	0.000	$IPCAmom_{t-1}$	0.654	7.959	0.000
$Selic_{t-1}$	0.157	3.029	0.003				
$Selic_{t-2}$	-0.169	-3.465	0.001				
$EMBI_{t-4}$	0.054	4.469	0.000				
FX_{t-2}	0.010	4.158	0.000				
FX_{t-3}	-0.008	-3.373	0.001				
R^2	0.726	Adj. R^2	0.704	R^2	0.427	Adj R^2	0.420

The first model, **IPCA_ADL1**, is the model that is estimated from previous research. The model includes the first lag of the dependent variable, an intercept, and the first lag of the one-year interest rate.

The model **IPCA_ADL2** includes the first two lags of the Selic rates and the 4 months lag of the EMBI spread. It is expected that the Selic rate time lags have fundamental information for the IPCA index since the Selic rate is adjusted based on the IPCA index expectation which in turn has effect on the effective IPCA index. Since the absolute coefficients for the Selic rate variables are quite similar, these explanatory variables can be interpreted as the one month lag of the first difference of the Selic rate. The 4 months lag of the EMBI spread seems also to give highly significant information for the IPCA index. As mentioned in Section 6.3, the EMBI spread give effect on the inflation through the exchange rates. All coefficients have the expected signs. The relation based on IPCA_ADL2 can be then interpreted as follows; a movement of the Selic rate in the previous month and a change of the EMBI spread have a positive movement in IPCA inflation level.

In model **IPCA_ADL3**, the exchange rate variable time lags are added as explanatory variables which give a model with a better goodness of fit, compared to the **IPCA_ADL2**. Since the $EMBI_{t-4}$ variable is still significant after the lags of exchange rate variable are added to the regression, this can be interpreted as the reflection of the direct and the indirect effects of EMBI spread on the IPCA index.

In case of the exchange rate, the two months and three months lags seem to give significant information for the IPCA. The coefficients of the lags of the Selic rate do not have much difference in absolute term. This is also the case for the coefficients of the lags of the exchange rate. Therefore, the relation can be interpreted as follows; a change in the EMBI spread four months before, a movement of Selic rate in the previous month, and movement of the exchange rate two months before have a positive movement to the IPCA inflation level.

The models IPCA_ADL2 and IPCA_ADL3 were re-estimated using the first difference data as explanatory variables for the exchange rate and the Selic rate. The estimation results are given in Table 8-6.

Table 8-6: The adjusted estimation model IPCA_ADL2 and IPCA_ADL3 for IPCA index.

IPCA_ADL2				IPCA_ADL3			
Expl. Var	**Coefficient**	**t-Stat.**	**p-value**	**Expl. Var**	**Coefficient**	**t-Stat.**	**p-value**
(Intercept)	0.002	1.966	0.053	(Intercept)	0.001	2.033	0.046
$IPCAmom_{t-1}$	0.416	4.234	0.000	$IPCAmom_{t-1}$	0.440	4.833	0.000
$IPCAmom_{t-2}$	-0.394	-4.055	0.000	$IPCAmom_{t-2}$	-0.327	-3.571	0.001
$\Delta Selic_{t-1}$	0.213	4.153	0.000	$\Delta Selic_{t-1}$	0.167	3.409	0.001
$EMBI_{t-4}$	0.063	5.321	0.000	$EMBI_{t-4}$	0.056	4.980	0.000
				ΔFX_{t-2}	0.009	3.789	0.000
R^2	0.660	Adj. R^2	0.637	R^2	0.713	Adj. R^2	0.690

The summary of the residuals obtained from these models are given in Figure 8-16. It is observed that there is a high peak in December 2002 in the residuals obtained from the models IPCA_ADL1 and IPCA_Benchmark. This gives an idea about the lack of information in these models for explaining the high inflation in that period. This can also be observed in Figure 8-17 up to Figure 8-20, graphs of the fitted values based on each model plotted with the effective value of the IPCA index. Given the correlograms, the residuals from model IPCA_ADL2 seem to be non-autocorrelated; the autocorrelations and the partial autocorrelations are significantly zero. The

residuals from other models seem to have some (partial) autocorrelations that are significantly non-zero.

The correlations of coefficients for the models are given in Appendix J. The correlations of coefficients for IPCA_ADL2 and IPCA_ADL3 are all below 0.8 which does not imply having serious problems with multicollinearity.

Figure 8-16: Information about the residuals of the models for the IPCA index.

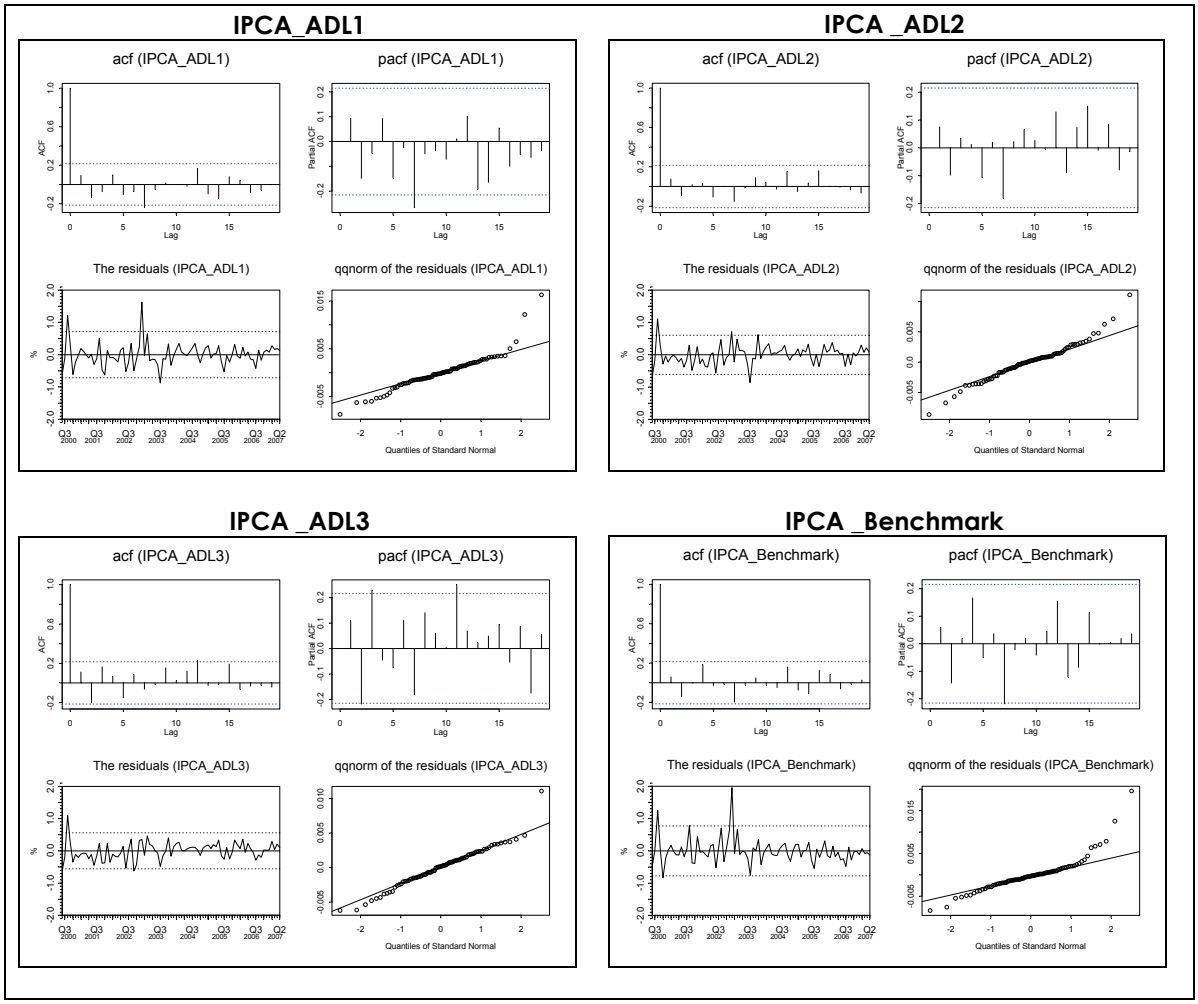


Figure 8-17: The fitted vs. the effective IPCA monthly index (IPCA_ADL1).

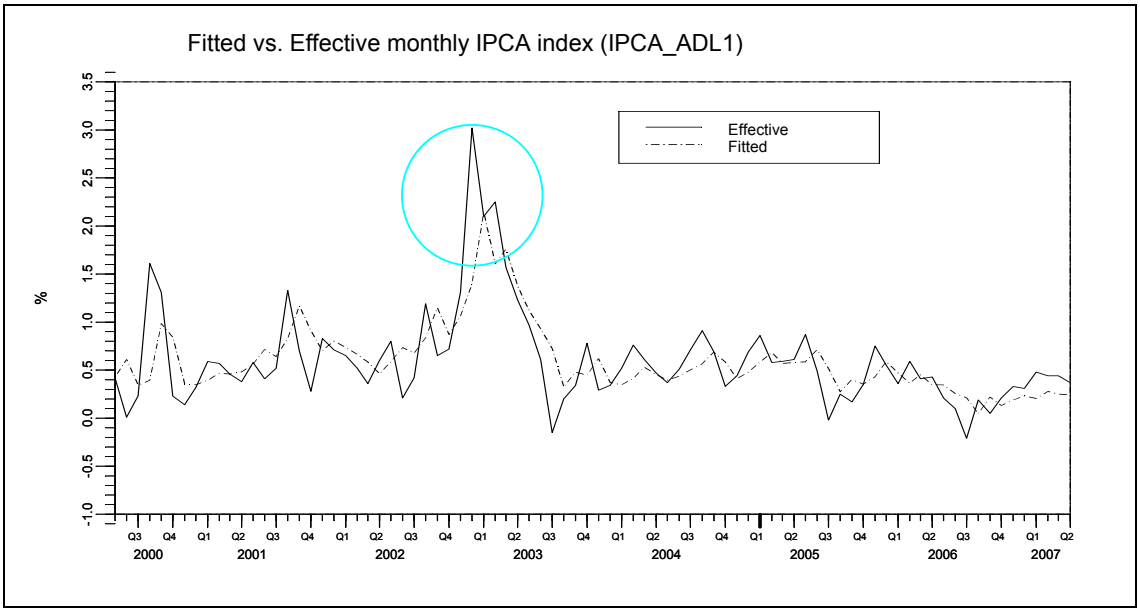


Figure 8-18: The fitted vs. the effective IPCA monthly index (IPCA_ADL2).

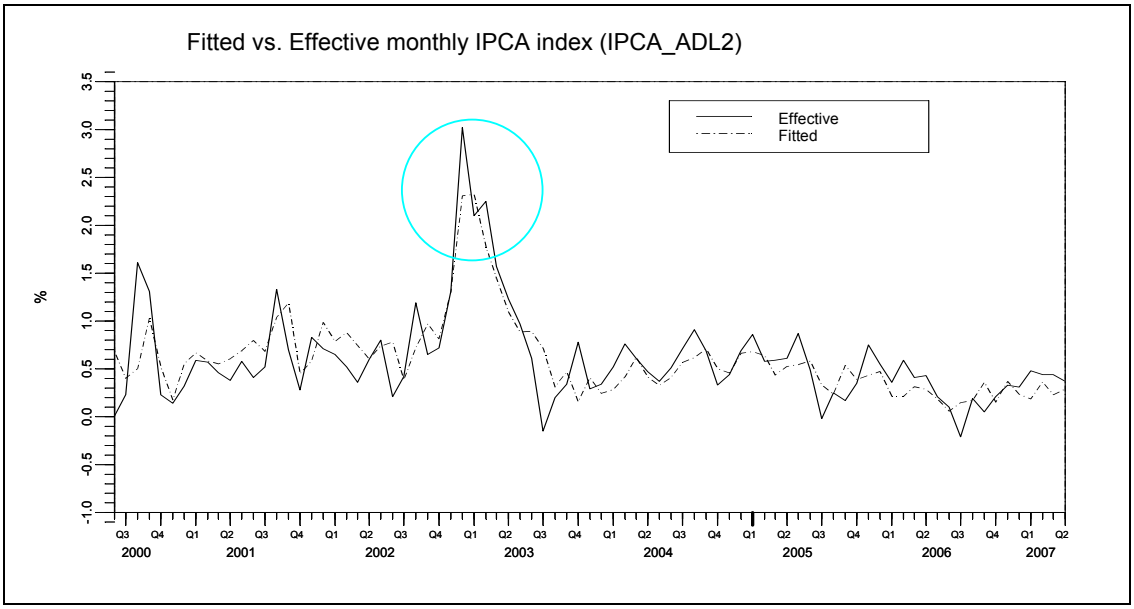


Figure 8-19: The fitted vs. the effective IPCA monthly index (IPCA_ADL3).

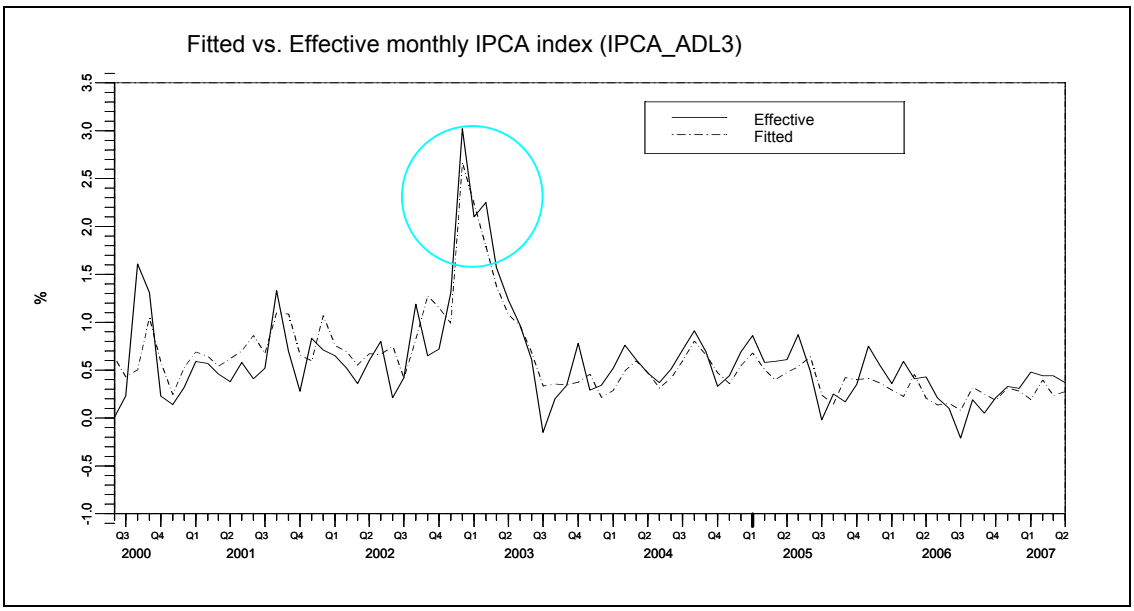
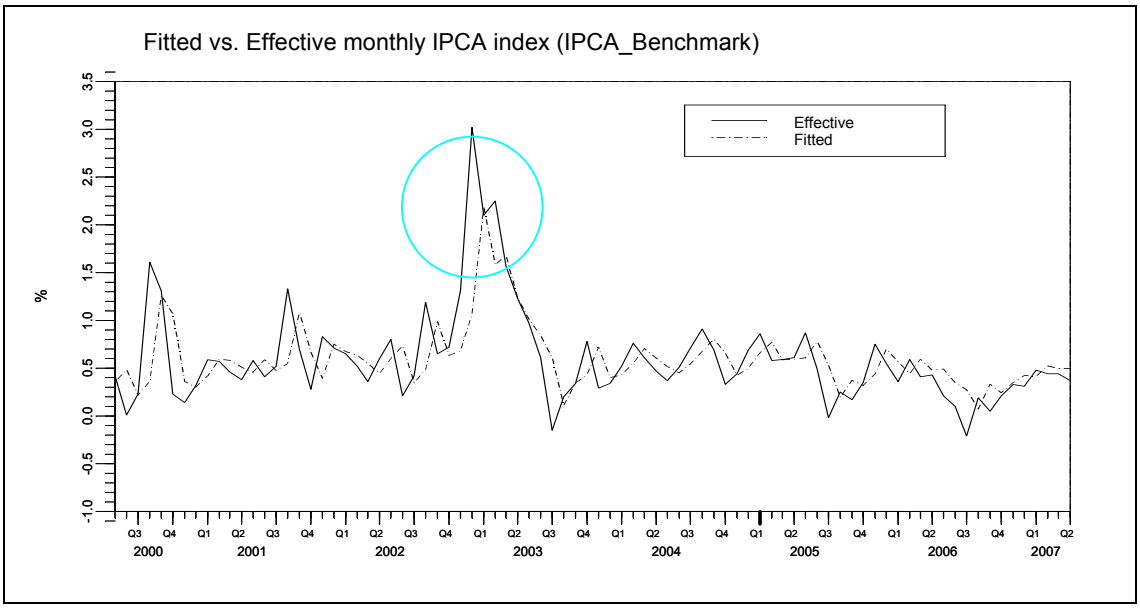


Figure 8-20: The fitted vs. the effective IPCA monthly index (IPCA_Benchmark).



8.2.2 Out-of-sample forecasting comparison: the IPCA index.

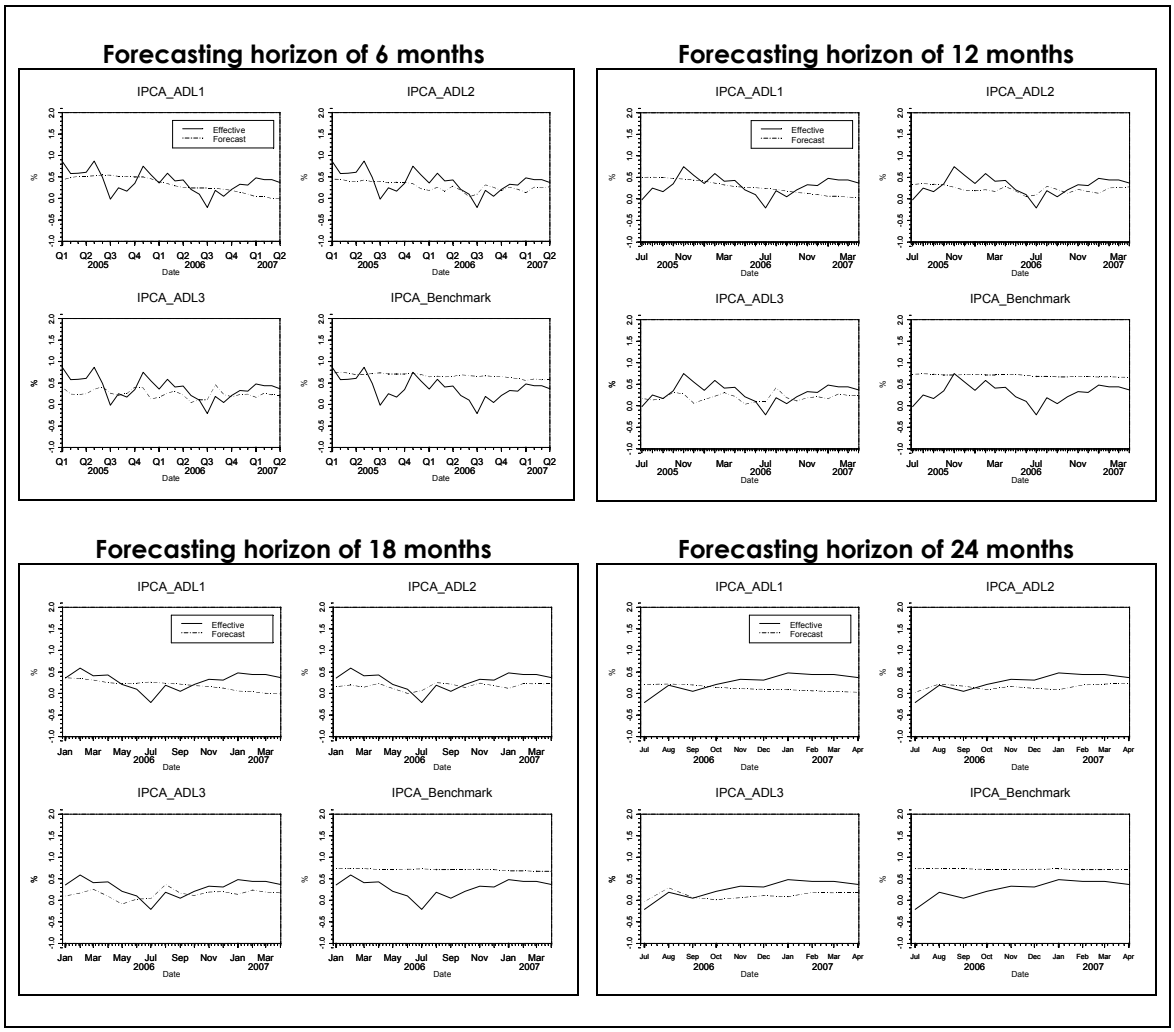
The same forecasting procedure as explained in Section 8.1.2 is done for the out-of-sample forecasting for IPCA index. The first estimated model for 'out-of-sample' forecasting is based on 50 data points (adjusted), that is corresponding to the data period from January 2000 until July 2004. The forecasting ability of the models is compared in terms of the root mean square forecasting error (RMSE), which is given in Table 8-7.

Table 8-7: Forecasting comparison: RMSE of the models for IPCA index.

	RMSE			
	6 months	12 months	18 months	24 months
IPCA_ADL1	2.71E-03	2.67E-03	2.64E-03	2.95E-03
IPCA_ADL2	2.37E-03	2.27E-03	2.11E-03	2.08E-03
IPCA_ADL3	2.61E-03	2.31E-03	2.35E-03	2.29E-03
IPCA_Benchmark	3.74E-03	4.46E-03	4.62E-03	5.07E-03

Based on the results, the three models slightly outperform the benchmark model in forecasting. Among the presented three models, the RMSE based on IPCA_ADL2 is the smallest with the time horizons up to two years while IPCA_ADL3 performs slightly better than IPCA_ADL1. However, if the forecasted values are plotted with the effective IPCA index as given in Figure 8-21, it is clearly observed that the plot of forecasted values based on the model IPCA_ADL3 follows the pattern of the effective IPCA while the forecast value based on IPCA_ADL2 is flatter. Since the IPCA index itself is not that volatile, the RMSE based on IPCA_ADL2 is smaller than the RMSE based on IPCA_ADL3.

Figure 8-21: The forecasted values based on the models are plotted with the effective monthly IPCA index, for the forecasting horizons of 6, 12, 18, and 24 months.



8.2.3 Coefficients stability

The models IPCA_ADL2 and IPCA_ADL3 perform slightly better than the other models for the IPCA index in in-sample as well as in out-of-sample forecasting with the forecasting horizons of up to two years that gives impression of coefficients stability. The models for the 'in-sample' forecasting and 'out-of-sample' forecasting were estimated by using the data including the crisis period in 2002.

In Figure 8-22 until Figure 8-25, the parameters of the models IPCA_ADL2 and IPCA_ADL3 are plotted against the date of the last observation included for estimating the parameters.

Figure 8-22: The parameters of the models IPCA_ADL1 plotted against the date of the last observation included in the 'in-sample' data.

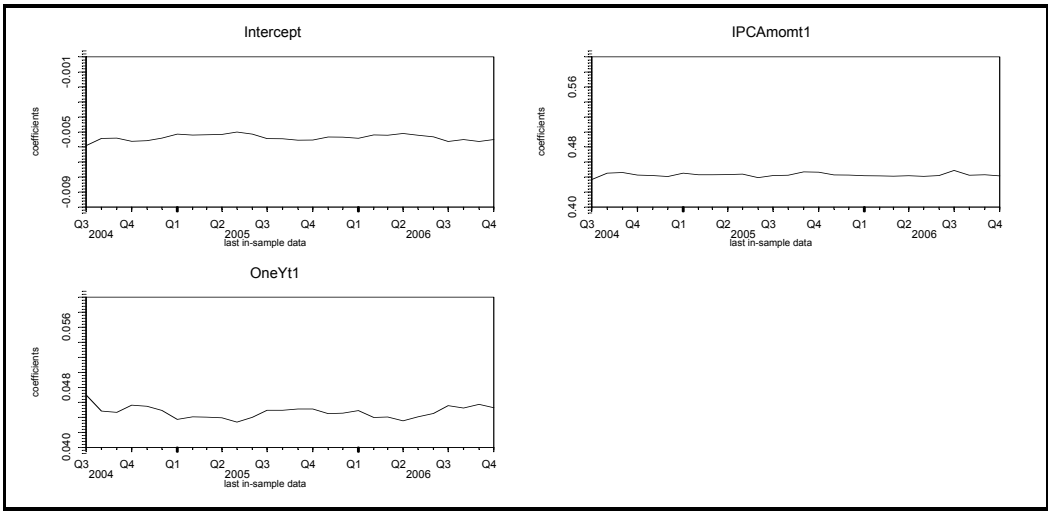


Figure 8-23: The parameters of the models IPCA_ADL2 plotted against the date of the last observation included in the 'in-sample' data.

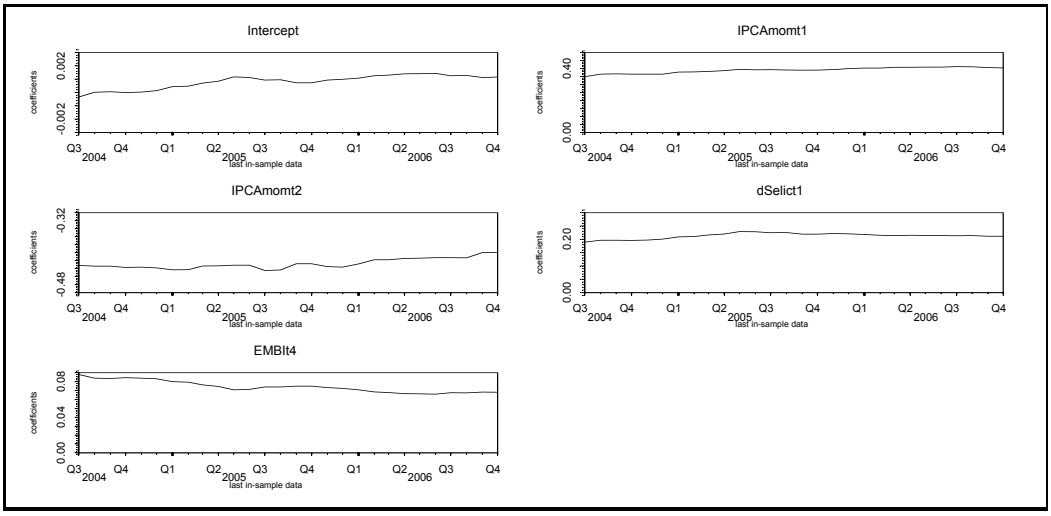


Figure 8-24: The parameters of the models IPCA_ADL3 plotted against the date of the last observation included in the 'in-sample' data.

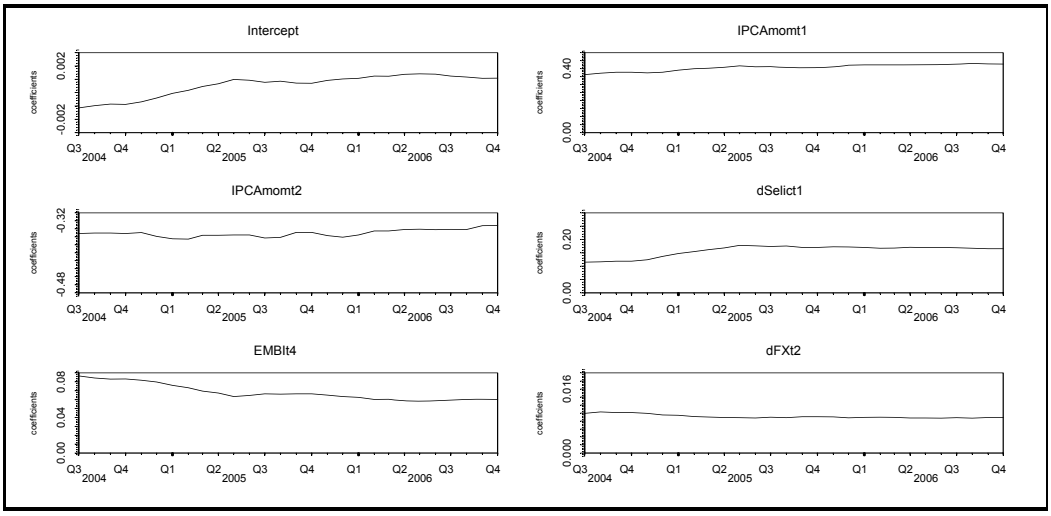
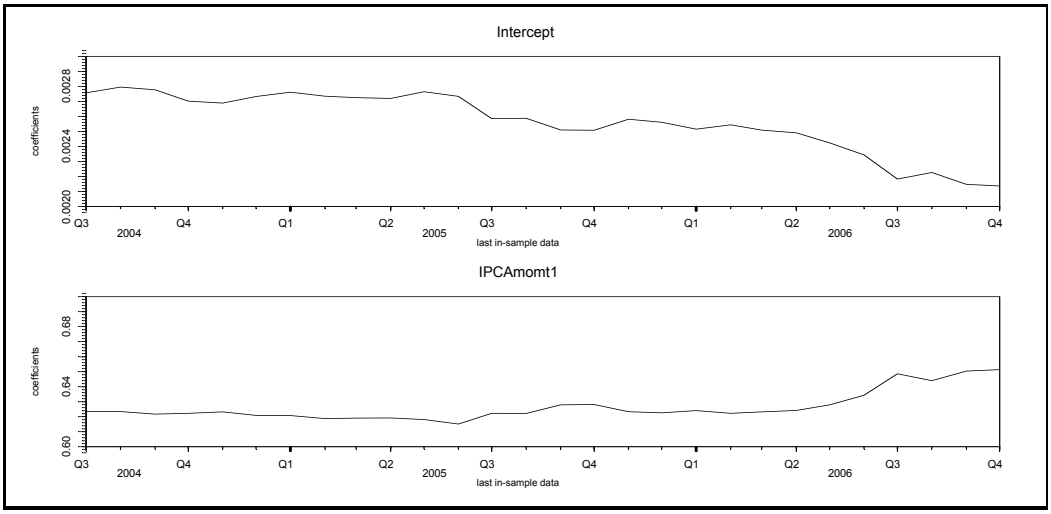


Figure 8-25: The parameters of the models IPCA_Benchmark plotted against the date of the last observation included in the 'in-sample' data.



In order to compare the coefficients stability more easily, the standard deviation and the mean of the estimates of the coefficients are given in Table 8-4. As in the case of the 'out-of-sample' forecasting procedure, the first estimate is based on a data sample period up to July 2004. Based on this information, the coefficients in model IPCA_ADL2 and IPCA_ADL3 are not necessarily more stable than the coefficients of the other models.

Table 8-8: The mean and standard deviation of the coefficient estimations.

IPCA_ADL1			IPCA_ADL2		
Coefficients	**St. Dev.**	**Mean**	**Coefficients**	**St. Dev.**	**Mean**
(Intercept)	2.05E-04	-0.005	(Intercept)	5.12E-04	0.001
$IPCAmom_{t-1}$	2.40E-03	0.443	$IPCAmom_{t-1}$	1.75E-02	0.390
$OneY_{t-1}$	7.97E-04	0.045	$IPCAmom_{t-2}$	1.27E-02	-0.427
			$\Delta Selic_{t-1}$	1.03E-02	0.213
			$EMBI_{t-4}$	6.61E-03	0.074
IPCA_ADL3			**IPCA_Benchmark**		
Coefficients	**St. Dev.**	**Mean**	**Coefficients**	**St. Dev.**	**Mean**
(Intercept)	8.14E-04	0.001	(Intercept)	2.01E-04	0.003
$IPCAmom_{t-1}$	2.05E-02	0.407	$IPCAmom_{t-1}$	9.93E-03	0.627
$IPCAmom_{t-2}$	9.10E-03	-0.351			
$\Delta Selic_{t-1}$	2.08E-02	0.158			
$EMBI_{t-4}$	9.12E-03	0.068			
ΔFX_{t-2}	4.83E-04	0.009			

8.2.4 Conversion of the models into annual terms: the IPCA index.

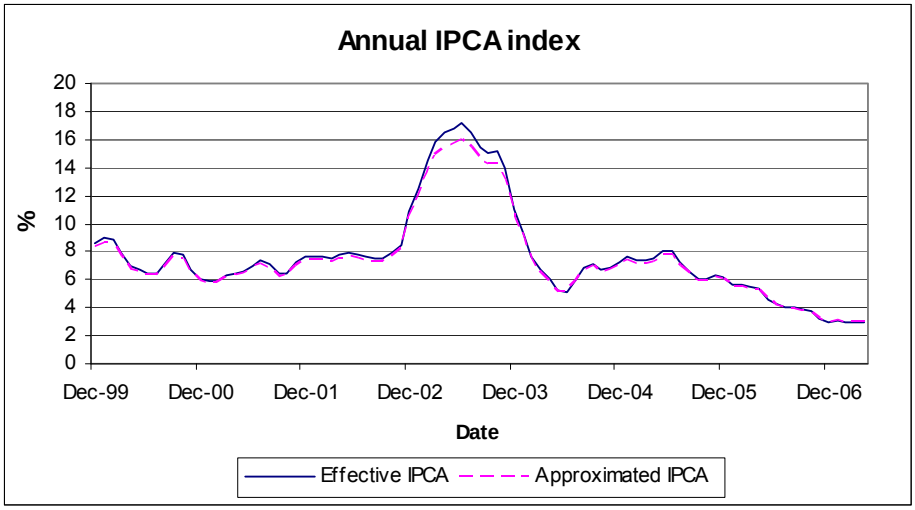
Based on the results from the previous section, only the models IPCA_ADL2 and IPCA_ADL3 will be further analyzed. The approximation of the annual IPCA index using **(8-8)** is plotted with the effective annual IPCA index in Figure 8-26. Compared to the case of the IGP-M index, the approximation of the annual IPCA index in the crisis situation (high inflation) is much closer to the real value since the IPCA index is less volatile than the IGP-M index. Given the parameters (see Table 8-5), the IPCA_ADL2 in terms of the monthly inflation index is given by

$$IPCAmom_t = 0.002 + 0.416\, IPCAmom_{t-1} - 0.394\, IPCAmom_{t-2} + 0.213\, \Delta Selic_{t-1} + 0.063\, EMBI_{t-4}. \quad \textbf{(8-14)}$$

A change in the annual IPCA index based on the model IPCA_ADL2 is given by

$$\Delta IPCAyoy_t = 0.416\, \Delta IPCAyoy_{t-1} - 0.394\, \Delta IPCAyoy_{t-2} + 0.213\, (\Delta Selic_{t-1} - \Delta Selic_{t-13}) + 0.063\, (EMBI_{t-4} - EMBI_{t-16}). \quad \textbf{(8-15)}$$

Figure 8-26: The approximated and the effective annual IPCA index.



The model IPCA_ADL3 in terms of monthly inflation index is given by

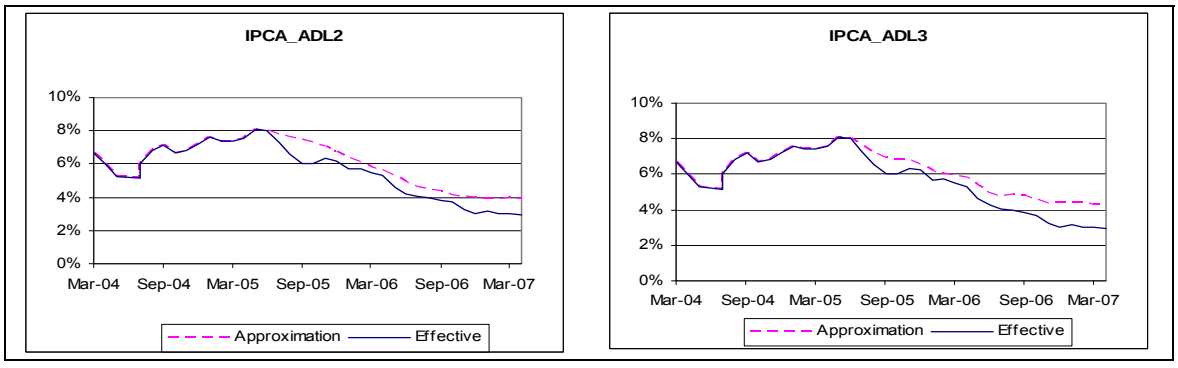
$$IPCAmom_t = 0.001 + 0.440\ IPCAmom_{t-1} - 0.327\ IPCAmom_{t-2} + 0.167\ \Delta Selic_{t-1} + 0.056\ EMBI_{t-4} + 0.009\ \Delta FX_{t-2}. \quad \textbf{(8-16)}$$

A change in the annual IPCA index based on model IPCA_ADL3 is then

$$\Delta IPCAyoy_t = 0.440\ \Delta IPCAyoy_{t-1} - 0.327\ \Delta IPCAyoy_{t-2} + 0.167\ (\Delta Selic_{t-1} - \Delta Selic_{t-13}) + 0.056\ (EMBI_{t-4} - EMBI_{t-16}) + 0.009\ (\Delta FX_{t-2} - \Delta FX_{t-14}). \quad \textbf{(8-17)}$$

The annual IPCA approximations using equation **(8-15)**, obtained from model IPCA_ADL2, and equation **(8-17)**, obtained from model IPCA_ADL3 are plotted with the effective annual IPCA index in Figure 8-27. The approximations are obtained using the annual IPCA data up to June 2005 and observable data for the exchange rate, the Selic rate, and the EMBI spread.

Figure 8-27: Annual IPCA approximation from data period July 2005 until April 2007.



8.3 Conclusion

In this study, a set of variables that are analyzed as inflation determinants in the previous chapter are empirically proven for their ability to explain the two inflation indexes in Brazil, the IGP-M and the IPCA index. Four models were presented above for each index. The first model is the ADL model estimated from the previous analysis by using the one-year nominal interest rate as one of the explanatory variables, the second and the third model were models built by using other explanatory variables. The fourth model is a benchmark models which is a simple autoregressive process that only includes the past values of the index itself as explanatory variables. The focus is on empirically assessing the out-of-sample forecasting abilities of the chosen explanatory variables for each inflation index.

In-sample and out-of sample forecasts were performed. The in-sample forecast is a one step ahead forecasting based on the model that is estimated using the full sample, which is the data period from January 2000 until April 2007, and kept constant throughout the forecasting exercise. For the dynamic 'out-of-sample' forecasts the coefficients are estimated with rolling regressions. At each new *h* steps ahead forecasting the sample is extended by one further observation and the coefficients are re-estimated. The first estimated models for 'out-of-sample' forecasting is based on 50 data points (adjusted), that is corresponding to the data period from January 200 until May 2004 for IGP-M index and the data period from January 2000 until July 2004 for IPCA index. The 'out-of-sample' forecasts are done by using the estimated model and index time series data up to the last observation used for estimating the model and the other **observed** explanatory variables. In both indexes, the first three models outperform clearly the benchmark models based on its goodness of fit and the forecasting ability.

Among the first three models for the IGP-M index, IGPM_ADL2 and IGPM_ADL3, where the term spread, and the exchange rate in IGPM_ADL3, are used as explanatory variables, perform better than IGPM_ADL1; the adjusted R^2 of these two models are much higher than the adjusted R^2 of the model IGPM_ADL1. These two models also perform better in 'out-of-sample' forecasting. The RMSE of these two models are circa 40% lower than the RMSE of IGPM_ADL1 for forecasting horizons of 6, 12, and 24 months and circa 10% for forecasting horizon of 18 months. These results confirm that the term spread and the exchange rate embody significant information for forecasting future IGP-M index in real time. Based on 'out-of-sample' forecasting ability, the difference between IGPM_ADL2 and IGPM_ADL3 in their RMSE is very small. However, it is observed that the 'out-of-sample' forecasts based on the model IGPM_ADL3 follow more the pattern of the effective IGP-M index. In this case, the model IGPM_ADL3 would be preferable to the model IGPM_ADL2.

Comparing the models for the IPCA index with the models obtained for IGP-M index, the performance of the IPCA models is much less in the terms of their goodness of fit. The adjusted R^2 of the IPCA models are lower compared with the adjusted R^2 of the IGP-M models. Among the first three models for the IPCA index, IPCA_ADL3 has the highest adjusted R^2. Based on the 'out-of-sample' forecasting ability, it is clearly observed that the forecasted values based on the model IPCA_ADL3 have more similar pattern as the effective IPCA index, while the forecasted values based on IPCA_ADL2 have a flatter pattern. In this case, the model IPCA_ADL3 would be preferable to the model IPCA_ADL2.

The time horizons of the 'out-of-sample' forecasts presented in this section are 6, 12, 18, and 24 months. A closer look at the coefficients variation through the rolling

regression is done by calculating the standard deviation and the mean of the estimated coefficients variation. Looking at the coefficients variation of the model IPCA_ADL3 and IGPM_ADL3, some of the coefficients tend to converge to some point while some of the coefficients have a down/up trend as more data is used to estimate the coefficients. Therefore, to maintain the 'out-of-sample' forecasting ability of the models, it is recommended to re-estimate the parameters of the models within a short time horizon.

The models are converted into models in terms of annual basis for inflation since the annual inflation index is of interest. Due to this conversion, the models in terms of annual basis for inflation would be less confidence. The annual inflation is approximated by summing the past 12 monthly inflation indexes. The standard error of the converted model would also 12 times bigger than the standard error of the models built in terms of monthly inflation index. The way of obtaining the confidence interval is described in Appendix A.

9 Impact analysis

In the previous chapter, models for IGP-M and IPCA index are evaluated. Based on the stress scenarios for the yield curve and the corresponding inflation developments based on the models, one could analyze whether the inflation-linked bonds indeed fit to the ALM perspectives for the situation in Brazil. In other words, the aim of the impact analysis is to analyze what would happen with the market value risk and the earnings risk when the inflation-linked bonds are integrated in the ALM framework, and compare these results with the case when the inflation-linked bonds are not integrated.

So far, the focus has been on the forecast errors resulting from the model specification. The question how to forecast the explanatory variables is out of the scope of this project. To be able to do the impact analysis, stress scenarios for the explanatory variables have to be obtained which correspond to the stress scenarios for the yield curve. These stress scenarios for the explanatory variables are proposed based on the principal component analysis. This will be analyzed in Section 9.1 of this chapter. The impact analysis will be done by using a simple balance sheet in which inflation-linked bonds are positioned at the asset side.

9.1 Principal Component Analysis (PCA)

A set of stress scenarios for the yield curve are used in the ALM framework for measuring both market value risk and earnings risk. These yield curve scenarios are proposed based on the principal component analysis (PCA). PCA is a statistical technique which attempts to describe the behavior of a range of correlated random variables, in this case, the various interest rates with different maturities, in terms of a small number of uncorrelated principal components. It is a way of identifying the interrelationships among the variables and, basically, the idea is to develop an idea of the movements of the variables that may arise in the future based on the historical behavior. For more information, the reader will be referred to [3].

Models resulted from the evaluation explain the relationship between the inflation and the interest rates. Besides the interest rate indicators, the models also include other variables as explanatory variables. Therefore, the corresponding stress scenarios for other variables, i.e., the exchange rates and the EMBI spread, need to be obtained.

For this impact analysis, PCA will be performed to examine the behavior and the evolution of the interest rate for different time to maturities, the exchange rate, and the EMBI spread over the period from January 2000 to April 2007. The data that are used are the same as the data used in the previous analysis and they are all monthly data. Thus, the variables that are included in the principal component analysis are the Selic rate, the yield curve that is assumed to be described by 10 points, located at 1, 2, 3, 6, and 9 months, 1, 2, 3, 4, and 5 years, the exchange rate, and the EMBI spread.

The analysis is thus based on monthly movements in a given period and is performed by using the S-Plus script that is used for proposing the yield curve scenarios for ALM analysis [4]. Small adjustments have to be made since the script is meant for daily data as inputs, while the data used in this project is monthly data.

Several options have to be made before performing the analysis. The analysis can be based on the either relative, absolute, or logarithmic monthly changes to make sure that the input is (weakly) stationary and the type of matrix from which the eigenvectors and its corresponding eigenvalues are calculated can be either the correlation or the covariance matrix. For this analysis, a relative type of monthly change is chosen as input of the data. The results of the principal component analysis depend on the scales at which the variables are measured. Therefore, if the variables are measured in different units, the variables should be standardized before the PCA is carried out. By standardizing the input data, all variables have the same variation, i.e., a standard deviation of 1. This standardization can be done by using the correlation matrix, instead of the covariance matrix. Since the variables included in the PCA have different units of measurements, the correlation matrix is used[5].

The results of the PCA are given in Table 9-1. The first three principal components are found to explain most of the variation in the given variables over the period. The first, second and the third principal components explain 70.17%, 16.46% and 6.07% of the variation, which is in total 92.70% of the variance.

Table 9-1: The standard output from the performed PCA.

	Princ. Comp. 1	Princ. Comp. 2	Princ. Comp. 3
Eigenvalue	9.122	2.140	0.789
Importance (%)	70.17	16.46	6.07
Cum. Importance (%)	70.17	86.63	92.70

The observations with respect to which the scenarios are calculated are the observations from April 2007. For this impact analysis, the movements captured from the first and the second principal components will be used as stress scenarios.

The movements captured from these two principal components summarize the variation in the variables of 86.63%. A change in the level factor, that is the first principal component, corresponds to roughly a parallel shift in the yield curve. The upward and downward move of the yield curve for a time horizon of one year, obtained from the first principal component of the PCA, are plotted in Figure 9-1. The corresponding changes in EMBI spread and the exchange rates for a time horizon of one year are an upward or downward move of 84 basis points and 0.38 BRL/USD, respectively. These ramp up and ramp down scenarios of the variables will be used for measuring the earnings risk. Since the market value risk scenarios are immediate shocks, the ramp up and ramp down scenarios of the variables for measuring the market value risk will be based on a change with a time horizon of 1 month. These scenarios are given in Table 9-2.

[5] http://www.stat.psu.edu/~jglenn/stat505/15_princomp/07_princomp_alternative.html

Figure 9-1: Shift scenarios resulting from the PCA.

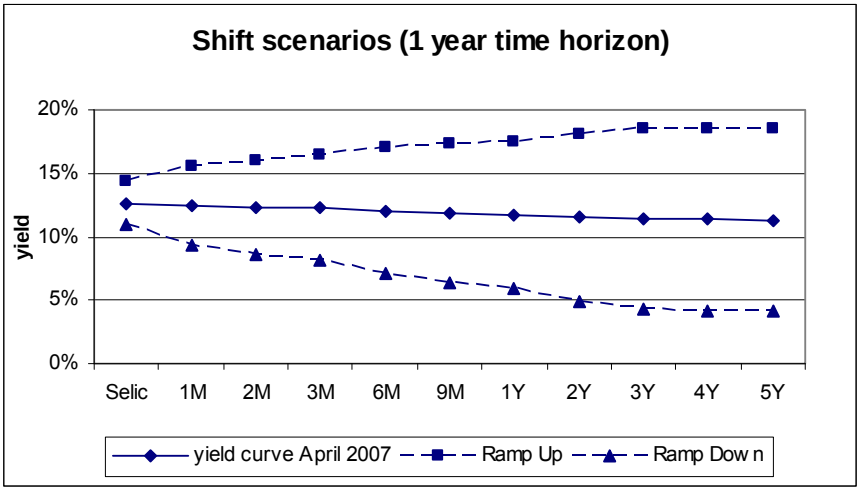


Table 9-2: The shift scenarios, with moves in terms of basis points except for FX (BRL/USD).

	Shift up scenario (one month Δ)	Shift down scenario (one month Δ)	Shift up scenario (one year Δ)	Shift down scenario (one year Δ)
Selic	50.11	-50.09	173.58	-173.52
1M	91.63	-91.62	317.40	-317.40
2M	107.50	-107.50	372.40	-372.40
3M	119.33	-119.35	413.37	-413.43
6M	141.92	-141.93	491.64	-491.66
9M	157.51	-157.52	545.64	-545.66
1Y	167.03	-167.03	578.59	-578.61
2Y	191.34	-191.33	662.82	-662.78
3Y	204.36	-204.37	707.93	-707.97
4Y	207.09	-207.07	717.40	-717.30
5Y	207.61	-207.60	719.17	-719.13
FX	0.11	-0.11	0.38	-0.38
EMBI	24.23	-24.21	83.93	-83.87

A change in the slope factor is explained by the second principal component. The counter clockwise and the clockwise rotations of the slope of the yield curve for a time horizon of one year are plotted in Figure 9-2. The corresponding changes in the EMBI spread and the exchange rates for a time horizon of one year are an upward or downward move of 58 basis points and 0.30 BRL/USD, respectively. These twist scenarios of the variables will be used for measuring the earnings risk. The twist scenarios of the variables for measuring the market value risk will be based on a change with a time horizon of one month. These scenarios are given in Table 9-2.

Figure 9-2: Twist scenarios resulting from the PCA.

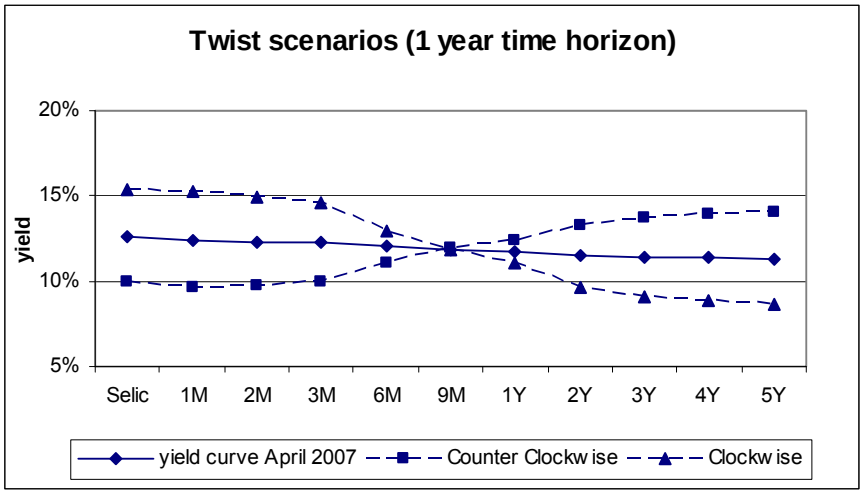


Table 9-3: The twist scenarios, with moves in terms of basis points except for FX (BRL/USD).

	Counter Clockwise scenario (one month)	Clockwise scenario (one month)	Counter Clockwise Scenario (one year)	Clockwise scenario (one year)
Selic	-78.12	78.11	-270.62	270.58
1M	-81.75	81.75	-283.20	283.20
2M	-75.72	75.72	-262.30	262.30
3M	-66.32	66.30	-229.73	229.67
6M	-27.15	27.15	-94.06	94.04
9M	1.40	-1.38	4.84	-4.76
1Y	19.25	-19.26	66.69	-66.71
2Y	51.91	-51.93	179.82	-179.88
3Y	66.40	-66.39	230.03	-229.97
4Y	73.35	-73.35	254.10	-254.10
5Y	78.14	-78.12	270.67	-270.63
FX	0.09	-0.09	0.30	-0.30
EMBI	16.87	-16.85	58.43	-58.37

The earnings risk scenarios for the explanatory variables in the ADL models based on the movements in the level and the slope factors are plotted in Figure 9-3. The past values of the variables are plotted from July 2005 to April 2007. From April 2007 to April 2008, the variables develop gradually with a change according to the shift scenarios and twist scenarios from PCA within one year (the third and the fourth columns of Table 9-2) and after that they will remain constant.

The IGP-M and the IPCA index developments for measuring the earnings risk which are obtained using the stress scenarios and the equations explaining the IGP-M and the IPCA indexes based on the ADL models are plotted in Figure 9-4. As observed, the developments of both inflation indexes follow the yield curve development. However, comparing the development of the two inflation indexes, the deviation movement of the IGP-M index has a larger range than the deviation movement of the IPCA index, as is expected based on the historical movement. Given the stress scenarios for the yield curve and other explanatory variables, the IPCA index development is stable compared to the IGP-M index development. In the ramp up and the ramp down

scenarios, the IGP-M index moves around 5% up and down, respectively, within two years. In the counter clockwise and the clockwise scenarios, the IGP-M index moves around 8% up and down, respectively, within two years. In the case of the IPCA index development, the upward and downward movements are within 1%.

Figure 9-3: The earnings scenarios based on the movements in the level and slope factors for the following variables; the Selic rate, the slope of the yield curve, the EMBI spread, and the exchange rate.

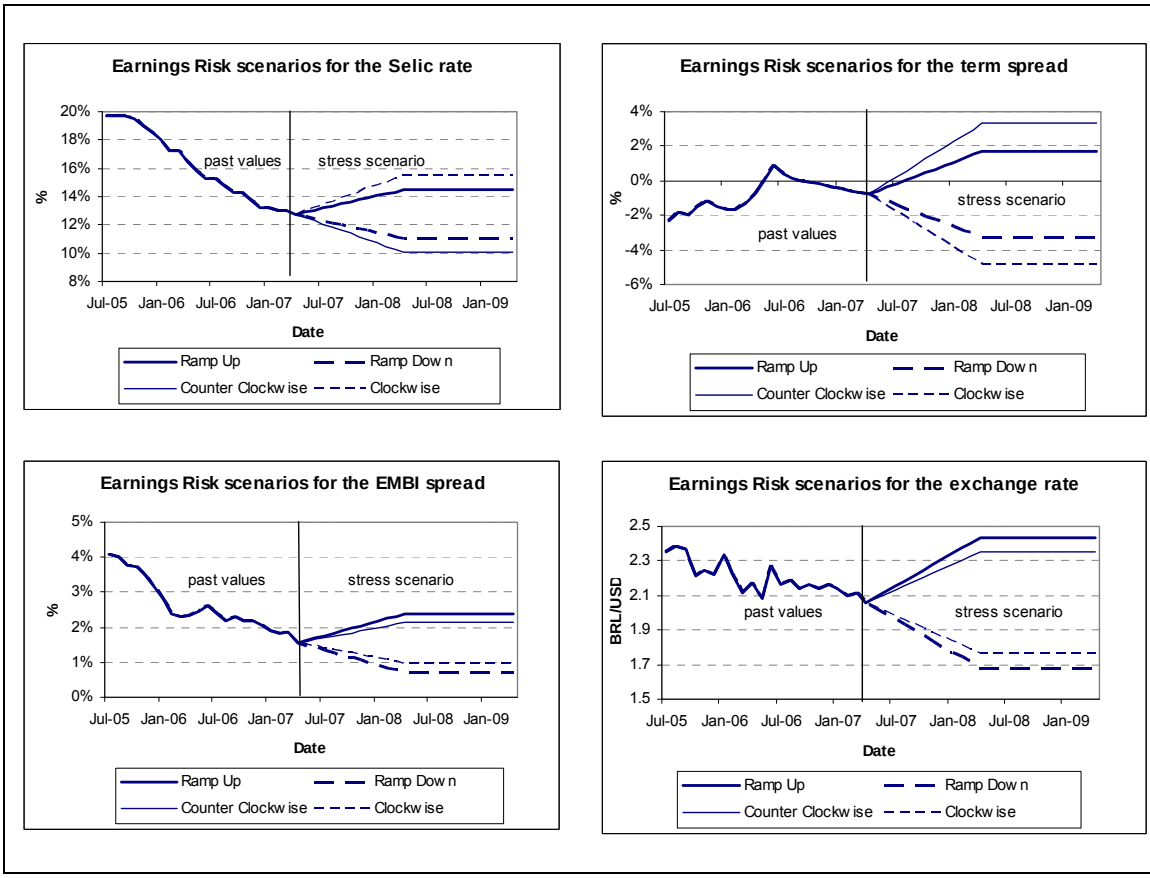
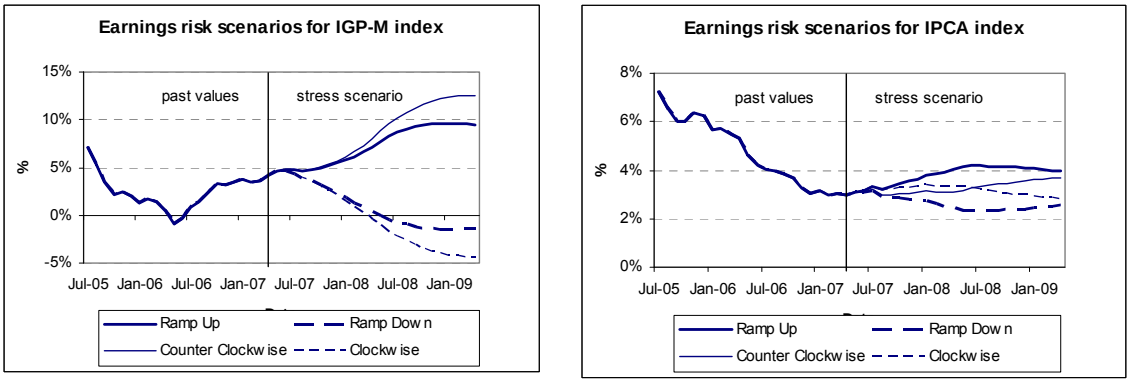


Figure 9-4: The Earnings Risk scenarios for the IGP-M and the IPCA indexes using the ADL models and the Earnings Risk scenarios for the explanatory variables obtained from PCA.



The Market Value Risk scenarios for the explanatory variables in the ADL models based on the movements in the level and the slope factors are plotted in Figure 9-5. The past values of the variables are plotted from July 2005 to April 2007. From April 2007 to April 2008, the variables develop gradually with a change according to the shift scenarios and twist scenarios from PCA within one month (the first and the second columns of Table 9-2) and after that they will remain constant. The corresponding Market Value Risk scenarios for the IGP-M and the IPCA indexes are plotted in Figure 9-6.

Figure 9-5: The Market Value risk scenarios based on the movements in the level and slope factors for the following variables; the Selic rate, the slope of the yield curve, the EMBI spread, and the exchange rate.

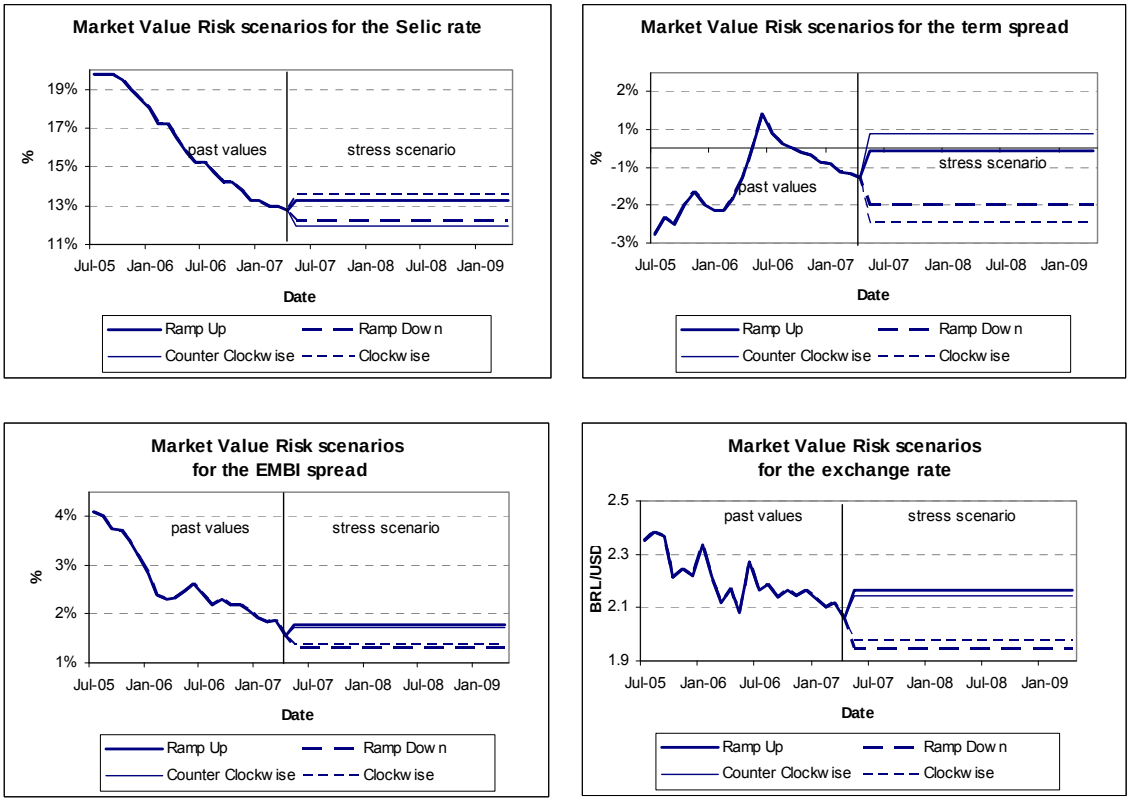
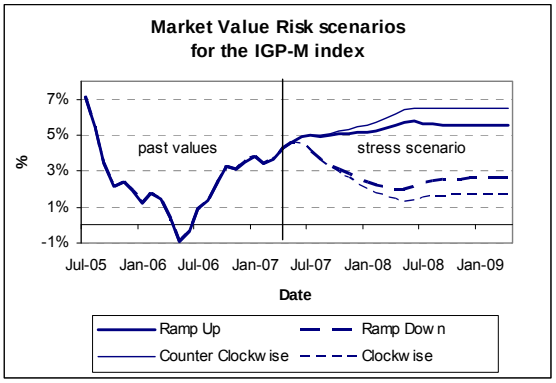
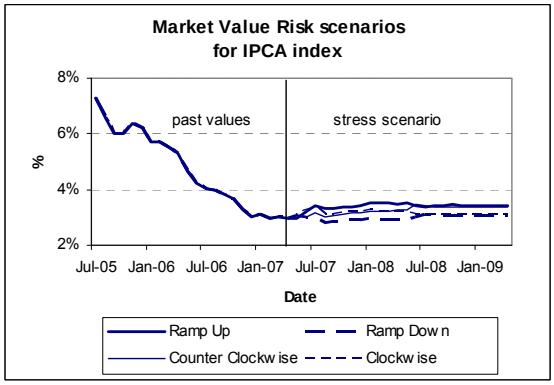


Figure 9-6: The Market Value risk scenario for the IGP-M and the IPCA indexes using the ADL models and the Market Value risk scenarios for the explanatory variables obtained from PCA.





9.2 Impact analysis using simple balance sheet

To analyze whether the inflation-linked bonds indeed fit to the ALM perspectives for the situation in Brazil, three balance sheets will be introduced. The balance sheet (1) integrates the **NTNB** bond position (**IPCA** index-linked bond) on the asset side, the balance sheet (2) integrates the **NTNC** bond position (**IGP-M** index-linked bonds) on the asset side, and the balance sheet (3) integrates the conventional bond position on the asset side. Given these three balance sheets and the stress scenarios for the yield curve and the corresponding inflation developments obtained in the Section 9.1, the earnings risk and the market value risk are calculated.

The impact of the interest rates and inflation movements on these three balance sheets from an ALM point of view will be analyzed and compared. The maturity of the bonds on the asset side of all balance sheets is two year. The liability position on all three balance sheets is one-year conventional bonds. The coupon rates for the conventional bonds are based on the observed yield curve at period April 2007. The coupon rates for the inflation-linked bonds are equal to the real rate of return of the conventional bonds. Let us assume that the expected inflation is equal to the observed inflation at period April 2007. The observed IGP-M and the observed IPCA indexes on April 2007 are 4.26% and 2.96%, respectively. The two-year interest rate on April 2007 is 11.49%. The coupon rates for NTNB and NTNC bonds are

$$(1+\text{coupon}_{NTNB}) = (1+11.49\%)/\ (1+2.96\%) \Leftrightarrow \text{coupon}_{NTNB} = 8.28\% \quad \textbf{(9-1)}$$

and

$$(1+\text{coupon}_{NTNC}) = (1+11.49\%)/\ (1+4.26\%) \Leftrightarrow \text{coupon}_{NTNC} = 6.93\% \quad \textbf{(9-2)}$$

respectively.

The balance sheets are illustrated in Table 9-4. The payment period of all bonds is semiannual. After one year, when the conventional bond on the liability side matures, a new production on the liability side is included in the earnings risk measurement, which is a one-year conventional bond whose coupon rate is equal to the one-year yield based on the stress scenario at that period.

Table 9-4: Three balance sheets that are used for the impact analysis.

Asset	Balance sheet (1)		Liability
8.28% NTNB bond (2 year)	1000	11.75% conventional bond (1 year)	800
		Equity	200
	1000		1000

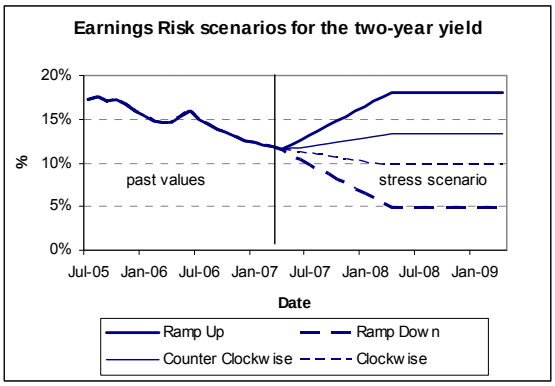
Asset	Balance sheet (2)		Liability
6.93% NTNC bond (2 year)	1000	11.75% conventional bond (1 year)	800
		Equity	200
	1000		1000

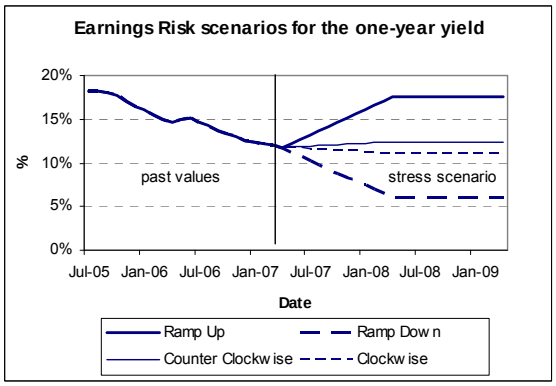
Asset	Balance sheet (3)		Liability
11.48% conventional bond (2 year)	1000	11.75% conventional bond (1 year)	800
		Equity	200
	1000		1000

9.2.1 Cumulative Earnings Risk

The IGP-M and the IPCA index developments used for measuring the earnings risk are plotted in Figure 9-4. The two-year and one-year interest rates scenarios are given in Figure 9-7. As observed, the movement of the one-year interest rate in the counter clockwise and the clockwise scenarios is small.

Figure 9-7: The earnings scenarios based on the movements in the level and slope factors for the following variables; two-year and one-year interest rates.





Recall that the cumulative earnings risk for a certain time horizon is obtained by calculating the relative change of the cumulative NII of the stress scenario compared to the base case scenario. The NII on the base case scenario is simply the NII when there are no interest rate changes. The monthly NII is difference between the accrued value of one month and the accrued value of the previous month. The accrued value is defined by the sum of the (indexed) principal and the accrued interest that has not been paid. In Figure 9-8, the developments of the NII per month based on various stress scenarios on three balance sheets are given.

The **balance sheet (1)** includes a two-year NTNB bond (IPCA index-linked bond) with a coupon rate of 8.28% on the asset side and a one-year conventional bond with a coupon rate of 11.75% on the liability side. As observed in Figure 9-4, the movements of the IPCA index have the same direction as the long end of the yield curve. However, the movements of the IPCA index are much smaller than the movements of the long end of the yield curve. Since the IPCA index development is more or less stable in all stress scenarios, the earnings from the asset side position based on the stress scenarios and the base case scenario do not differ that much. Up to one year, there is no change in the coupon rate of the conventional bond in all stress scenarios. Therefore, the monthly NII on the stress scenarios and the base case scenario are more or less the same up to one year.

After one year, the liability position will be repriced. Based on the ramp up scenario, the coupon rate of the conventional bond will rise to 17.53% that corresponds to an upward move of 5.79%, while in the base case scenario the coupon rate remains the same. This would cause a lower monthly NII in the ramp up scenario. Based on the ramp down scenario, the coupon rate of the conventional bond will decrease to 5.96% that corresponds to a downward move of 5.79%. This would then cause a higher monthly NII in the ramp up scenario. Now consider the counter clockwise and the clockwise scenarios. The movements of one-year interest rate over one year time horizon are an upward or downward move of 0.67%. Due to this small change on the coupon rate, the monthly NII based on the counter clockwise and the clockwise scenarios do not differ much from the monthly NII in the base case scenario.

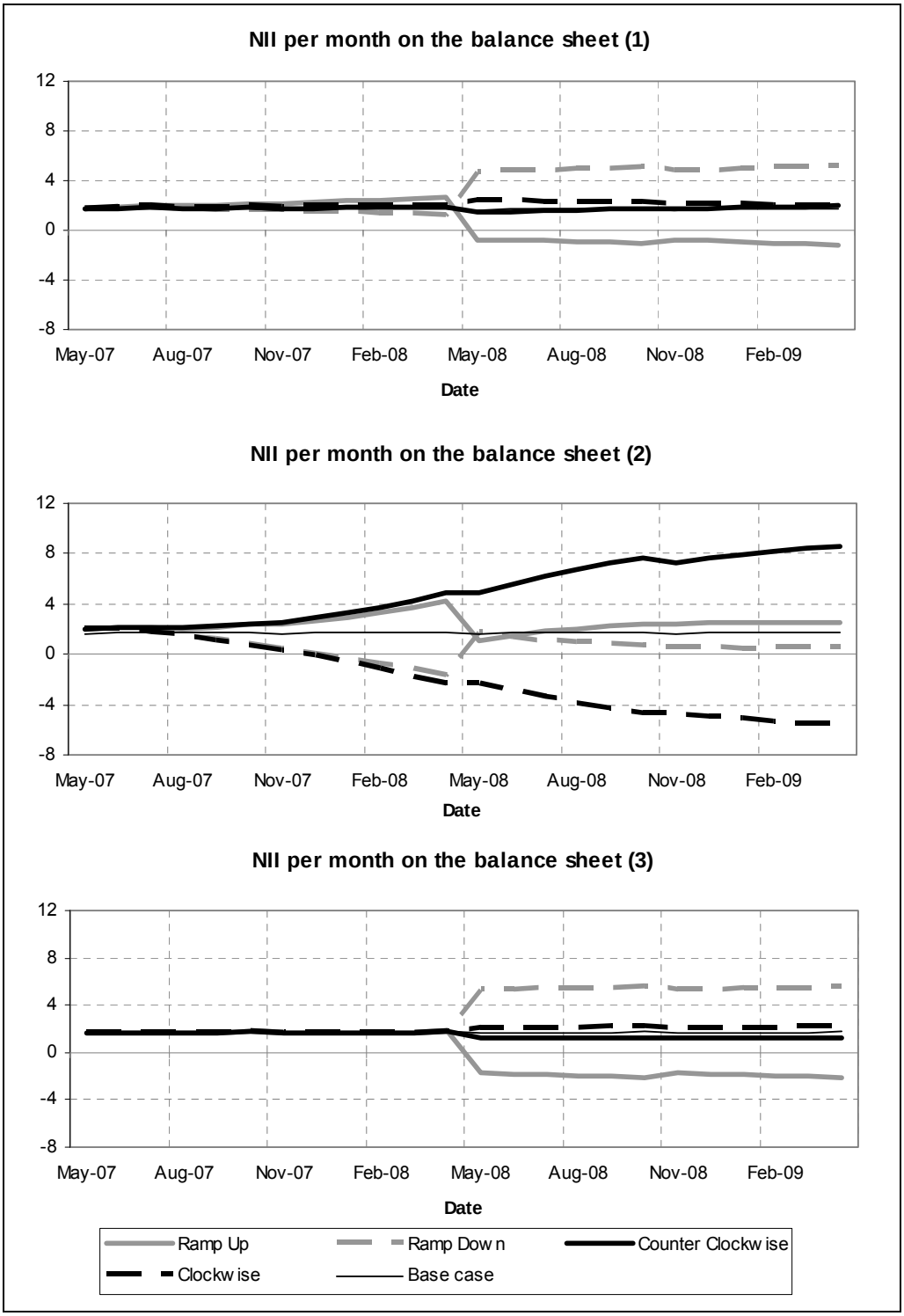
The **balance sheet (2)** includes a two-year NTNC bond (IGP-M index-linked bond) with a coupon rate of 6.93% on the asset side and a one-year conventional bond with a coupon rate of 11.75% on the liability side. As observed in Figure 9-4, the movements of the IGP-M index have the same direction as the long end of the yield curve and the movements are much more volatile than the movements of the IPCA index. Consider the monthly NII up to one year. As the case in the balance sheet (1), the difference in monthly NII based on the stress scenarios and the base case scenario is

only caused by the changes in the earnings from the asset side position since the liability position will not be repriced within one year. It is observed that the IGP-M index is highly correlated with the long-term interest rate scenario. Investing in NTNC bond would lead to higher earnings if the IGP-M index increases since the principal is indexed. Therefore, the monthly NII in the ramp up and the clockwise scenarios are higher than the monthly NII in the base case scenario. On the other hand, the monthly NII in the ramp down and the clockwise scenarios are lower than the monthly NII in the base case scenario.

Consider the monthly NII after one year; the liability position will be repriced. The monthly NII on the ramp up scenario decreases since the coupon rate of the conventional bond on the liability side is higher. However, the monthly NII based on the ramp up scenario is still higher than the monthly NII based on the base case scenario due to the higher earnings from the asset side position. The coupon rate of the conventional bond on the ramp down scenario decreases, which leads to a lower cost. Therefore, the monthly NII goes up a bit after one year. In the case of the counter clockwise and the clockwise scenarios, the coupon rate of the conventional bond does not change that much, which causes the monthly NII continually go up and down, respectively.

The **balance sheet (3)** includes a two-year conventional bond (IGP-M index-linked bond) with a coupon rate of 11.48% on the asset side and a one-year conventional bond with a coupon rate of 11.75% on the liability side. Since both positions are not linked to inflation index, the coupon payments are equal in both stress and base case scenarios until the new production on the liability side is included.

Figure 9-8: The NII per month on the three balance sheets based on the stress scenarios.

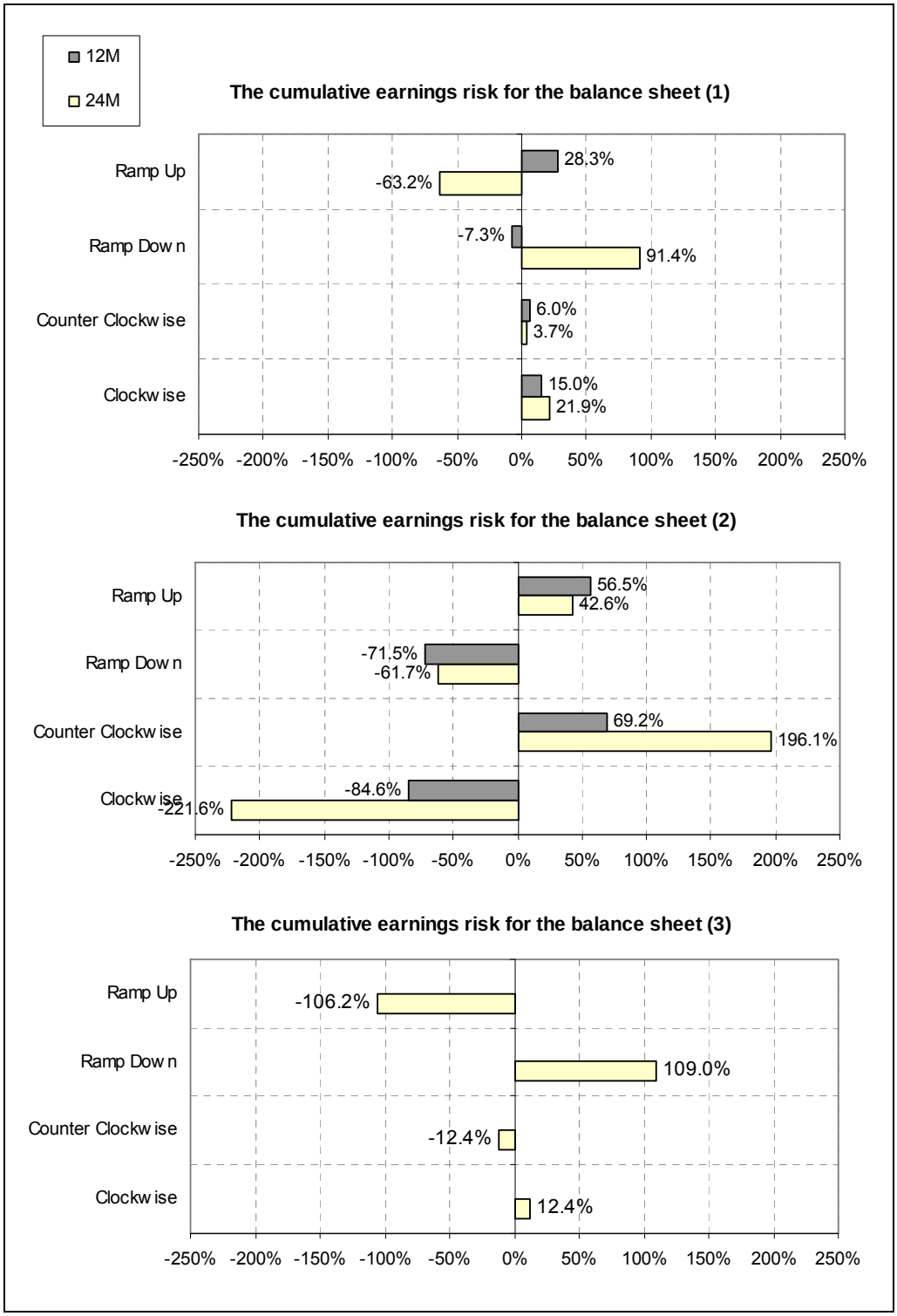


The cumulative earnings risks for time horizons up to two years for the three balance sheets are given in Figure 9-9. Overall, the earnings risk on the balance sheet (2) is higher than the earnings risk on the balance sheet (1) due to the higher volatility of IGP-M index and a stable IPCA index.

Let us look at the earnings risk in the situation where the interest rates follow the ramp up and counter clockwise scenarios. Taking the positions as in the balance sheet (2) would have a positive earnings risk. This is expected since the increasing interest rate scenario is accompanied with an increasing IGP-M index development. Investing in NTNC bond, which is an IGP-M index-linked bond, provides a protection against the possible upward shock in the interest rates. On the other hand, investing in NTNB bond, which is an IPCA index-linked bond, as in the balance sheet (1) would give less positive results on earnings risk since the IPCA index development is much more stable than the IGP-M index development. This would mean that investing in NTNB bond would offset the negative effects of the increasing interest rate much less than investing in NTNC bond. However, both positions, as in balance sheet (1) and balance sheet (2), would be more preferable to the position as in the balance sheet (3).

In the situation where the interest rates follow the ramp down and clockwise scenarios, taking the position as in balance sheet (2) could be considered as the worst alternative. This is because the IGP-M index develops to a lower level than is expected which makes the earnings on the inflation-linked bond is lower than the earnings on the conventional bond. Taking the position as in the balance sheet (1) would give a positive change in earnings since the IPCA index is more or less stable.

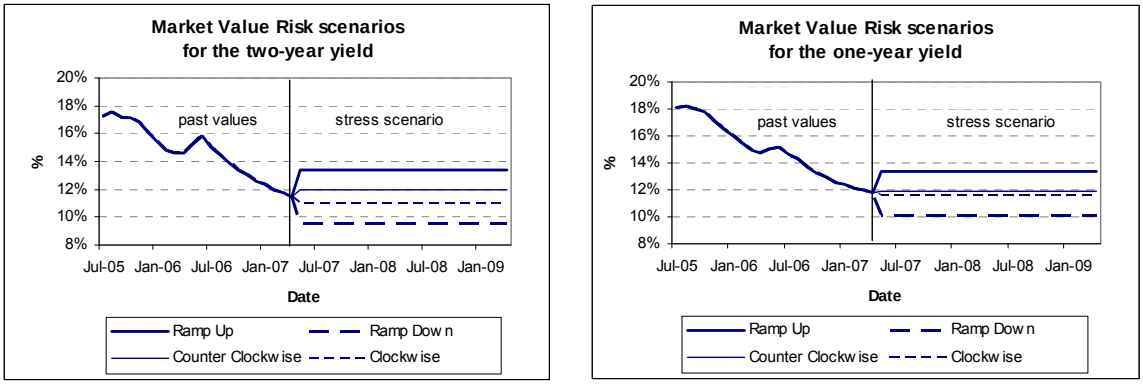
Figure 9-9: The cumulative earnings risk for the three balance sheets.



9.2.2 Market Value Risk

The IGP-M and the IPCA index developments used for measuring the market value risk are plotted in Figure 9-6. These scenarios are based on the models IGPM_ADL3 and IPCA_ADL3 and the market value risk scenarios for the explanatory variables from PCA. The market value risk scenarios for the two-year and one-year interest rates are given in Figure 9-10. As observed, the movement of the one-year interest rate in the counter clockwise and the clockwise scenarios is small.

Figure 9-10: The Market Value Risk scenarios based on the movements in the level and slope factors for the following variables; two-year and one-year interest rates.

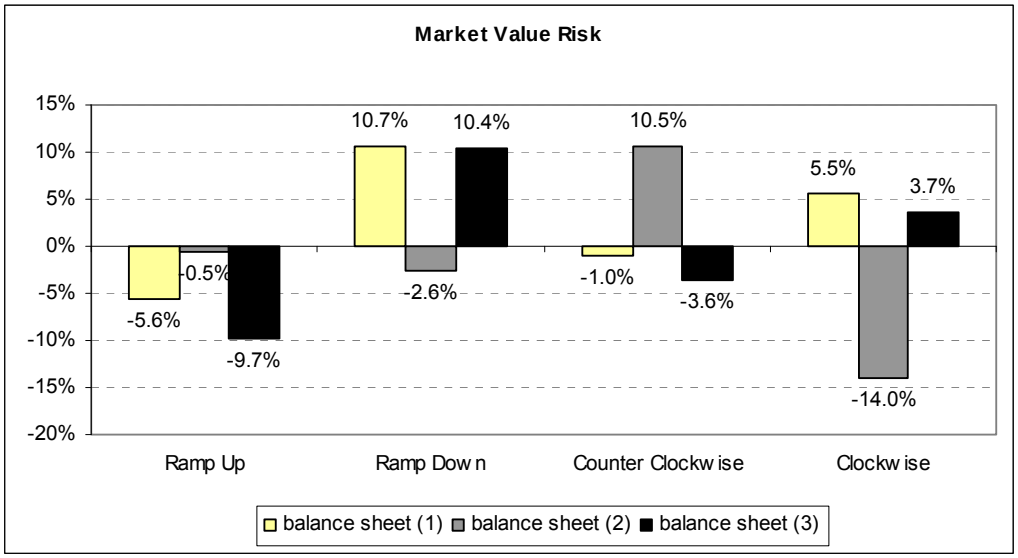


Recall that the market value of equity (MVE) is the difference between the net present value of all cash flows from assets and the net present value of all cash flows from liabilities. To show the sensitivity of the market value of equity to changes in interest rate, the market value of equity on the base case scenario is compared to the market value of equity on the stress scenarios. The results of the market value risk are depicted in Figure 9-11.

As mentioned in Section 3.2.2, the interest rates and the inflation movement affect the market value of equity directly in different ways. The inflation movement has a direct impact on the cash flows and the interest rate movement has a direct impact on the discount factor. In the ramp up and the counter clockwise scenarios, the discount factors are smaller that would lead to lower net present values. However, the corresponding IGP-M index development rises to a higher level that leads to higher cash flows on the asset side of the balance sheet (2). This would offset the negative effect of the interest rate hikes. On the other hand, investing in NTNB bonds as in balance sheet (1) would offset a much smaller parts of negative effects of the interest rate hikes, due to a stable development of IPCA index.

When the interest rates follow the ramp down and the clockwise scenarios, which correspond to a downward shock in the long-end of the yield curve, the IGP-M index would develop to a lower level that would lead to smaller cash flows. This explains the negative change in the market value of equity on the balance sheet (2). On the other hand, taking a position as in the balance sheet (1) would give us a market value risk that does not differ much from the market value risk when taking a position as in the balance sheet (3).

Figure 9-11: Market value risk of the three balance sheets for specified stress scenarios.



10 Appendix

A Confidence interval for the inflation scenarios

In Section 9.1, the inflation development from period April 2007 until April 2009 is obtained based on the chosen models and the stress scenarios for macro-economic explanatory variables. In this section a way of obtaining confidence intervals for the inflation development (see Figure 9-4 and Figure 9-6) will be discussed. An approach that is used to compute the confidence interest is the bootstrap. Consider time period T as April 2007, and the inflation development that is of interest is from period T+1 until T+24.

To construct the 95% confidence interval of inflation development, its sampling distribution is needed. Let λ be a general function the inflation development at time T+1 until T+24, and $S = \hat{\lambda} - \lambda$, with distribution Ψ under the estimated parameters based on the sample January 2000 until April 2007 and $\hat{\lambda}$ is the estimated inflation development. Let's first assume that the distribution Ψ is known. Then, the 95% confidence interval for λ would be

$$\Psi^{-1}(0.025) \le \hat{\lambda} - \lambda \le \Psi^{-1}(0.975) \Leftrightarrow$$
$$[\hat{\lambda} - \Psi^{-1}(0.975), \hat{\lambda} - \Psi^{-1}(0.025)].$$

But the distribution Ψ is unknown. This distribution will be approximated based on the bootstrap, by generating $S_1^*, \ldots, S_B^*$ using Monte Carlo simulation. Note that

$$\Psi^{-1}(0.025) \approx S^*_{(\lfloor B\times(0.025)\rfloor)}; \quad \Psi^{-1}(0.975) \approx S^*_{(\lfloor B\times(0.975)\rfloor)}.$$

The procedure to generate a Monte Carlo sample $S_1^*, \ldots, S_B^*$ is the following. The procedure illustrated below is meant for the IGP-M development based on the model IGPM_ADL3.

1. Estimate the parameters of the model based on the data sample from period January 2000 until April 2007. This is done in Section 8.1.1. The estimated model is given below.

$$IGPMmom_t = 0.002 + 0.580\ IGPMmom_{t-1} + 0.071\ Slope_{t-1} + 0.011\ (\Delta FX_{t-1} + \Delta FX_{t-2}).$$

2. The residuals of the regression $\{\varepsilon_i\}$, which is the difference between the effective monthly IGP-M index and the fitted IGP-M index based on the model, can be determined.

3. Approximate the (mean zero) noise distribution by the empirical distribution of the centered residuals $\{\tilde{\varepsilon}_i\}$, $\tilde{\varepsilon}_i = \hat{\varepsilon}_i - \frac{1}{N}\sum_i \hat{\varepsilon}_i$, where *N* is number of the residuals.

4. Compute the inflation development $\lambda = \{IGPMyoy_{T+1|T}, \ldots, IGPMyoy_{T+24|T}\}$ based on the converted IGPM_ADL3 model given below, the data samples up to time period T, and the stress scenario for the ***Slope*** variable for period T+1 until T+24.

$\Delta IGPMyoy_t = 0.580\ \Delta IGPMyoy_{t-1} + 0.071(Slope_{t-1} - Slope_{t-13}) + 0.011(\Delta FX_{t-1} - \Delta FX_{t-13} + \Delta FX_{t-2} - \Delta FX_{t-14})$.

5. Compute the inflation development $\{IGPMyoy_{T+1|T}, \ldots, IGPMyoy_{T+24|T}\}$ as in step 4, but by adding the residuals in every step of the forecasting to obtained $\hat{\lambda}$.

$\Delta IGPMyoy_t = 0.580\ \Delta IGPMyoy_{t-1} + 0.071(Slope_{t-1} - Slope_{t-13}) + 0.011(\Delta FX_{t-1} - \Delta FX_{t-13} + \Delta FX_{t-2} - \Delta FX_{t-14}) + (\varepsilon_t - \varepsilon_{t-12})$.

The residuals ε_t and ε_{t-12} are the bootstrap samples generated from the empirical distribution of $\{\tilde{\varepsilon}_i\}$.

6. Compute $S^* = \hat{\lambda} - \lambda$.

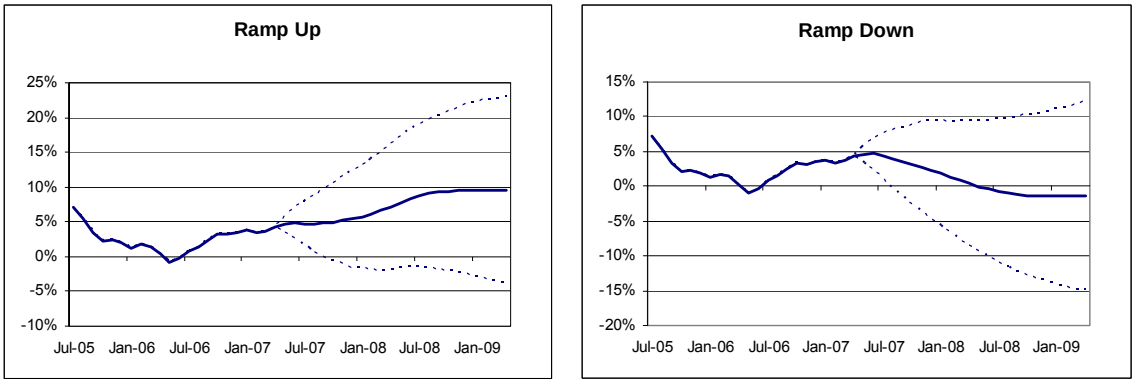
7. Repeat steps 5 and 6 for B times; say 5000, to get an approximation of distribution Ψ. To obtain the 95% confidence interval, the simple method is by taking 2.5% and 97.5% quantiles of the B replication $S_1^*, \ldots, S_B^*$ as the lower and upper bound respectively.

Hence, by replacing the quantiles of Ψ by the Monte Carlo approximated quantiles of the bootstrap approximation, the confidence interval for λ is given by

$$[\hat{\lambda} - S^*_{(\lfloor B\times(0.975)\rfloor)}, \hat{\lambda} - S^*_{(\lfloor B\times(0.025)\rfloor)}]$$

The confidence interval for the inflation development based on the bootstrap are plotted below, along with the inflation developments itself.

Figure 10-1: The IGP-M development based on the model IGPM_ADL3, along with the 95% confidence interval obtained using the bootstrap.



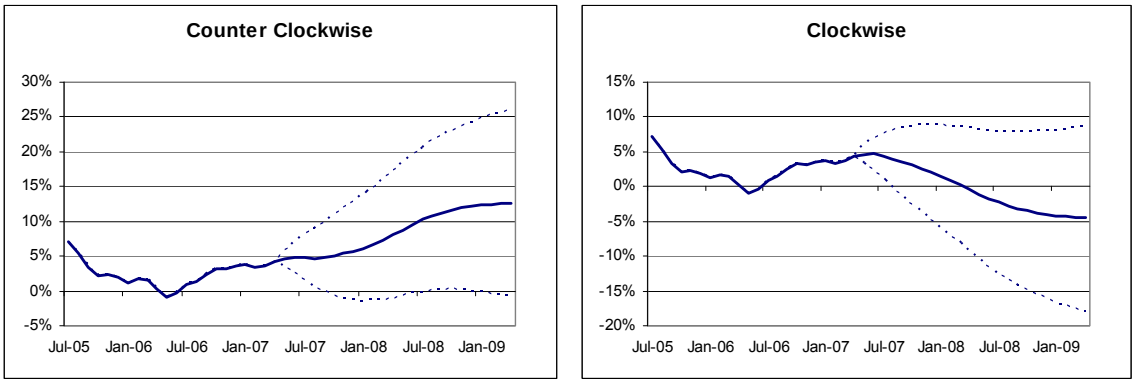
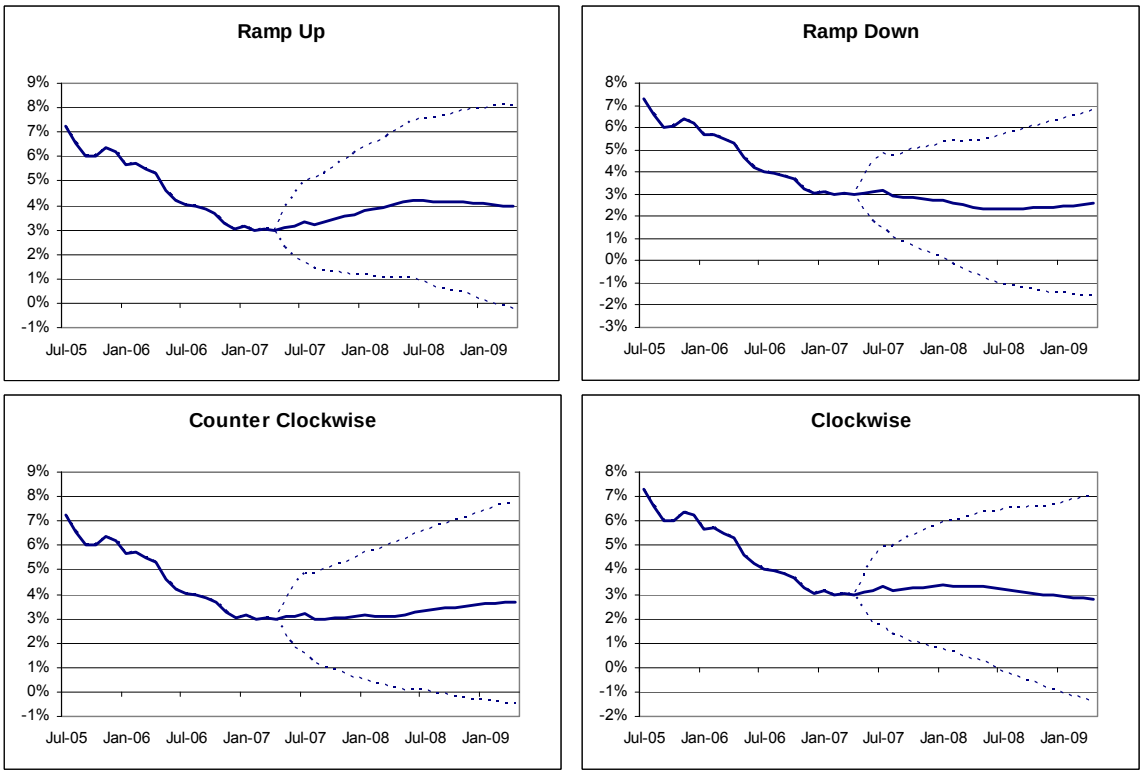


Figure 10-2: The IPCA development based on the model IGPM_ADL3, along with the 95% confidence interval obtained using the bootstrap.



B Example of payments on inflation-linked bonds

To give an illustration of payments on indexed bonds, consider a one-year inflation-linked bond. Table 10-1 gives the main characteristics of the bond.

Table 10-1: Main characteristics of the bond.

Principal amount	1,000
Fixed coupon payment (% per year)	8%
Time until maturity	one year
Payment	semi-annual

The inflation is considered to be positively related to the nominal interest rate on annual basis. The parameters that describe the inflation and interest rate developments are summarized in Table 10-2. The linear relation between the inflation and the interest rate is formulated in Equation 4-1. The interest rate is considered to increase to 12% at the end of the year and the increasing movement is gradual over one year.

Table 10-2: Inflation and interest rate development parameters.

yield at the settlement date	10%
β	0.5
C	-3%
change in yield in one year	2%

In Table 10-3 below, the development of the bond during the twelve months is given. The second column shows the annual base movement on the interest rate each month which is constant. The third column shows the annual base interest rate in each month. It is simply the annual base interest rate of the previous month plus the annual movement of the interest rate. The fourth column gives the compounded monthly base interest rate.

The next column gives the annual inflation that is given by multiplying the annual base interest rate by β, which is 0.5 and adding c, which is -3%. The next column is the compounded monthly base inflation.

The fixed coupon rate is 8% which corresponds to a monthly fixed coupon rate of 0.64%. The principal is monthly indexed. In the accrued value column, the principal plus the accrued interest of the bond that has not been paid is indexed. In other words, the accrued value is calculated as follows.

$$\text{Accrued Value}_t = \text{Accrued Value}_{t-1}\ (1+\text{inflation}_t)\ (1+\text{coupon}_t)$$

The subscript denotes the month. At month 6 the accrued (indexed) coupon will be paid, that is the first cash flow. Automatically, the accrued value in month 6 will be reduced by the first cash flow which leaves only the principal amount at the origination of the bond. After six months the bond starts accruing again at the original principal amount. At month 12 the indexed principal and the second accrued coupon will be paid.

The last column gives the NII. This is obtained by taking the difference between the accrued value of one month and the accrued value of the previous month.

Table 10-3: Development of the bond during 12 months.

month	Interest rate (%)			Inflation (%)		Coupon (%)	accrued value	cash flows	NII
	gradual move	annual base	monthly base	annual base	monthly base				
0		10.00%	0.80%	2.00%			1,000.00		
1	0.17%	10.17%	0.81%	2.08%	0.17%	0.64%	1,008.16		8.16
2	0.17%	10.33%	0.82%	2.17%	0.18%	0.64%	1,016.47		8.30
3	0.17%	10.50%	0.84%	2.25%	0.19%	0.64%	1,024.90		8.44
4	0.17%	10.67%	0.85%	2.33%	0.19%	0.64%	1,033.48		8.58
5	0.17%	10.83%	0.86%	2.42%	0.20%	0.64%	1,042.20		8.72
6	0.17%	11.00%	0.87%	2.50%	0.21%	0.64%	1,000.00	51.07	8.87

7	0.17%	11.17%	0.89%	2.58%	0.21%	0.64%	1,008.58		8.58
8	0.17%	11.33%	0.90%	2.67%	0.22%	0.64%	1,017.29		8.72
9	0.17%	11.50%	0.91%	2.75%	0.23%	0.64%	1,026.16		8.86
10	0.17%	11.67%	0.92%	2.83%	0.23%	0.64%	1,035.17		9.01
11	0.17%	11.83%	0.94%	2.92%	0.24%	0.64%	1,044.32		9.16
12	0.17%	12.00%	0.95%	3.00%	0.25%	0.64%	0.00	1,053.64	9.31

C Error Correction Models

In the previous analysis, the error correction models are mentioned to find the long-run relationship between the explanatory variables and the dependent variable. Generally, the error correction models are meant for non-stationary data in order to capture both long and short-term dynamics in a single statistical model. However, the property of error correction models when used with stationary data is investigated in the paper Keele, L **[6]**. The equivalence between the autoregressive distributed lag (ADL) models and error correction (ER) models is demonstrated in this paper. In other words, the error correction model can be derived from the autoregressive distributed lag model. The advantage of the error correction model is that it explicitly shows the equilibrium relationship between the variables as well as the short-term effects of the explanatory variables, if they exist.

In the case of time series data, a change in explanatory variables may affect the dependent variable immediately, or the effect may be delayed, occurring in the future across several time periods as is the case in many relations among macro-economic variables. There are at least three possible combinations of dynamic effects; the first one is the presence of contemporaneous effects of explanatory variables, where the explanatory variables affects the dependent variable immediately but that effect does not persist into the future. The second one is the presence of contemporaneous effects as well as an equilibrium effect that persists across future time periods and decays at some rate. The third possible dynamic effect is when there are no contemporaneous effects, but instead there is a presence of equilibrium effect that occurs across future time periods.

The derivation of the ER model from ADL model will be illustrated below. Consider an ADL model in the following form,

$$Y_t = \alpha_0 + \alpha_1 Y_{t-1} + \alpha_2 Y_{t-2} + \beta_1 X_{t-1} + \varepsilon_t . \qquad \textbf{(10-1)}$$

The model IGPM_ADL2 has the same form as **(10-1)** with the IGP-M monthly index as Y_t and the term spread as X_t. The short run effect of the term spread is readily estimated in the model by the coefficient β_1, which gives the immediate effect of a change in X at some given t. However, the long run equilibrium effects are given by the expected value of Y_t. Let $y^* = E(Y_t)$ and $x^* = E(X_t)$ for all t. If the two processes moved together without error, in the long run, they would converge to the following equilibrium values:

$$y^* = \alpha_0 + \alpha_1 y^* + \alpha_2 y^* + \beta_1 x^* . \qquad \textbf{(10-2)}$$

Solving for $\mathbf{y^*}$ in terms of $\mathbf{x^*}$ yields

$$y^* = k_0 + k_1 x^* \qquad \textbf{(10-3)}$$

where,

$$k_0 = \frac{\alpha_0}{1-\alpha_1-\alpha_2}, \quad k_1 = \frac{\beta_1}{1-\alpha_1-\alpha_2}. \tag{10-4}$$

This equation represents the long run relationship between Y and X. Any deviation from equilibrium, $y^* - (k_0 + k_1 x^*) \neq 0$, should induce a change back to the equilibrium in the next period.

Obtaining the ER model from **(10-1)** is done by the following procedure. First, taking the first difference of Y would give

$$\Delta Y_t = \alpha_0 + (\alpha_1 - 1)Y_{t-1} + \alpha_2 Y_{t-2} + \beta_1 X_{t-1} + \varepsilon_t. \tag{10-5}$$

Next, adding and subtracting $\alpha_2 Y_{t-1}$ from the right hand side would give

$$\Delta Y_t = \alpha_0 + (\alpha_1 + \alpha_2 - 1)Y_{t-1} - \alpha_2 \Delta Y_{t-1} + \beta_1 X_{t-1} + \varepsilon_t. \tag{10-6}$$

To have explicitly a long-run term, the equation can be reformulated as follows

$$\Delta Y_t = \gamma_0 + \gamma_1 (Y_{t-1} - \gamma_2 X_{t-1}) + \gamma_3 \Delta Y_{t-1} + \varepsilon_t, \tag{10-7}$$

where

$$\gamma_0 = \alpha_0; \quad \gamma_1 = \alpha_1 + \alpha_2 - 1; \quad \gamma_2 = \frac{\beta_1}{1-\alpha_1-\alpha_2}; \quad \gamma_3 = -\alpha_2. \tag{10-8}$$

Note that the parameter γ_2 is the long-run multiplier, which is equal to k_1 from **(10-4)**. The term $(Y_{t-1} - \gamma_2 X_{t-1})$ describes the long-term effects of the relationship. The term γ_1 is interpreted as the speed at which Y adjusts to any discrepancy between Y and X in the previous period. The error correction is interesting because it gives the equilibrium effect explicitly so that analysts can discriminate between the relations of two of more variables that have short versus long-run behaviour.

D Ordinary Least Square (OLS) technique.

Ordinary Least Square is used to estimate the parameters from a linear regression based on the least square method. The least square method is a technique for fitting a straight line through a set of points in such a way that the sum of squared vertical distances from the observed points to the fitted line is minimized. For an illustration, a linear regression for two variables y_t and x_t is given by

$$y_t = \beta_0 + \beta_1 x_t + \varepsilon_t \tag{10-9}$$

The assumption is that there exists linear relationship between the variable to be predicted y_t (this is called the dependent variable) and the explanatory variable x_t that is selected to determine its relationship to the dependent variable. The dependent variable in this case is the inflation index, and the explanatory variables are the other macro-economic variables or the auto regressions of the dependent variable. In this model it is assumed that the macro-economic variables that are chosen to be the explanatory variables are exogenous - that is does not depend on the dependent variable.

E Schwarz criterion

$$SIC = \ln(\hat{\sigma}) + \frac{2k}{T} \tag{10-10}$$

where $\hat{\sigma}$ is the residual variance, *k* is the total number of parameters estimated and *T* is the sample size.

F Wald test

Wald test is used to test whether the coefficients estimated without restrictions come close to the coefficients specified by the null hypothesis. The test statistic measures how close the unrestricted estimates of the coefficients to the proposed value of the coefficients (coefficient restriction) under the null hypothesis. The test statistic compares the residual sum of squares computed with and without the restrictions imposed. With assumption that the residuals are independent and identically normally distributed, the test statistics can be compared to F-statistic.

If the proposed values of the coefficients are valid, there should be little difference in the two residual sum of squares and the F-value should be small. For the Wald test, Eviews 3.1 is used.

G R^2

A mathematical term describing the proportion of variability in a data set $\{y_t\}$ that is explained by a statistical model. In this definition, the term "variability" is defined as the sum of squares. In other words, R^2 is a statistic that will give some information about the goodness of fit of a model.

$$R^2 = 1 - \frac{SS_E}{SS_T} \qquad \textbf{(10-11)}$$

where SS_E is the sum of squared errors, and SS_T is the total sum of squares. That is

$$SS_E = \sum_t (y_t - \hat{y}_t)^2; \quad SS_T = \sum_t (y_t - \bar{y})^2. \qquad \textbf{(10-12)}$$

where $\hat{y}_t$ is the fitted values based on a statistical model, $\bar{y}$ is the mean of the data.

H Adjusted R^2

As the R^2 never falls when additional regressors are added to the equation, the adjusted R^2 takes into account the loss of degrees of freedom associated with adding extra variables

$$\overline{R}^2 = 1 - \left(\frac{T-1}{T-k} (1 - R^2) \right) \qquad \textbf{(10-13)}$$

where T is the sample size and k is the total number of parameters in the model.

I Derivation of Augmented Dickey Fuller model

Consider an autoregressive process $\{y_t\}$ of order p, AR(p)

$$y_t = \gamma_1 y_{t-1} + \gamma_2 y_{t-2} + \ldots + \gamma_p y_{t-p} + \varepsilon_t. \qquad \textbf{(10-14)}$$

Then the regression model can be written in terms of first differences as follows,

$$\Delta y_t = \beta y_{t-1} + \alpha_1 \Delta y_{t-1} + \alpha_2 \Delta y_{t-2} + \ldots + \alpha_{p-1} \Delta y_{t-p+1} + \varepsilon_t, \qquad \textbf{(10-15)}$$

where

$$\beta = \left(\sum_{i=1}^{p} \gamma_i \right) - 1, \qquad \textbf{(10-16)}$$

and

$$\alpha_j = -\sum_{k=j+1}^{p} \gamma_k \,. \qquad \textbf{(10-17)}$$

For an illustration of the derivation, consider an AR(3) process $\{y_t\}$

$$y_t = \gamma_1 y_{t-1} + \gamma_2 y_{t-2} + \gamma_3 y_{t-3} + \varepsilon_t \,.$$

To get the term Δy_{t-2} on the right hand side, add term $\gamma_3 y_{t-2} - \gamma_3 y_{t-2}$ on the right hand side, which leads to the following formulation

$$\begin{aligned} y_t &= \gamma_1 y_{t-1} + \gamma_2 y_{t-2} + \underline{\gamma_3 y_{t-2} - \gamma_3 y_{t-2}} + \gamma_3 y_{t-3} + \varepsilon_t \\ &= \gamma_1 y_{t-1} + (\gamma_2 + \gamma_3) y_{t-2} - \gamma_3 \Delta y_{t-2} + \varepsilon_t \end{aligned}$$

Further, add term $(\gamma_2 + \gamma_3) y_{t-1} - (\gamma_2 + \gamma_3) y_{t-1}$ to get the term Δy_{t-1} that leads to the following formulation

$$\begin{aligned} y_t &= \gamma_1 y_{t-1} + \underline{(\gamma_2 + \gamma_3) y_{t-1} - (\gamma_2 + \gamma_3) y_{t-1}} + (\gamma_2 + \gamma_3) y_{t-2} - \gamma_3 \Delta y_{t-2} + \varepsilon_t \\ &= (\gamma_1 + \gamma_2 + \gamma_3) y_{t-1} - (\gamma_2 + \gamma_3) \Delta y_{t-1} - \gamma_3 \Delta y_{t-2} + \varepsilon_t \end{aligned}$$

Finally, subtract both sides with y_{t-1} that leads to the following formulation

$$\Delta y_t = (\gamma_1 + \gamma_2 + \gamma_3 - 1) y_{t-1} - (\gamma_2 + \gamma_3) \Delta y_{t-1} - \gamma_3 \Delta y_{t-2} + \varepsilon_t$$

J Correlation of coefficients

Table 10-4: The correlation of coefficients in model IGPM_ADL1

	(Intercept)	$IGPMmom_{t-1}$	$IGPMmom_{t-2}$
$IGPMmom_{t-1}$	0.2126		
$IGPMmom_{t-2}$	0.1517	-0.7061	
$OneY_{t-1}$	-0.9649	-0.2691	-0.2144

Table 10-5: The correlation of coefficients in model IGPM_ADL2

	(Intercept)	$IGPMmom_{t-1}$	$IGPMmom_{t-2}$
$IGPMmom_{t-1}$	-0.1845		
$IGPMmom_{t-2}$	-0.2102	-0.7473	
$Slope_{t-1}$	0.0452	-0.4855	0.0823

Table 10-6: The correlation of coefficients in model IGPM_ADL3

	(Intercept)	$IGPMmom_{t-1}$	$Slope_{t-1}$
$IGPMmom_{t-1}$	-0.4799		
$Slope_{t-1}$	-0.0747	-0.5882	
$\Delta FX_{t-1} + \Delta FX_{t-2}$	0.1985	0.1496	-0.6236

Table 10-7: The correlation of coefficients in model IGPM_Benchmark

	(Intercept)	$IGPMmom_{t-1}$
$IGPMmom_{t-1}$	-0.1861	
$IGPMmom_{t-2}$	-0.2149	-0.8119

Table 10-8: The correlation of coefficients in model IPCA_ADL1

	(Intercept)	$IPCAmom_{t-1}$
$IPCAmom_{t-1}$	0.3106	
$OneY_{t-1}$	-0.9409	-0.5592

Table 10-9: The correlation of coefficients in model IPCA_ADL2

	(Intercept)	$IPCAmom_{t-1}$	$IPCAmom_{t-2}$	$\Delta Selic_{t-1}$
$IPCAmom_{t-1}$	-0.0289			
$IPCAmom_{t-2}$	-0.172	-0.3461		
$\Delta Selic_{t-1}$	0.4686	-0.0805	-0.2792	
$EMBI_{t-4}$	-0.558	-0.4367	-0.3018	-0.1286

Table 10-10: The correlation of coefficients in model IPCA_ADL3

	(Intercept)	$IPCAmom_{t-1}$	$IPCAmom_{t-2}$	$\Delta Selic_{t-1}$	$EMBI_{t-4}$
$IPCAmom_{t-1}$	-0.0305				
$IPCAmom_{t-2}$	-0.1735	-0.3254			
$\Delta Selic_{t-1}$	0.4599	-0.095	-0.3135		
$EMBI_{t-4}$	-0.5442	-0.441	-0.3261	-0.0775	
ΔFX_{t-2}	-0.0247	0.069	0.1936	-0.2491	-0.1806

Table 10-11: The correlation of coefficients in model IPCA_Benchmark

	(Intercept)
$IPCAmom_{t-1}$	-0.7674

11 Glossary

ALM	Asset Liability Management
acf	autocorrelation function
adf	Augmented Dickey Fuller test
ADL	Autoregressive Distributed Lag model
ALCO	Asset Liability Committee
AR	Autoregressive model
b.p.	basis points
COPOM	Monetary Policy Committee
EMBI	Emerging Market Bond Index spread
ER	Error Correction model
FX	The nominal REAL/ USD exchange rate
GALM	Group Asset Liability Management
GDP	Gross Domestic Product
IGP-M	The Brazil's general price index
IGPMmom	Monthly percentage change in IGP-M index
IGPMyoy	Annual percentage change in IGP-M index
IPCA	The Brazil's consumer price index
IPCAmom	Monthly percentage change in IPCA index
IPCAyoy	Annual percentage change in IPCA index
MEFR	Macro-economic Financing ratio
MVE	Market Value of Equity
NII	Net Interest Income
NMC	National Monetary Council
NTNB	IPCA index-linked bond
NTNC	IGP-M index-linked bond
OLS	Ordinary Least Square technique

OneY	the nominal one-year interest rate
pacf	partial autocorrelation function
PCA	Principal Component Analysis
RMSE	Root Mean Square Error
Selic	the overnight interest rate
SIC	Schwarz Information Criterion
Slope	The difference between nominal two-year and three-month interest rates
VEC	Vector Error Correction model

12 List of Tables

13 List of Figures

14 References

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