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Master Project Business Analytics

Optimal charging/discharging strategies for batteries in smart energy grids

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November 16, 2016

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Preface

This thesis was written as a final requirement to obtain the Master's degree in Business Analytics at the Vrije Universiteit Amsterdam. The goal of the programme Business Analytics is to become able to recognize and solve in-company problems by applying a combination of methods based on mathematics, computer science and business management. This thesis is the result of my internship as part of the Cyber Security and Robustness department at TNO on the problem of battery control in smart energy grids.

There are several people I would like to thank for their guidance and support during this graduation internship. First and foremost, I would like to thank my primary supervisor at TNO, Pia Kempker, for introducing me to this subject. I am especially grateful for the way in which she scrutinized my work, without her this thesis would be riddled with errors.
Next, I would like to thank my secondary supervisor at TNO, Hans van den Berg, for his continued support and especially for making additional time available for me when I needed it most.
My thanks also go out to Leon Kester for the thought invoking sparring sessions.
Furthermore, I would like to thank my supervisor from the VU, Sandjai Bhulai, for his guidance and supervision, notably for his help on my grammar and punctuation. And finally, I would like to thank Rob van der Mei for being my second reader.

Sven van der Kooij
The Hague, October 2016

Abstract

The intermittent nature of the ever increasing renewable electricity production demands an ever increasing flexibility in the electrical grid. With the price of the most flexible device dropping rapidly, batteries are close to large-scale implementation. In this thesis, we start by formulating a Stochastic Dynamic Programming (SDP) algorithm to control a stand-alone battery that is acting as a price taker in an hour-ahead electricity market. By using a simulation with realistic data we find that our algorithm significantly outperforms two lower bound methods and gets within 92% of the theoretical upper bound. We then extend the SDP algorithm to minimize the costs of a household with a home battery. Through simulation we show that this algorithm still outperforms both lower bound methods and this time gets within 93% of the theoretical upper bound. By applying this algorithm to a situation where the battery affects the price without being aware of it, we show the robustness of our algorithm. Finally, by adapting the algorithm to include a scenario where the battery is aware of its influence on the price, we show that the choice of price function significantly influences the actions of the battery and with it the effectiveness of its peak shaving.

Contents

Chapter 1

Introduction

1.1 Company Description

This thesis is written as part of my internship at the Netherlands organisation for applied scientific research, TNO. TNO was founded by Dutch law in 1932 in order to assist both the Dutch government and industry. Before TNO was founded only large corporations were capable of developing state of the art knowledge through their own R&D facilities. This gave them a competitive advantage over small and medium enterprises. In order to level the playing field and give every enterprise a place to invest in research, TNO was started.
More than 80 years later TNO has grown into one of the leading Research and Technology Organizations in Europe. Whereas TNO started as an organization to support Dutch government and industry, it now also supports European government and global leading companies and organizations. Its place in society is now firmly established in between universities and industry.
Even though the scope of TNO has increased greatly, the founding vision is still a central part of operations at TNO, as can be seen in its mission statement:

> TNO connects people and knowledge to create innovations that boost the competitive strength of industry and the well-being of society in a sustainable way.

How this mission will be pursued is defined in the following five transitions on which research will focus:

- Industry: from economic stagnation to growth in high-technology industry;
- Healthy Living: from illness and treatment to health and behaviour;
- Defence, Safety & Security: from a wide range of threats to controllable risks;
- Urbanisation: from urbanisation bottlenecks to urban vitality;
- Energy: from conventional sources to sustainable energy systems.

The organisation is subdivided into two expertise areas: Technical Sciences, and Earth, Life and Social Sciences. This research is part of the former, which is subdivided into four clusters: ICT, Nano Technology, Solid & Fluid Mechanics, and Observation, Weapon & Protection Systems. The ICT cluster is the cluster where this research has been conducted. Finally, this cluster consists of five expertise groups, one of which, Cyber Security & Robustness (CSR), is the expertise group where this research was performed. CSR is the result of a recent merger between the departments Cyber Security, of around 30 people, and Performance of Networks and Systems (PoNS), of approximately 20 people.

The funds for the research conducted at TNO are provided by the government and other clients. The results are presented to clients as ideas, consultancy, scientific publications, or prototypes of products.
Even though TNO is a nonprofit organization, profits are sometimes made. When this happens

the additional money is used to fund internal research projects. This research is part of such a research project, with the title Self Organising Smart Energy Networks (SOSENS). SOSENS is a collaboration between the research groups former PoNS, Distributed Sensor Systems, and Service Enabling Management. The project is part of the aforementioned transition in Energy: *from conventional sources to sustainable energy systems.*

1.2 Problem Description

Until recently nearly all uncertainty in the energy system came from the uncertainty in demand. This uncertainty has been the driving force behind the electricity network design as we know it today. The uncertainty generated by the demand side is compensated by flexibility on the production side of the network. However, with the introduction of renewable energy sources like wind and solar, uncertainty is introduced on the production side and the capacity for the production side to be flexible is reduced greatly. The current system is still capable of dealing with the current levels of uncertainty, however, as the renewable energy penetration increases, it becomes harder and more costly to compensate for the increased volatility solely on the production side.

The increased uncertainty introduced by renewable energy sources is not the only threat to the electricity network. Another development that threatens the design of the current electricity grid is the increased use of Electric Vehicles (EVs). According to [10], "a good rule of thumb is that a typical EV is likely to double the electricity consumption of a typical home"(p. 64). Not only do EVs increase the energy consumption of households significantly, they also have limited availability for charging. Most EVs become available for charging when they are plugged in as people come home. This time coincides with a peak in energy demand, since people generally use electrical appliances from the moment they get home until the time they go to bed. If EVs would also start charging around this time, the peak would increase drastically and would be much harder and more costly to deal with.

In order to solve these problems the concept of smart grids is introduced. The European Technology Platform SmartGrids defines the smart grid as follows:

> "A Smart Grid is an electricity network that can intelligently integrate the actions of all users connected to it - generators, consumers and those that do both - in order to efficiently deliver sustainable, economic and secure electricity supplies."[1]

The principle idea behind smart grids is that they allow the demand side to respond and react to uncertainty and changes in the system. For example, if there is more wind than was predicted, home batteries can be used to store this excess energy. And if, later on, there is a period of less wind or higher than expected demand the battery can be set to discharge. This adjusting behaviour on the demand side is called Demand Response (DR). Batteries are not the only method of DR, you could also think of electric vehicles charging with excess solar energy, your freezer freezing a little bit more when there is too much wind, or delaying the start time of your washing machine when other demand is at its peak.

In order to enable DR, a change in the current electricity market design is required. One of the core questions in the design of the new market is whether or not the utility companies should have control over the demand side flexibility provided by DR. Some researchers believe that the control of this flexibility should be in the hands of the device owners, while others believe that utility companies should hold control over those devices. Some researchers even go so far as to put the control of the entire grid in the hands of regulators and transmission companies. James A. Momoh said for example:

> "Meanwhile, security measures will render individual pricing signals opaque to end-users; restricting access assures that customers will not have an adequate basis on

[1] `http://www.globalsmartgridfederation.org/smart-grids/`

> which to game the system and/or only regulators and transmitters having real-time access to critical data."[23]

We believe that the decision on the control of end-user devices should lie with the end-user. An electricity grid should not be designed such that it can only support centrally controlled flexibility, nor should the option be made impossible by design. The end-user should be able to decide whether to give up control, and with it privacy, for, for example, monetary gains, or to keep privacy and control in-house.
In this research we will mainly focus on three situations. In the first, we will be looking at a battery that has its own controller which acts on price signals from the market. In the second we will focus on a home where the battery is controlled by a central in-house controller. And in the last we will consider an island setting where either one battery, or a collection of batteries, is big enough to have an influence on the price. When an actor has an influence on the price, he is known as a price maker, otherwise he is a price taker.

1.3 Research Questions/Goals

Since we are now at a point in time where the transition to smart grids is starting to become a reality, an important question becomes what the design of the smart grid should become and how such a design can be found. One possibility is to start with a system design first and to look at how each part would fit in such a system second. In this case the base system is usually created with a clear goal in mind, like protecting the privacy of the households, or minimizing the communication required in the system. The upside of this method is that it keeps the core of the system relatively simple and the goal clear. However, when the components are fit around this core principle, it is hard to see what the influence has been of our design decisions on the solution.
Another possibility is to start by considering the core of the system to be a black box and to start by finding the best strategies for the components in this system. Once we have this, we can begin to unravel the black box and design the core of the system. This way we have an understanding of the impact of our design decisions before we design the system.
We aim to support the latter possibility and consider the core of the system to be a black box. In this research we will be looking at the viewpoint of a battery in several different settings. In each setting we will start by looking at a way to find the best strategy for the battery. After this we will be looking at the impact of several system characteristics on the profitability of batteries. These characteristics range from the size and efficiency of the battery, to the different forms of taxation or the way electricity prices are computed. This leads to the following main research questions:

- How to optimally control an electrical energy storage in the smart grid context given forecasts with uncertainty?
- What are the main characteristics that determine the profitability and applicability of electrical energy storages in the smart grid context given forecasts with uncertainty?

An important factor in these questions is the fact that not only will we be using forecasts, but we will also take their uncertainty into account. In each of the settings we will be looking at, we start by using this information to formulate a method of finding the best strategy for a battery specific to that setting. After this we will look at the setting specific influences of the characteristics.
The first setting we will be looking at is a stand-alone battery. This battery can be a large battery situated on a grid level, or a smaller battery operating independently on a household level. The only actions this battery can perform are buying and selling electricity from the grid. The size of this battery is assumed to be sufficiently small, such that it does not have any impact on the electricity price and is a price taker. The goal of looking at this situation is to get a general understanding of the impact of the characteristics on the profitability of the battery in a basic situation.
The next situation we will be looking at is the battery on a household level. The home battery is

starting to rise in popularity, especially with the recent introduction of the Tesla Powerwall[2]. In addition to the buying and selling of electricity from the grid in the previous situation, this battery will now also be able to provide electricity directly to the household it is connected to. We would like to find out what changes for the battery strategy in this new situation.
The last situation we will be looking at is the situation where either the battery is big enough to influence the electricity price, or there are sufficiently many batteries using our strategy that together they affect prices. This change makes the battery a price maker and we again would like to know what changes this brings to the influence of the design parameters on the battery performance.
Even though the organization of the grid will be viewed as a black box throughout these situations, we will start this thesis with a literature research on decentralised coordinated planning methods as part of the tasks set out in the SOSENS project.
In order to answer the main research questions by achieving the goals set out above, we seek to answer the following questions in this research:

- What are some of the decentralised coordinated planning methods proposed in the literature?
- How do the characteristics affect the profitability of a stand-alone battery?
- What are the main characteristics for the profitability of a home battery?
- What are the main characteristics for the applicability of batteries that are price makers?

1.4 Related Research

Research into the optimal control of batteries spans a wide range of disciplines and settings. The optimal control of batteries can refer to the internal battery control systems, which attempt to optimize the operation of the battery. Or it can refer to an external control system that makes use of the charging capabilities of the battery to optimize an external objective. The focus of our research is on the latter.
Within this framework, batteries can be used to optimize a variety of different objectives. The main difference between these objective is the timescale on which they operate. Batteries can be used for frequency control on the extremely short timescale, or on the balancing market on a slightly larger timescale. Larger still are the timescales involved when looking at the hour- or day-ahead markets. And the largest ones are the timescales where business cases are made for storage systems that trade and store energy to make use of seasonal changes. In this research we will focus on the hour-ahead markets.
A further distinction in research topics can be made by looking at the location of the storage system. The main settings in which research is conducted are:

- Grid level batteries in micro grids;
- Wind or solar farms combined with a battery;
- Home batteries.

The research on batteries in a micro grid setting has two points of primary focus. The first is determining the optimal size of a battery when trying to minimize the costs within the micro grid [4][5]. While the second is focused on the optimal control of a battery when the micro grid may be disconnected from the main grid at any moment due to instability [13]. In order to take many different participants of the micro grid into account, the main algorithms used in this setting are based on stochastic Mixed Integer Linear Programming (MILP).
Other researchers have focused on the optimal control of a battery in combination with a wind, or solar farm [8][28][14]. In this situation the objective usually is to create an output profile that is as flat as possible, or as close to a previously made prediction as possible.
Finally the research on the optimal control of a home battery can be divided into different solution methods. Researchers in [2] consider a battery in a system with stochastic prices. The goal is to

[2] `https://www.tesla.com/powerwall`

optimally bid with the battery in the day-ahead and the hour-ahead markets. In the day-ahead market, the battery is able to offer both energy and reserve. The committed energy is always used, while the reserve may or may not be used by the system operator. The Stochastic Programming formulation proposed is limited, because the hour-ahead market is only used to instantly compensate for any discrepancy between the committed reserve and the used reserve.
The authors of [18] consider a system where the main objective is to optimally control an independently owned battery in the hour-ahead market. They decide to solve the problem using Approximate Dynamic Programming (ADP), because they consider the computation time of the SDP to be too long. They compare their solution to the solution of the SDP and show that the solution of the SDP is slightly better than the solution of the ADP, however, the ADP solution is computed much faster.
Finally the authors of [1] propose an SDP approach to solve the problem of optimally controlling a home battery using an extensive battery degradation model. The stochastic part of their SDP is the energy produced by the PVs connected to the home and the demand of the household. They consider this information unknown even for the next period. This means that the realisation of the stochastic variables does not influence the decision made at each time step.

So far, all research mentioned concerns batteries that act as price taker in the electricity market. Below we will discuss some of the research related to price maker batteries.
The authors of [11] have formulated the problem of price maker energy storages as a quadratic programming problem. Using the Lagrangian method they derive an optimal charging strategy based on the average forecasted demand and the current demand. They limit themselves to a system without renewable energy. Their solution requires a quadratic cost function to work and they do not look into the effects of the accuracy of the forecast on their outcome.
The authors of [6] use linear programming to optimally control a battery bank in the spot and balancing market. They assume perfect forecasts and afterwards apply a penalty of 11% to account for the error that using forecasts would have resulted in. They combine this information with a real options method, simulating the advances in battery technology, to determine the ideal moment in time to invest in storage systems.
Another method of solving this type of problem is proposed by the authors of [26]. They use Approximate Dynamic Programming (ADP) to solve the problem of controlling multiple storage devices in a system with wind, demand, and price uncertainties.
The authors of [15] use bi-level programming to investigate what would happen to the social welfare of a system if multiple selfish battery operators operated in such a system. They show that under current pricing rules the batteries will hold back some capacity in order to keep the prices from dropping too much. This is done to maximize the profit of the batteries, but it may also result in network congestion that could have been avoided if the battery used its full flexibility.

The main contributions of this research are to present a base Stochastic Dynamic Programming (SDP) approach which can be applied to each of the settings presented above. The base approach consists of a stand-alone battery in a system with uncertainty in the electricity price. This stand-alone battery is only capable of buying and selling energy to the grid.
Furthermore, two extensions to this SDP formulation will be presented to demonstrate the applicability of the SDP. The first extension is the modeling of a home battery. In this setting the home battery is located in a home with fixed household demand and an Electric Vehicle (EV). We present a theoretical model in which a combined strategy for the battery and the EV is determined, taking into account the uncertainty in both the electricity price as well as in the household demand. In order to get numerical results, an adapted version is used where both the household demand and the strategy for the EV are considered to be fixed and known.
In the second extension, the base model is applied to an islanded system. In this system we consider an island with a battery, 200 households, and wind-, solar-, and diesel-generators. While previously the battery was too small to make a difference on the price, this time the battery is a price maker. We will show that we can still use an adapted version of the base model when the battery knows it influence on the price. We will also show that, even when the battery is unaware of its influence on the price, the solution of the SDP will stabilize. To our best knowledge, this is the first time two different price functions are compared for the price maker scenario.

1.5 Thesis outline

In this section we will explain the structure of the remainder of this thesis. Chapter 2 describes some of the coordinated planning methods that can be found in the literature. In Chapter 3 we propose an SDP algorithm for a stand-alone price taker battery in an hour-ahead market. We use a simulation based on realistic data to compare our results to two lower, and one upper bound. In Chapter 4 we extend the SDP algorithm to be applicable to a household with a home battery and compare our results again to an upper and two lower bounds. In Chapter 5 we evaluate this algorithm in an hour-ahead market where the battery is a price maker. Additionally, we formulate an adaptation of the SDP to include the influence of the actions on the price and evaluate the peak reduction capabilities of this algorithm using two different price functions. Finally, in Chapter 6 we will conclude this thesis and provide recommendations for further research.

Chapter 2

Literature

2.1 Coordinated planning methods

Coordinated planning methods can be subdivided into two categories, centralized and decentralized planning methods. In centralized planning methods there is a centralized controller that makes decisions about the usage of flexibility in the system on a device level. In decentralized planning methods however, this decision is made within the household itself.
So imagine that you would like to run your dish washer somewhere between 00:00 and 06:00, but you are indifferent about when exactly it will run in this interval. In the centralized planning method you would communicate this information to your energy provider, who will give a signal when it is time to start running. In contrast, in the decentralized system your device itself, or your smart-meter, could make this decision for you based, for example, on a price signal received from your energy provider.
This section will focus on the latter, decentralized planning methods.

2.1.1 Market Based

ABEM (Ahead- and Balancing Energy Market), as described in [17], is a two-settlement market mechanism. This means that there are two markets, one ahead market, and one balancing market. In the ahead market participants commit to buying and selling electricity at a future time period, which is the first market to be cleared quite a bit ahead of time.
Just before the time period is due, the balancing market is cleared. This market is used by flexible producers and consumers to sell flexibility to participants who require more energy than their already committed quantities. The novelty in this system is that the participants only submit one bidding function which contains both their bid and their flexibility. The system operator is then responsible for saving enough flexibility for the balancing to work. Because the balancing market is always cleared at a higher price than the ahead market, participants are encouraged to give a fair estimate of their flexibility.
The downside seems to be that the only flexibility offered is for consumers to consume less, or for suppliers to supply more. Therefore the only option for inflexible consumers and suppliers in the balancing market is to buy additional energy.

The PowerMatcher, as described in [20], is a smart grid coordination system focused around matching electricity supply and demand using price signals. The system is build up of a combination of several agents. Each device has its own device agent. Every time period this agent creates a bid function, which tells you how much energy the device is willing to buy for each price. Once the bid function is created, it is sent to the concentrator agent. This agent aggregates the bid functions of several different devices or other concentrator agents into one bid function. This new bid function is sent to the next agent in the tree and so on, until all bid functions reach the final agent, the auctioneer. The auctioneer accumulates all bids and when the time window is over, it creates an equilibrium price. This price is then sent back through the agents to the devices and

the devices act on the given price.
One of the downsides of the PowerMatcher is the fact that there is only coordinated planning for the next time window, and not several time windows in the future.

The authors of [24] describe a micro-market system that is supposed to be the link between a micro-grid, containing smart homes, and the current day-ahead market. In addition to the homes, the micro-grid is also connected to a community electricity storage.
The market system works as follows. The smart homes and appliances make decisions on their energy profiles based on expected future electricity price scenarios. They then send a price/power utilization pair to the micro-market for several different prices. The job of the micro-market is now to devise a strategy for the community battery, and to send a bid to the day-ahead market. After the day-ahead market is cleared, the micro-cluster clears its own market and communicates the local price to the individual households.
The system is designed such that local network constraints can be taken into account when the micro-market is cleared and can be updated by the distribution system operator as the situation changes. However, this system only includes the day-ahead market and completely ignores the external balancing market.

Up to this point all markets described were double-side auctions which were only cleared once, at the end of the trading window. Another type of auction is the continuous double-sided auction. This auction is cleared every-time a bid is higher than an ask-price. Such an auction is described in [3]. The goal of this market is, much like the micro-market in [24], to position itself between the micro-grid and the outside grid. The market trades in 15 minute intervals and stops 15 minutes before the start of the interval. During the trading period, households can submit bids and asks and when a bid exceeds an ask, it will be cleared. If, at the end of the trading window, there are any bids or asks left, they will be cleared for the outside market trading price.

2.1.2 Non-market Based

The authors of [9] propose the Distributed Energy Management (DEM) algorithm Priv-ADE. As the name suggests, this method is centered around preserving the privacy of participants. The general structure of the system is a circle with a server as the start- and end-point. In order to achieve electricity balancing, the system has to go through several rounds.
In the first round, the server sends a data packet through the circle of households. Each household adds their encrypted characteristics, like total demand and flexibility, to the packet using the Paillier crypto-system. Once the server receives the aggregated information of the system, it computes the flexibility needed to balance the system.
Next, an iterative scheme starts where the server sends out a data-packet containing the required flexibility per flexible household. The households react to this and add their adjusted demand. This scheme ends when either the required balance is reached, or the total flexibility is used up. It is not clear what would happen if flexibility is not sufficient to reach an equilibrium.
By aggregating and encrypting the demands, the privacy of individual households is extremely well preserved with this system. However, because the households are organized in a circle, this method does not scale well.

Triana is a method of optimizing demand and supply using steering signals as described in [7]. The idea is that each device makes a local forecast of its future demand. Based on this forecast it makes a prediction of its flexibility. The forecast of the demand and the flexibility are then sent to a global optimizer that creates an upper- and lower-bound for the energy profile of the device based on a specific criterion, such as to reduce peak load. This profile is sent to the device, the device tries to match the profile as well as possible, and it sends the results back to the global optimizer. This goes on until either satisfactory results are achieved, or until the difference between iterations is sufficiently small. The results will then be the current valid schedule, until the procedure is started again.

Several authors have tried to adopt a game-theoretic approach to solve the energy distribution problem. The authors of [22], for example, propose a method where households compete for electricity. Each household communicates its consumption schedule for the coming day to all other participants. Every time a household transmits its schedule, all other households have to update and transmit their schedule.
This approach is very limited as there is no uncertainty, the pricing function has to be strictly convex, and every household has to communicate its consumption pattern to all other households. On top of that, the solution is also achieved through an iteration scheme that converges to the optimum with no guarantee that this occurs in a reasonable amount of time.

In [25] the authors provide another game-theoretic solution which does not have the requirement that every household has to communicate its consumption pattern to all other households. In this system an iteration scheme is presented where the utility company communicates a price to the households. After this the households reply with their optimal strategy, the utility company updates the price, and the cycle starts again until convergence.
In the case study they present, the Nash-equilibrium is achieved in a reasonably small number of iterations. However no guarantee is given for fast convergence, the pricing function has to be strictly convex, and no uncertainty is taken into account.

Chapter 3

Stand-alone Battery Model

3.1 System Description

The first model we consider is illustrated in Figure 3.1. The idea behind this model is that a home has a battery that can be charged or discharged in discrete steps in both time and charge. However, due to the general nature of the model, this model can also be used to describe other storage devices that are capable of buying and selling a commodity with restricted storage.

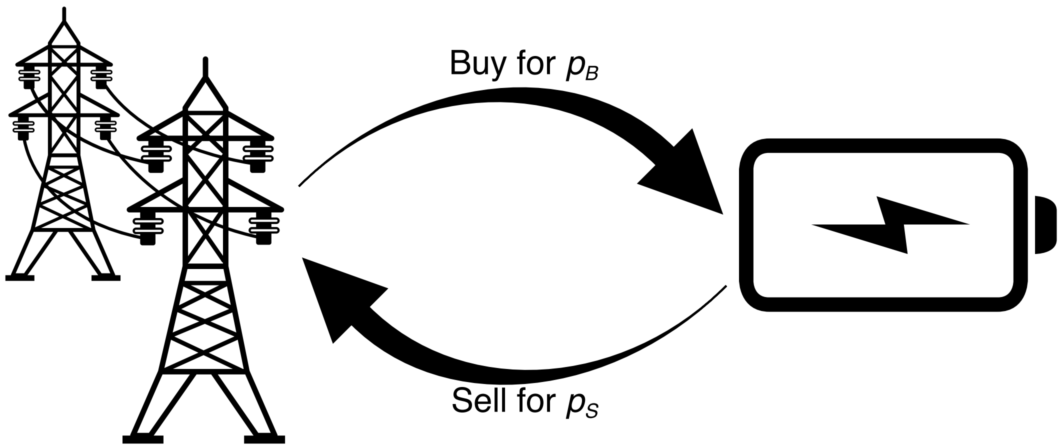


Figure 3.1: A schematic representation of the stand-alone battery model

3.1.1 Electricity Market

The battery buys and sells its electricity from a market. The battery is considered to be a price taker in this market. This means that the actions of the battery are insignificant compared to the system as a whole and therefore do not have any influence on the outcome. In this market the battery can decide several minutes before the start of an hour how much energy it will be using. We assume that when such a decision is made, this will always be fulfilled and therefore we do not take trading on the balancing market into account.

We assume energy can be bought for $p_B = \left(1 + (\gamma - 1)\,\mathbb{1}_{\{p_t > 0\}}\right) p_t + \tau$ and sold for $p_S = p_t$ at time t. When prices are negative, consumers do not have to pay value added taxes (VAT) and therefore $1 + (\gamma - 1)\,\mathbb{1}_{\{p_t > 0\}}$ represents price added VAT and τ represents fixed costs. These fixed costs can consist of a collection of various costs, for example a fixed tax per unit of energy, or a network fee for using the electricity network per unit of energy. For example, in the Netherlands the fixed costs are $\tau = 0.1007$€/kWh, $\gamma = 1.21$ due to the VAT of 21%, and the typical price of energy is around five cent per kWh.

3.1.2 Battery

Ten years ago the standard for most rechargeable batteries was lead-acid. Nowadays the standard home battery, like the Tesla Powerwall, is composed of lithium-ion. Since we consider a futuristic scenario, we do not know which battery technology will become the new standard. Some contenders are improvements to the current lithium-ion technology, like lithium-air or lithium-iron-phosphate. Others are based on entirely new techniques, like the liquid metal battery or the flow battery. Each of these technologies have their pros and cons, some batteries are very durable but slow, while others charge extremely fast but are very inefficient. Because of this uncertainty, we made the choice to keep the battery model general.
The battery has a total capacity given by x_{max} in kWh and a maximum charging speed of u_{max} in kW. Another way to denote the maximum charging speed of a battery is by its C-rate. A C-rate of 1 corresponds to a battery that can fully charge in one hour, while a C-rate of 2 means the battery can go from empty to full in just 30 minutes. Thus the C-rate is one divided by the number of hours it takes to charge the battery from empty to full.
Another battery parameter is the round-trip efficiency given by η. The round-trip efficiency denotes the percentage of energy that is returned after it has been stored in the battery. Most batteries operate on DC-current, while the net operates on AC-current. So in order to get energy into the battery and back out of the battery, this energy has to be converted, and during this conversion some of the energy is lost. The other way to lose energy is in the chemical process of storing and withdrawing energy. Both these losses are captured in the round-trip efficiency.
Aside from the efficiency, the battery depreciation costs also have to be taken into account. Batteries are a commodity which is used up and their lifetime is given as a number of charging cycles. A single cycle is defined as charging 100% of the battery's capacity and discharging that same amount, however, this does not have to occur in that order. You could, for example, charge your battery to full, discharge 50%, charge it back to full and discharge another 50% to complete a single cycle. This means that the cost associated with the cycles can be translated into a cost per unit of charging.

3.2 Methods

The overall goal of the stand-alone model is to decide at each point in time t what to do with the battery in order to maximize the expected profit, based on predictions of future prices, p_t. In this model we assume that the maximum charging speed equals the maximum discharging speed of the battery and without loss of generality we set this speed equal to 1. The set of all possible actions is then given by $U = [-1, 1]$, with $u < 0$ representing discharging and $u > 0$ representing charging of the battery.
In order to get computationally attractive discrete storage levels, the choice was made to settle the round trip efficiency (η) when discharging, as opposed to taking $\sqrt{\eta}$ into account when charging and $\sqrt{\eta}$ into account when discharging. This means that only the charging can happen at full speed and discharging happens at $\sqrt{\eta}$ times the maximum speed.
The next costs we have to take into account are the cycle costs per unit of charging (κ). While we do not take the charging efficiency into account to make the storage levels more attractive, we will take it into account when it comes to the charging cycle costs. As mentioned before, $1 - \sqrt{\eta}$ of the energy is lost in charging and therefore never reaches the battery, so the cycle costs when charging the battery are $\sqrt{\eta}\kappa$.
Combining the efficiency, cycle costs and price gives us the following reward function at time t given action $u_t \in U_t$ and price p_t:

$$R(u_t, p_t) = -u_t \left(p_t \left(1 + (\gamma - 1)\,\mathbb{1}_{\{p_t>0\}}\right) + \sqrt{\eta}\kappa + \tau\right) \mathbb{1}_{\{u_t>0\}} - \eta u_t p_t \mathbb{1}_{\{u_t<0\}}. \tag{3.1}$$

If the price in each period were known, the problem of finding the charging scheme that maximizes profit could be found using dynamic programming with the states being equal to the state of charge of the battery, x_t. This problem is given by:

$$V_t^*(x_t) = \max_{u_t \in U_t} \left[R(u_t, p_t) + V_{t+1}^*(x_t + u_t) \right], \tag{3.2}$$

with $U_t = [\max(-1, -x_t), \min(1, x_{max} - x_t)]$, such that we never discharge more than is currently in the battery and never attempt to charge beyond the maximum. If we use the real prices p_t^* for $t = 1, \ldots, T$, with T being the end of the total considered time period and if we use $V_{T+1}^*(x) = 0$, we get the optimal value of charging by solving Equation (3.2).

In reality however, prices for upcoming periods are not known with certainty. Therefore we assume that there are forecasts for p_t available up to L time units ahead at every point in time. In addition, we assume the distribution of the error of these forecasts to be independent with a known probability density function, $f_{P_t}(p_t)$.
In order to take this uncertainty into account, we have to extend the state space of the dynamic program in (3.2). Instead of using only the current charge of the battery, x, we have to include the current price, p. Hence the state at time t is given by (x_t, p_t).
Let $V_t(x_t, p_t)$ be the maximum expected profit in the interval $t, \ldots, \min(t + L, T)$, given the state at time t is (x_t, p_t). The highest expected value at time t can be determined by iteratively solving the following Stochastic Dynamic Programming equations for all prices:

$$V_{t+L+1}(x, p) = cx, \text{for some price } c, \tag{3.3}$$

$$V_{T+\alpha}(x) = 0, \forall \alpha \in \mathbb{N}. \tag{3.4}$$

Next, for times $\tau = t, \ldots, t + L$ and state (x_τ, p_τ) we have:

$$V_\tau(x_\tau, p_\tau) = \max_{u_\tau \in U_\tau} \left[R(u_\tau, p_\tau) + \mathbb{E}_{P_{\tau+1}}(V_{\tau+1}(x_\tau + u_\tau, P_{\tau+1})) \right], \tag{3.5}$$

with $U_\tau = [\max(-1, -x_\tau), \min(1, x_{max} - x_\tau)]$. In order to find the optimal decision using these equations, we use backwards recursion. Since we know $V_{t+L+1}(x, p)$ we can calculate $V_{t+L}(x, p)$, and with that we can calculate $V_{t+L-1}(x, p)$, and so on until we reach $V_t(x, p)$.
Given the definitions of $R(u, p)$ and $V_{t+L+1}(x, p)$, it is straightforward to calculate the maximum for $V_{t+L}(x, p)$ for given x and p. In order to calculate $V_{t+L-1}(x, p)$ however, we have to compute the expectation of a maximum as shown in the equation below. Let $a = t + L$, then Equation (3.5) becomes for $\tau = t + L - 1$:

$$\begin{aligned} V_{a-1}(x_{a-1}, p_{a-1}) &= \max_{u_{a-1}} \left[R(u_{a-1}, p_{a-1}) + \mathbb{E}_{P_a}(V_a(x_{a-1} + u_{a-1}, P_a)) \right] \\ &= \max_{u_{a-1}} \left[R(u_{a-1}, p_{a-1}) + \mathbb{E}_{P_a}\left(\max_{u_a} \left[R(u_a, P_a) + V_{a+1}(x_{a-1} + u_{a-1} + u_a, p_{a+1}) \right] \right) \right]. \end{aligned}$$

So in order to calculate $V_{a-1}(x_{a-1}, p_{a-1})$ we first have to compute $\mathbb{E}_{P_a}(V_a(x_a, P_a))$. Define:

$$\begin{aligned} \hat{V}_a(x_a) &= \mathbb{E}_{P_a}(V_a(x_a, P_a)) \\ &= \mathbb{E}_{P_a}\left(\max_{u_a} \left[R(u_a, P_a) + \hat{V}_{a+1}(x_a + u_a) \right] \right). \end{aligned}$$

In order to solve this expectation of a maximum, we would like to split the expectation into parts, where in each part a unique decision u_a leads to the maximum. With $u_a \in U = [-1, 1]$ it is difficult to split the expectation as there are an infinite number of decisions. However, as we prove in Appendix A, if x_a is an integer, the maximum is achieved for $u_a \in \{-1, 0, 1\}$. This makes it possible to split the expectation into a finite number of parts. Buying energy ($u_a = 1$) is optimal when we expect to earn more in the future than it costs to buy energy now, so when $R(1, p) < \hat{V}_{a+1}(x_a + 1) - \hat{V}_{a+1}(x_a)$. Selling energy ($u_a = -1$) is optimal when we can get more money now than we expect to get for the same energy in the future, so when $R(-1,p) > \hat{V}_{a+1}(x_a) - \hat{V}_{a+1}(x_a - 1)$. If we do not want to charge, nor discharge, we do nothing ($u_a = 0$). Next, define $R^{-1}(u,\delta)$ as the preimage of $R(u,p)$ with respect to the price, such that $R^{-1}(u, R(u,p)) = p$. And define:

$$\delta_1 = \hat{V}_{a+1}(x_a) - \hat{V}_{a+1}(x_a - 1),$$
$$\delta_2 = \hat{V}_{a+1}(x_a) - \hat{V}_{a+1}(x_a + 1).$$

Then splitting the three different cases, we get:

$$\hat{V}_a(x_a) = \int_{-\infty}^{R^{-1}(1,\delta_2)} f_{P_a}(p)\left(R(1,p) + \hat{V}_{a+1}(x_a+1)\right)\mathrm{d}p \tag{3.6}$$
$$+ \int_{R^{-1}(1,\delta_2)}^{R^{-1}(-1,\delta_1)} f_{P_a}(p)\,\hat{V}_{a+1}(x_a)\,\mathrm{d}p \tag{3.7}$$
$$+ \int_{R^{-1}(-1,\delta_1)}^{\infty} f_{P_a}(p)\left(R(-1,p) + \hat{V}_{a+1}(x_a-1)\right)\mathrm{d}p, \tag{3.8}$$

with $f_{P_a}(p)$ the probability density function of the price at time a. There are two extreme cases, one when the battery is full, and when when it is empty.
In case the battery is full it is impossible to charge, therefore the value of charging becomes $\hat{V}_{a+1}(x_a+1) = -\infty$, and $\delta_2 = \infty$. In order for the reward of buying to be infinite, the price has to be minus infinity, so $R^{-1}(1,\delta_2) = -\infty$ and Summand (3.6) disappears.
And if the battery is empty and we cannot discharge, the value of discharging becomes $\hat{V}_{a+1}(x_a-1) = -\infty$, so $\delta_1 = \infty$. In order for the reward of selling energy to be infinite, the price has to be infinite as well, so $R^{-1}(-1,\delta_1) = \infty$ and Summand (3.8) disappears.
Using these calculations we can deduce the expected best decision to be taken at the current time t by solving the following equation:

$$\hat{U}_t(x_t) = \arg\max_{u_t}\left(R(u_t,p_t) + \hat{V}_{t+1}(x_t+u_t)\right).$$

A schematic representation of this calculation for a battery of size 2 and current charge 1 is given in Figure 3.2.

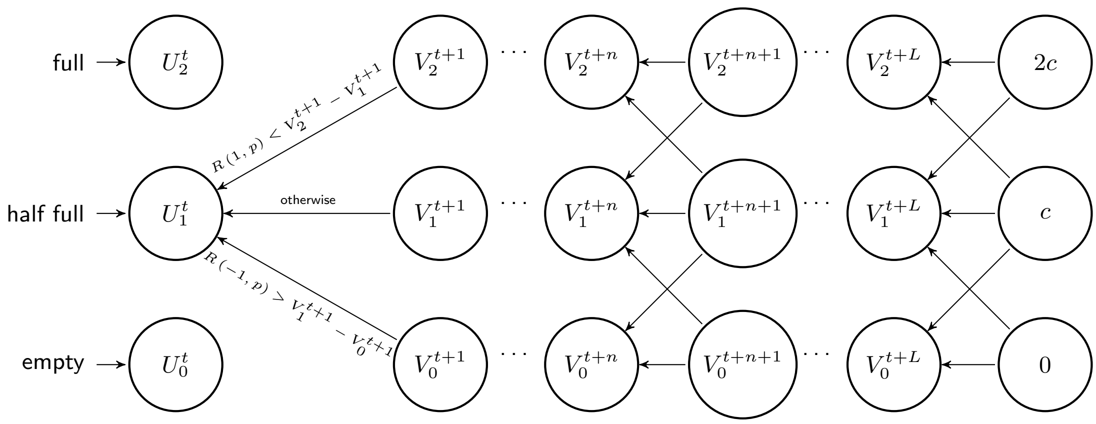


Figure 3.2: A schematic representation of the calculations used to make a decision at time t for a battery of size 2 and current charge 1

Now that we have calculated the expected best decision for time t, we apply this decision, update the State of Charge (SOC) of the battery, and move forward to time $t+1$. Then we update our predictions for the price and recalculate everything to make a decision for time $t+1$.

The total value of the expected best decisions is given by:

$$V_{total} = \sum_{t=1}^{T} R\left(\hat{U}_t\left(x_0 + \sum_{i=1}^{t-1}\hat{U}_i, p_t\right), p_t\right).$$

3.2.1 Lower bound

In order to show the added value of using forecasts in our models, we will use some models which do not make use of forecasts as lower bounds.

Daily Pattern

This first method to achieve a lower bound for the stand-alone model is to look at the average daily price trends taken over the entire year. If it would take n hours to charge the battery from empty to full, the strategy is to find the n cheapest hours in the average pattern. These hours will be used every day to charge the battery from empty to full. Similarly the n most expensive hours in the average day will be the hours during which the battery will be discharged. Figure 3.3 shows the average daily price pattern taken over the entire year with the strategy that corresponds to a battery that fully charges in 4 hours. So in this example the battery would be charged every day between 2:00 - 5:00 and from 14:00-15:00, and discharged daily between 18:00 and 21:00 and 22:00-23:00.

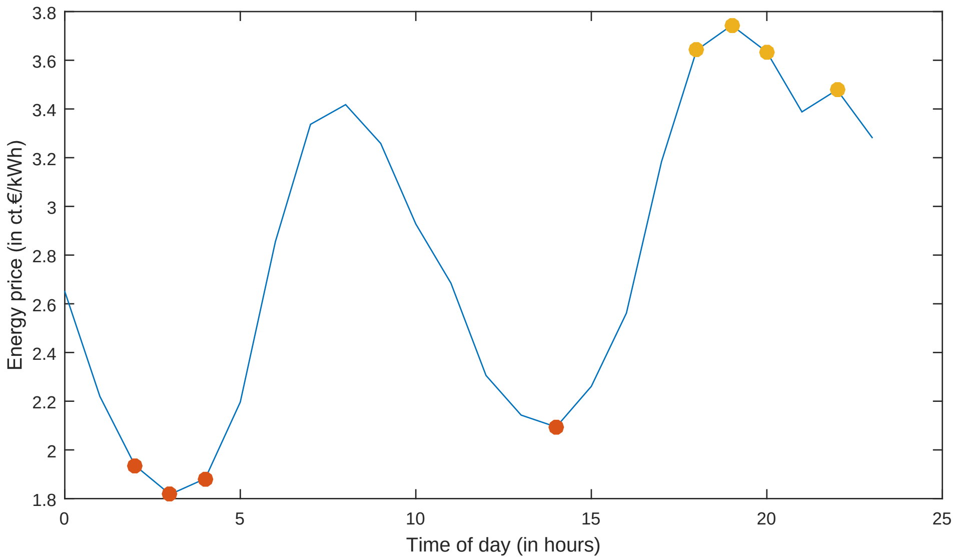


Figure 3.3: The average daily energy price pattern over the year 2015 with the four cheapest and four most expensive hours

Threshold

Another method of finding a lower bound is to use threshold values. If the price of energy is below a certain threshold, energy will be bought, and if the price is above another threshold, energy will be sold. We would like to choose these thresholds in such a way that we will never lose money on our investment. Therefore, the thresholds are chosen symmetrically around the mean electricity price over the year 2015, such that the return on investment is zero if we buy for the lower threshold and sell for the upper.

Figure 3.4 gives a graphical representation of this problem if our prices would be normally distributed with mean μ. And to put it in terms of the cost function from Equation (3.1):

$$
\begin{aligned}
R(1, \mu - x) &= R(-1, \mu + x) \\
(\mu - x)\gamma + \sqrt{\eta}\kappa + \tau &= -\eta(\mu + x) \\
(\eta - \gamma)x &= -\left((\gamma + \eta)\mu + \sqrt{\eta}\kappa + \tau\right) \\
x &= \frac{(\gamma + \eta)\mu + \sqrt{\eta}\kappa + \tau}{\gamma - \eta}
\end{aligned}
$$

So we will be buying electricity for prices below $\mu - x = \frac{2\eta\mu + \sqrt{\eta}\kappa + \tau}{\eta - \gamma}$ and we will be selling electricity for prices above $\mu + x = \frac{2\gamma\mu + \sqrt{\eta}\kappa + \tau}{\gamma - \eta}$.

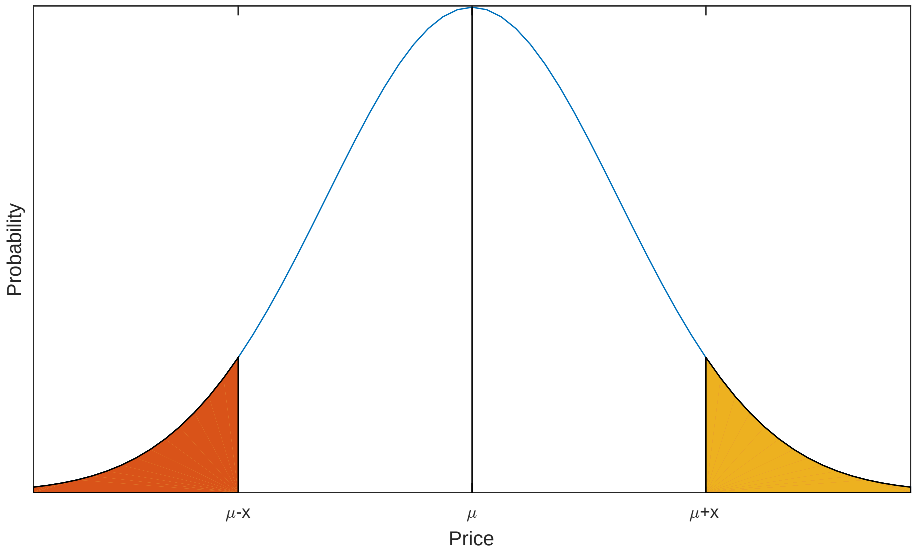


Figure 3.4: A graphical representation of a normal distribution with average μ and two values equidistant from μ

3.3 Data

3.3.1 Electricity Market

For this research we consider a futuristic scenario with increased renewable energy penetration. The reason for this is that the design of the current electricity network is already starting to show cracks and the expectation is that it will start to fail when more renewable energy gets added to the system.

In order to change the penetration levels of renewable energy, we would like to have data on the energy price, the renewable penetration, and the link between the two. For our models we also need forecasts of the prices, preferably updated every hour.

Data from Germany was available to us, but this consisted of only the hourly price data. So this set was missing forecasts, data on renewable energy, and the relation between the price and the renewable energy production, which made it very hard to use for our futuristic scenario.

Other data that was available to us came from Belgium [12]. This data set contained data for every 15 minutes on the total load, the wind energy production and the solar energy production. It also contained day-ahead predictions of all these measurements. These day-ahead predictions were predictions for every 15 minutes of an entire day and were made once a day at 11 am the day prior. Missing, however, were the electricity prices and the relation between the renewable energy production and the prices.

The decision was made to use the data from Belgium and to create the relationship between the

renewable energy production, the price and the demand ourselves based partly on the German data.
In the remainder of this section the data from the Belgian and German network will be explored, cleaned, and used to generate prices for the Belgian network.

Data Exploration

In order to get an idea of what the data looks like, we will first look at the data from week 24 of 2015 for each data set. Figure 3.5 shows the total load every 15 minutes during this week. A clear daily pattern can be discerned, where the demand during the day is a lot higher than the nightly demand. Another thing that is clearly visible is that energy demand seems to be a lot lower during the weekend when compared to weekdays. A final thing of note is the small peak in demand between 22:00 and 23:00 every day, it is not clear what causes this slight peak in demand. Generally speaking, the forecast predicts the realization quite well and the total demand seems to be fairly predictable one day in advance. The Mean Absolute Error (MAE) of this data is 943 kWh.

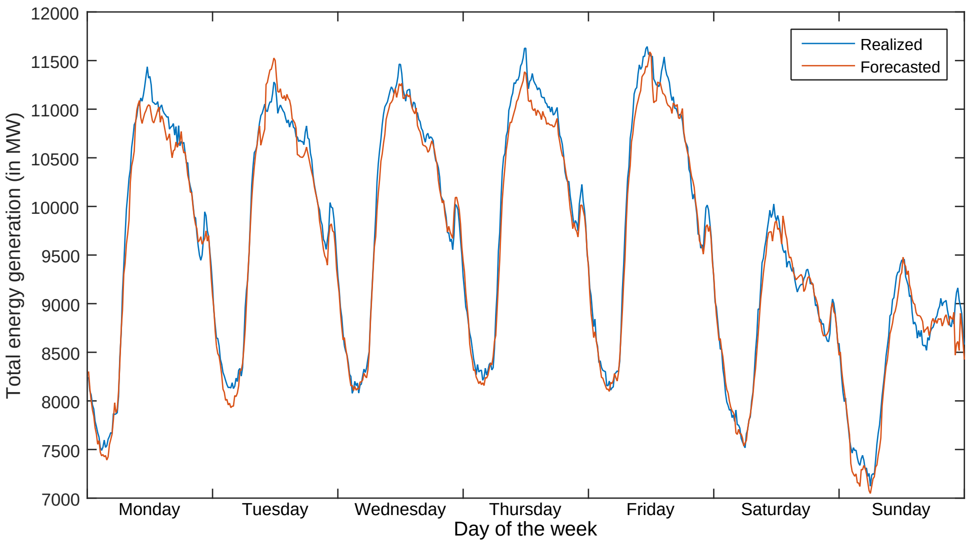


Figure 3.5: The realized total load and its forecast every 15 minutes in week 24 in 2015, taken from [12]

Figure 3.6 shows the total generated solar energy during this same week 24. Here we can see again that there is a clearly discernible daily pattern, as we would expect. However, it seems to be harder to predict the solar power, compared to the total demand. We can see that the erratic behaviour of the realization is absent from the prediction, especially when the sun is at its peak. This can be caused by the general nature of solar energy, or by the fact that the predictions are made a day in advance and are therefore incapable of predicting overcast accurately. The MAE of the solar energy forecast is 228 kWh which is lower than the 943 kWh error of the total demand. However, the average generated solar energy is only 3.4% of the total demand, therefore if we scale the error to the total demand, we get an error of 6706 kWh.
Therefore, the error is overall bigger than the error in the demand forecast, but they happen at predictable times of day.

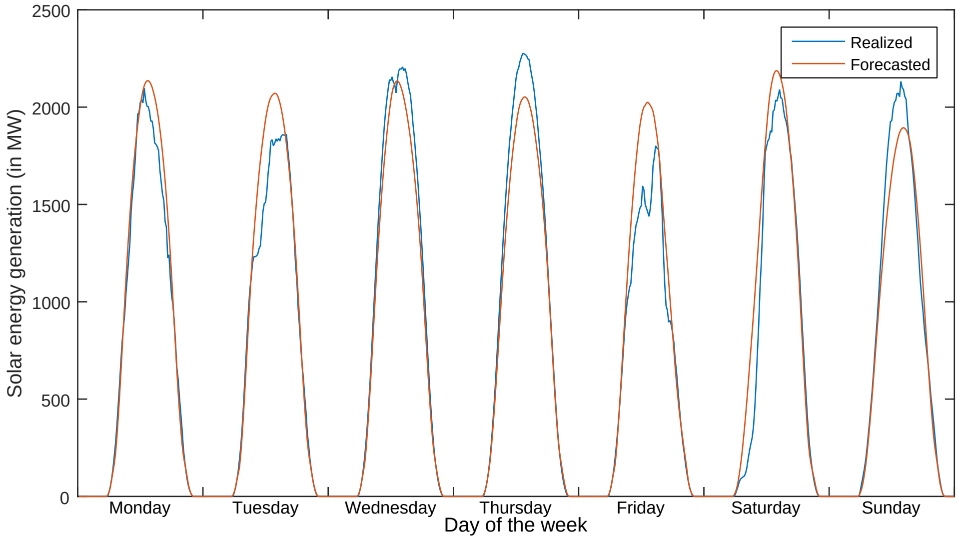


Figure 3.6: The realized total solar generation and its forecast every 15 minutes in week 24 in 2015, taken from [12]

Figure 3.7 shows the total generated wind energy and its prediction in, again, week 24. This plot is different from the others in the fact that it does not show a clear daily pattern. This lack of daily pattern means that there are not only errors in forecasting the height of peaks in generation, but also in the location in time of those peaks. This results in an MAE of 390 kWh, while wind energy is on average only 5.8% of the total demand, resulting in a scaled error of 6724 kWh. This error is only slightly higher than the error for solar energy at 6706 kWh. However the timing of when the errors occur is much less predictable and this, combined with the inherent erratic nature of wind energy, makes the error in the forecast of wind energy the most difficult to deal with.

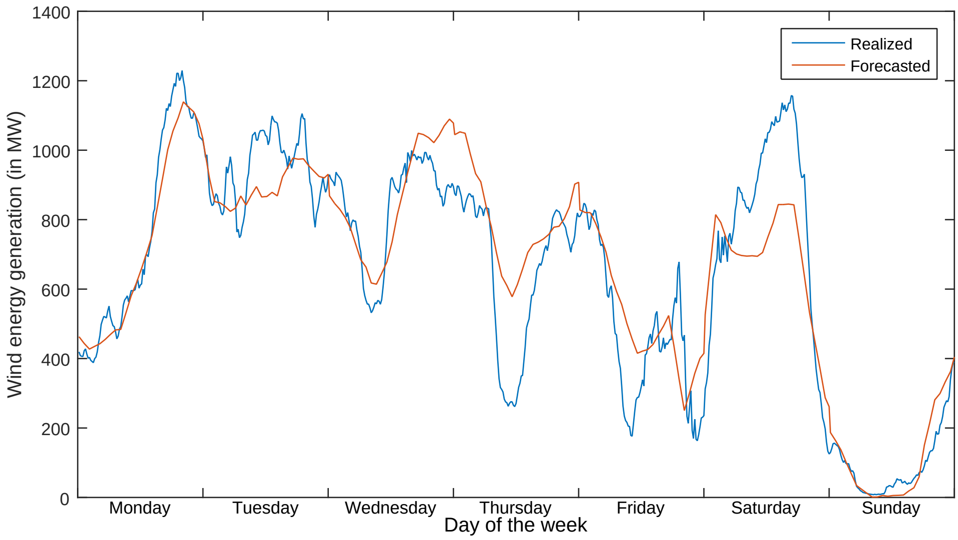


Figure 3.7: The realized total wind generation and its forecast every 15 minutes in week 24 in 2015, taken from [12]

Figure 3.8 shows the hourly electricity prices for the German market for week 24. No forecast was available to us and therefore it only contains the realized values. The data has a clear daily pattern that is considerably different in the weekends. The pattern shows a peak in price in both the morning and evening hours. The price is lowest in the middle of the night and valleys of different sizes can be found during the middle of the day, probably caused by a combination of generated solar power and maybe a slight reduction in demand.

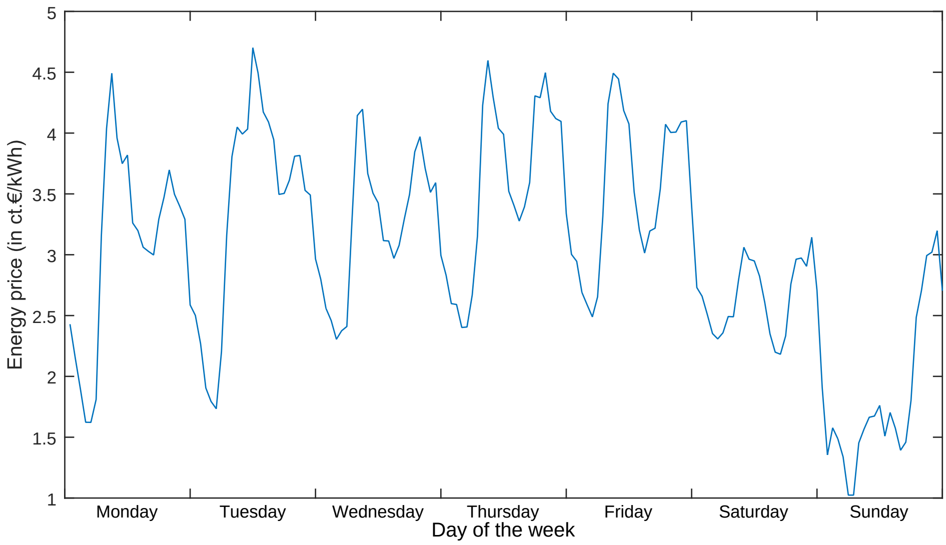


Figure 3.8: The hourly unit price of electricity in Germany in week 24 in 2015, taken from [12]

Data Cleaning

Table 3.1 shows the characteristics of the different attributes in the data set. All forecasts and the realization of the generated solar energy have only 4 missing values in the entire year. These 4 values correspond to the quarters in the additional daylight saving time hour on October 25th. None of the attributes had any data for this additional hour and the hour was therefore deleted from the data set.

The attribute with the most missing values is the realization of the total demand (load). Some of the missing values are small intervals spread out over the year that can be interpolated. However, there are two intervals that were deemed too big to interpolate. The first interval lasts from 00:00 June 23rd until 00:00 July 1st and the second lasts from 00:15 until 23:45 on July 31st. The first interval is composed of eight consecutive days and the second is almost one full day, the decision was therefore made to delete these days from the entire data set, as all the attributes are required to compute results. The deletion of entire days does result in an interruption of the weekly patterns. However, this part of the data was only used to create a model to reduce the prediction error and was not directly used in simulation.

The remaining missing values are in the realized Load and Wind generation and are all in relatively small intervals spread out over the year. In order to fill in these missing values we can use the fact that there are forecasts available for these times. However, we do not want to use these forecasts directly, as there should be an uncertainty in these values. Therefore the prediction error just before the missing interval is computed and the error just after the interval. Next, using these two errors, a linear interpolation is used to approximate the errors inside the interval and, using the errors and the forecasts, the missing realizations are replaced.

The German price data has a lower total number of data points, because this data is hourly data, while the Belgian data is for periods of 15 minutes. The missing values in the German data consist of the first five days of the year, which are missing, and the hour daylight saving time. The missing data does not affect the other data sets, since this data is only used to validate the generated price. As validation we only look at the average weekly pattern and therefore we just remove the missing values.

Table 3.1: Table of the data characteristics

	Number	Missing	min	max	mean
Load (realization)	35040	998	5744	13806	9978
Load (forecast)	35040	4	5741	13717	9902
Wind (realization)	35040	259	1	1705	572
Wind (forecast)	35040	4	1	1899	592
Solar (realization)	35040	4	0	2353	347
Solar (forecast)	35040	4	0	2266	345
Germany	8760	121	-79.94	99.77	29.971

Commodity Price Generation

Since the data for the electricity prices is missing from the Belgian data we have to create this data ourselves. The advantage of creating our own prices is that we can scale renewable energy penetration levels to represent future scenarios. The downside, however, is that our model may be missing a vital pattern or relationship. In order to reduce the discrepancy between the generated and real data, we will be using the electricity price data from the German market in order to validate our results.

In the literature many different models for price signals in smart grids are proposed. A price model that is positioned between a single-/double-tariff and the dynamic pricing model is a Time Of Use (TOU) price model. In a TOU model prices usually vary during the day, for example hourly, but are fixed from day to day for a longer period of time, usually several months to a year. The idea behind this model is that it is very expensive to have enough production capacity to meet peak demand. It is generally much cheaper to run production plants, like coal and nuclear, at a constant rate for long periods of time. The TOU pricing therefore aims at leveling the demand pattern and lowering the cost of electricity that way.

The problem with this method is that this does not take renewable energy production and actual demand into account. In order to take intermittent renewable energy and the actual demand into account, we propose a model that has a base price that is based around the idea of a TOU price for every time period. In addition to this base price a discount will be provided each time period for the renewable resources as a percentage of the demand at that time. Equation 3.9 shows this relationship.

$$p(t) = p_{base}\frac{E^t_{total}}{\bar{E}_{total}} - p_{solar}\frac{E^t_{solar}}{E^t_{total}} - p_{wind}\frac{E^t_{wind}}{E^t_{total}}, \tag{3.9}$$

with p_{base} a TOU related constant, p_{solar} and p_{wind} respectively the solar and wind constants, E^t_{total} the total consumed energy at time t, $\bar{E}_{total}$ the annual average total consumed energy, $E^t_{solar} = \min\left[E^t_{solar}, \max\left[E^t_{total} - E^t_{wind}, 0\right]\right]$ the consumed solar energy at time t, and $E^t_{wind} = \min\left[E^t_{wind}, E^t_{total}\right]$ the consumed wind energy at time t.

In order to create a base price structure, the values $p_{wind} = 9$, $p_{solar} = 8$, and $p_{base} = 10$ were chosen. The values are chosen the same as in [19], however the price structure is slightly different to account for the TOU pricing.

Turning this base price structure into a realistic commodity price which can be used for computations, is done by mapping the base price onto a range of price values taken from the German data using the linear transformation defined as:

$$p^{new}(t) = \left(p^{old}(t) - a_1\right)\frac{b_2 - a_2}{b_1 - a_1} + a_2,$$

with $[a_1, b_1] = [1.5, 12]$ the range of the base price as observed from the histogram of the base price, and $[a_2, b_2] = [-2, 6.5]$ the range of the German prices excluding extreme outliers, also observed from its corresponding histogram.

In order to compare the generated Belgian commodity price to the known German commodity price, the generated renewable energy in Belgium was scaled to match the generated renewable energy in Germany. In 2015, 21% of the consumed electrical energy in Germany was either solar or wind energy[1], in Belgium this number was 9.2%. Figure 3.9 shows the average week pattern of the German price, and the Belgian prices with 21% and 30% renewable energy penetration. The general structure of the Belgian prices with 21% renewables seems similar to the German prices, with slightly lower peaks during the morning and evening. The prices with 30% show a bigger difference between the morning/evening peak and the midday valley because of the increase in solar energy. A difference between both Belgian prices and the German price is the little peak between 22:00 and 23:00. This is the same peak as we have seen earlier in the demand, and it is not entirely clear what causes this peak. Elia, the company which provided the data, suggested that it might be caused by the start of the night tariff, but there was no certainty.

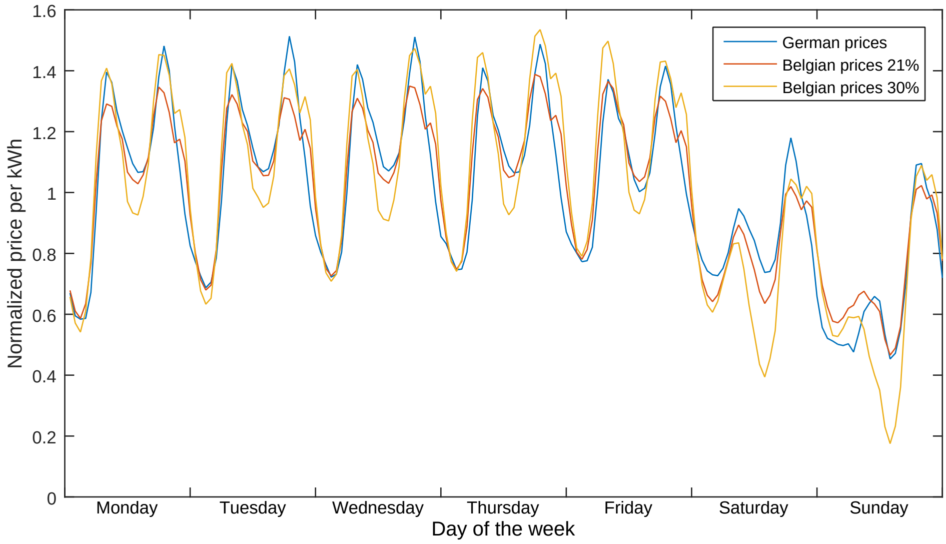


Figure 3.9: The average week pattern in 2015 for the hourly electricity price for the real German data, and the generated Belgian data with 21% and 30% renewable energy penetration

3.3.2 Forecasting

As described above, some forecasted data is available. However, this data lacks accuracy on the short term, as it is made at least 13 hours ahead of time. As described in the model in Section 3.2, we would like to move through time using a sliding window. However, because this data is made at least 13 hours ahead, we would like to make the forecasts more reliable as we get closer in time. In order to improve the forecasts without having access to updated wind and solar forecasts, we will be using the errors of previous time periods to predict the errors of future time periods. These predictions will then be used to update the forecasts.

Let ϵ_t be the forecast error of the original forecast made for time t, given by $\epsilon_t = p_t - \hat{p}_t$, with $\hat{p}_t$ the forecast and p_t the true price. We would like to create a linear model based on previous errors that is capable of predicting the error of the forecast that is i time units ahead. We will be using the errors made over the past three days, or 72 hours, to make this forecast. As training data for the forecasting model, we will be using the data from the 10th of April until the 31st of December 2015. This training is done by solving the following minimization problem:

$$\min_{x_i} \sum_{t=j}^{N} \left| \epsilon_t - \begin{pmatrix} \epsilon_{t-i} & \epsilon_{t-i-1} & \cdots & \epsilon_{t-i-71} \end{pmatrix} \begin{pmatrix} x_{i,1} \\ x_{i,2} \\ \vdots \\ x_{i,72} \end{pmatrix} \right|, \tag{3.10}$$

[1] http://www.erneuerbare-energien.de/EE/Navigation/DE/Service/Erneuerbare_Energien_in_Zahlen/Zeitreihen/zeitreihen.html

with j the 10th of April, N the 31st of December, i the number of hours we are predicting in the future, and x_i the resulting model.
When we want to update a prediction at time t for a time period i hours in advance, we simply calculate:

$$\hat{p}^*_{t+i,i} = \hat{p}_{t+i} + \begin{pmatrix} \epsilon_t & \epsilon_{t-1} & \cdots & \epsilon_{t-71} \end{pmatrix} \begin{pmatrix} x_{i,1} \\ x_{i,2} \\ \vdots \\ x_{i,72} \end{pmatrix}$$

.

3.4 Results

This section will discuss the results obtained using the model described in Section 3.2. In the model the size of the battery does not have an influence on its actions. A battery of 1 kWh with 1 kW maximum charging speed will have the same optimal actions as a battery of 10 kWh with 10 kW maximum charging speed, only 10 times smaller. This is because the actions of the battery only depend on the future electricity prices and how full it is relative to its capacity. Since these prices do not depend on the actions of the battery, both batteries have the same actions. For this reason, the results from this section can be scaled up or down to fit differently sized batteries, as long as the relative charging speeds of the batteries are the same. In other words, the strategies and resulting profits scale for batteries with the same C-rate.

3.4.1 Experimental Setup

The results are reached through simulation using the data discussed in Section 3.3. Additionally the default values for the parameters of the model in these simulations can be found in Table 3.2. The start of the test window is on the 4th of January 2015 at 00:00 and the test window ends on the 22nd of February 2015 at 24:00, which is a period of 50 days.

Table 3.2: The default values for the model parameters

Name	Symbol	Value
Efficiency	η	90%
Window length	L	36 hours
Length test window	T	50 days
Max charging speed	u_{max}	5 kW
Battery capacity	x_{max}	20 kWh
Starting charge	x_0	0 kWh
Tax rate	γ	21%
Fixed tax	τ	0
Battery operation costs	κ	0.01€/kWh
Window leftover value	c	$\frac{1}{8}\sum_{l=t+L+1}^{t+L+8}\hat{p}_l$

3.4.2 Impact of the General Parameters

Forecast Accuracy

One of the downsides of the generated price data described in Section 3.3.1 is that the forecasts are computed a day in advance. This means that they do not take the most recent information into account and therefore it does not matter whether we look at the data an hour, ten hours, or a day in advance, the forecast will remain the same.
In order to create a more realistic scenario we would like to update predictions as we go along. The problem, however, is that while in reality models are provided with updated weather models and predicted demands, we have no access to such data. Therefore we will update the predictions with the data that is available to us, the errors of the previous time periods. The general idea is

that if it was sunnier than predicted an hour before, chances are high that it will be sunnier than predicted the coming hours.
The errors in predictions of previous time periods are put in a linear model as described in Section 3.3.2 and the average results over a period of 50 days can be found in Figure 3.10. As expected, the biggest improvement is achieved for time periods that are closest to the current time period and it decreases rapidly as we get further in time. Interestingly we can still improve the forecasts even further than one day away.

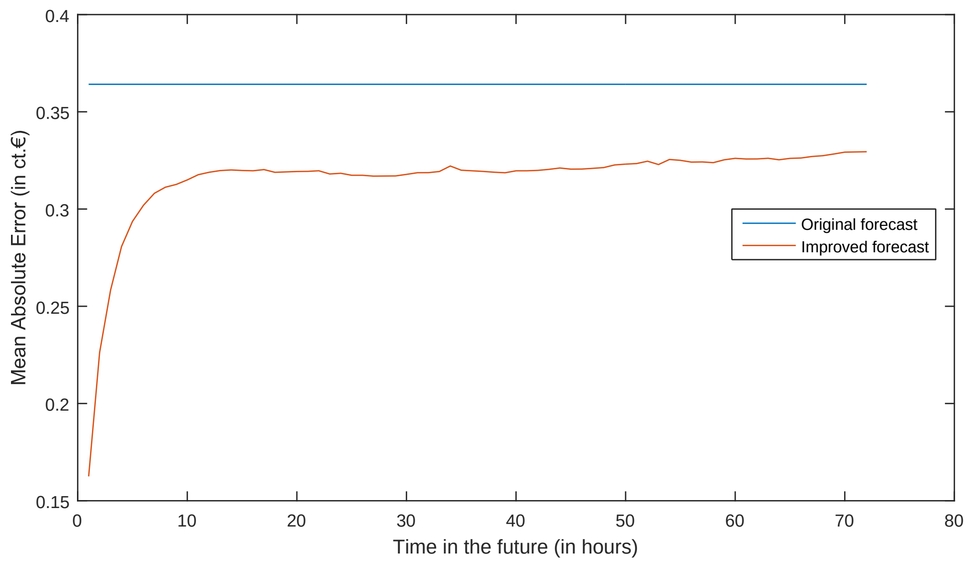


Figure 3.10: The Mean Absolute Error over fifty days for the original forecast and the improved forecast

We expect that this method of improving the forecast is worse than could be achieved in a realistic scenario where not only the previous errors, but also updated weather forecasts and demand profiles can be taken into account.
In order to determine the importance of improving the forecasts, we have changed the error size by multiplying the current error by a factor. The total profits for the simulation over 50 days can be found in Figure 3.11. In this figure we can see that for all charging speeds the profit reduces as the size of the error increases. However, there is a big difference between the decline in profit of the slower charging batteries compared to the fast charging batteries. The slower charging batteries appear to be more robust to increased error sizes. This is most likely because of the fact that the only strategies in this model are (dis)charging at the maximum rate, or not (dis)charging at all. If a really fast charging battery makes a bad investment, it commits a lot of funds to this bad investment. While a slower charging battery will most likely make the same mistake, but it cannot invest as many funds to it and is therefore more robust.
Thus slower charging batteries become relatively more valuable when the accuracy of the available forecasts decreases.

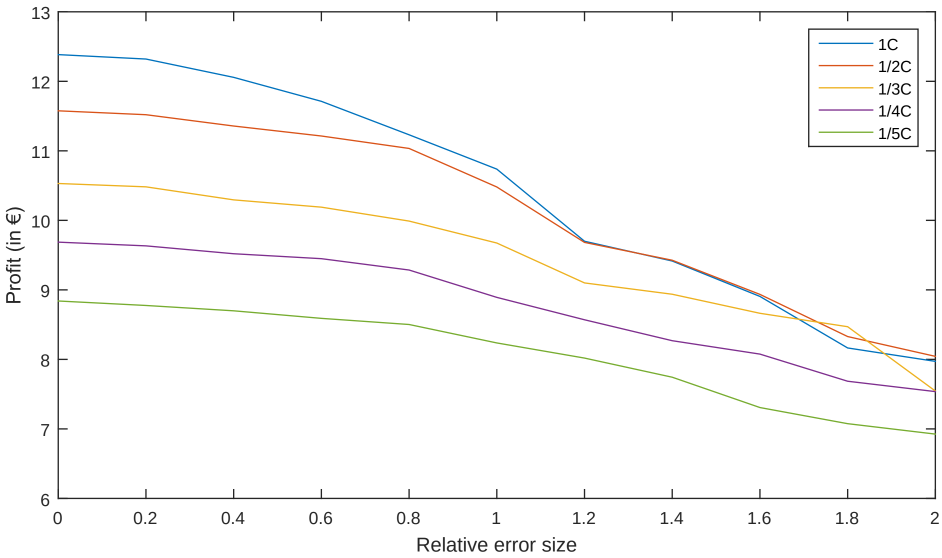


Figure 3.11: The profit for different error sizes relative to the error in the improved forecast for several charging speeds using the stand-alone model

Window Size

After having changed the price forecast, we would like to know the influence of other parameters on the overall profit. The influence of the window size is shown in Figure 3.12. This figure shows that different charging speeds converge to their maximum profitability at different rates. The fastest charging battery reaches its maximum profitability for a window size of 16 hours, while the slowest charging battery requires a window size of 36 hours. This is caused by the fact that a larger window size contains more information about the distant future than smaller window sizes. This increased information makes it possible to make more accurate predictions of future profits and improves the decisions up to a point. This point depends on the decisions to be made.
The fastest charging batteries, when full, have to decide whether they discharge now, or rather wait until a future point in time. This decision depends primarily on what will happen first in the future, one very cheap price, or one higher price.
On the other hand, the slowest charging batteries, when half full, have to decide whether to discharge, charge, or do nothing. This time the decision depends on whether multiple low price periods occur first, or multiple high price periods. Because multiple periods are required to formulate a strategy, the slower charging batteries require larger window sizes to reach the best decisions.
An interesting thing to note is the bump for the fastest and second fastest charging batteries at 16 hours. The profit for a window of 16 hours is higher for these batteries than the profit for a window of 72 hours. This is caused by the structure of the data set we used to create these lines. After simulating for 50 days starting January fourth at midnight, as is done to create the graph, the difference in profit for the fastest charging battery between 16 and 72 hour windows is 0.15€. If we extend the run-time to 100 days, the difference decreases to 0.05€. So the expectation is that if this model is run for an even bigger number of days this anomaly will cancel out and the profit for a 72 hour window will be at least as high as for a 16 hour window.

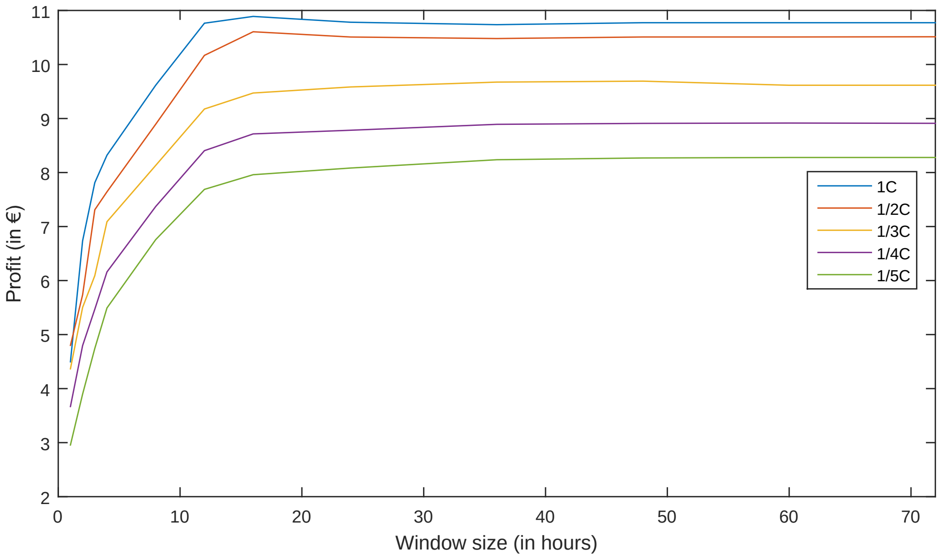


Figure 3.12: The influence of the window size on the profit for several charging speeds using the stand-alone model

3.4.3 Impact of the Battery Parameters

Charging Speed

In previous figures the influence of the charging speed on the profit has already been shown by providing different results for different charging speeds. However, in order to get a better understanding of the direct relationship between the profit and the charging speed, they are plotted against each other in Figure 3.13. This figure also shows the improvement made by updating the forecasts using a linear model as described in Section 3.4.2.

The faster charging batteries are clearly capable of gaining a higher profit than the slower charging batteries. However, comparing the reduced error with the original uncertainty, it seems that the faster charging batteries are also more heavily affected by uncertainty. For these faster charging batteries, the profit increases by up to 13% by using a linear model to reduce the error.

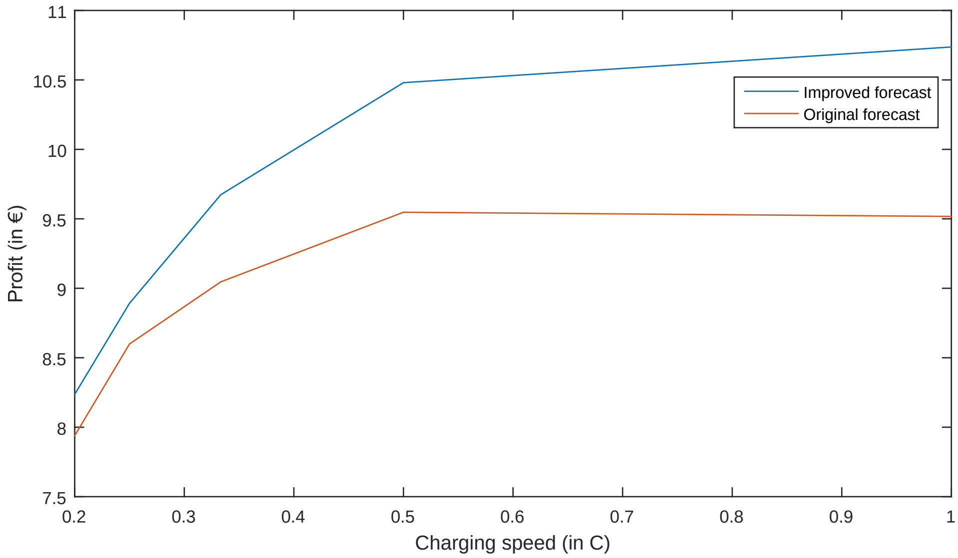


Figure 3.13: The influence of the charging speed and improving the forecast on the profit using the stand-alone model

Efficiency

Figure 3.14 shows the influence of the batteries round-trip efficiency on the overall profit. We can see that the profit increases faster as we get closer to 100% efficiency. As an example, Tesla states that the round-trip DC efficiency of their Powerwall is 92.5%. However, in order to convert this to the round-trip AC efficiency, which we are looking at, we have to take the loss during AC/DC and DC/AC conversion into account. If we assume the efficiency of the conversion to be 95% each way, this results in an overall round-trip AC efficiency of $.95 * .95 * .925 = .835$, or 83.5% efficiency. Looking at the figure again, we can see that the fastest charging speed shows volatility in its curve. The most likely cause for this is the lower number of actions performed during the 50-day simulation compared to the other charging speeds. Imagine a battery of 1C and a battery of 1/2C both fully charging and discharging once every day. The battery of 1 C will then charge one hour and discharge one hour every day for 50 days, giving a total of 100 hours where energy is traded. The battery of 1/2C will charge and discharge for 2 hours every day, for a total of 200 hours of energy trading. Now if there is an error in the prediction of one of the prices, it influence 1 out of 100 trading moments for the 1C battery, and 1 in 200 trading moments for the 1/2C battery. For this reason, the fastest charging battery is much more susceptible to forecast errors than the slower charging batteries.

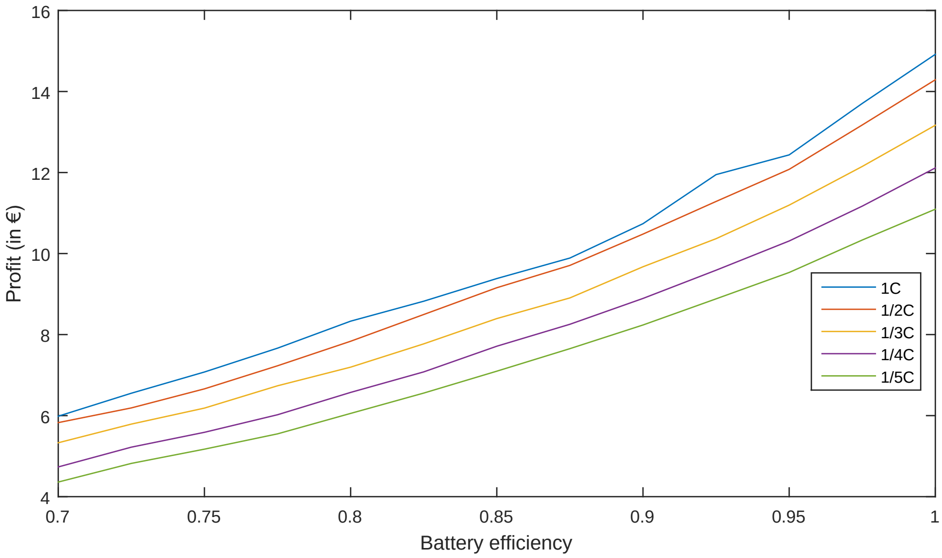


Figure 3.14: The influence of the round-trip efficiency on the profit using the stand-alone model

Cycle Costs

Another influential battery parameter are the cycle costs. Figure 3.15 shows the influence of this parameter on the profit. The higher the cycle costs, the slower the decline in profit. However, beyond a cost of five cent per kWh there is no longer any profit to be made. Since the cycle costs are a combination of the purchase cost of the battery and the number of cycles it is able to make during its lifetime, an improvement in either can drop these costs and improve the overall profit.

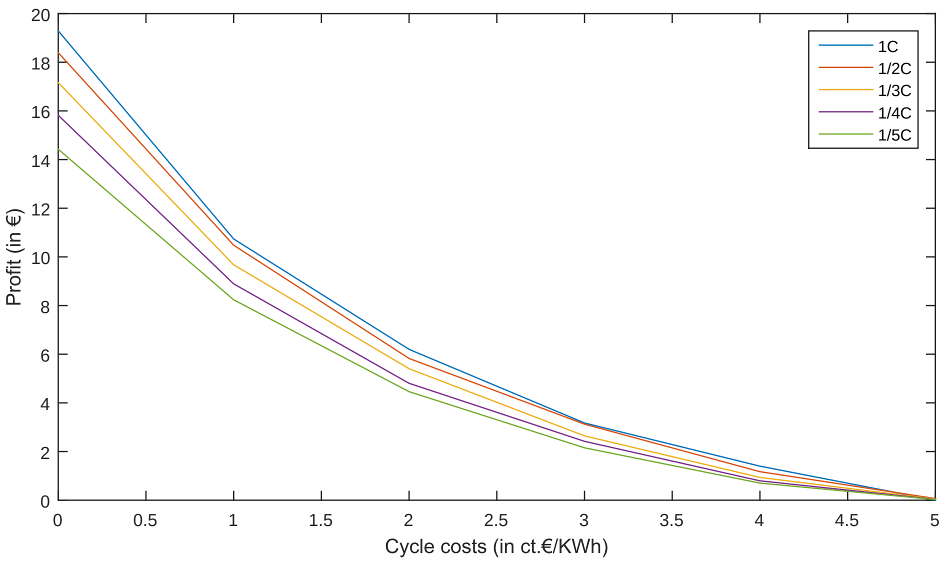


Figure 3.15: The influence of the cycle costs on the profit using the stand-alone model

3.4.4 Impact of the Tax Parameters

An important aspect of the future electricity grid will be the role the government will fulfill. As mentioned in Section 3.1.1 the electricity taxes in the current Dutch energy grid are 0.1007€/kWh and are partially compensated by a fixed tax return at the end of the month to promote the conservative use of energy. First we will explore the consequences of such a constant tax rate on the battery in a future energy grid. And in the section after that we will explore the possibilities provided by variable tax rates.

Constant Rate

Figure 3.16 shows the influence of a constant rate on the overall profit. In order to make a fair comparison between fixed and variable taxes, the variable taxes were set to 0. The shape of the figure is comparable to the shape of the previous figure (Figure 3.15 on cycle costs), as they are both fixed costs being added to the system. It is clear that the fixed tax system, which is currently in place in the Dutch energy market, is not viable for batteries. Even at four cents per kWh there is no longer any profit to be made, so something has to change in the current market if we want to profit from the flexibility provided by batteries.

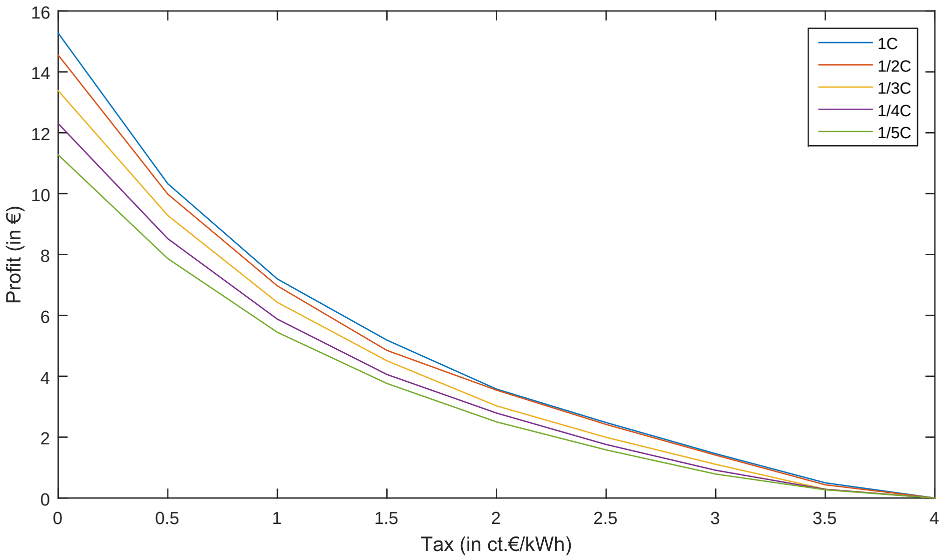


Figure 3.16: The influence of constant tax on the overall profit using the stand-alone model

Variable Rate

A solution to the taxation problem is a percentage based tax rate. Figure 3.17 shows that even with a tax rate of 100%, about 2/5th of the profit without any tax can be achieved. The big difference between this tax rate and the constant rate, is that the percentage based rate results in a small absolute tax when the price is low and a big absolute tax when prices are high. This is beneficial to batteries because they only buy when prices are low and this also incentivizes people to shift their electricity use to periods of low demand and/or high renewable energy generation.

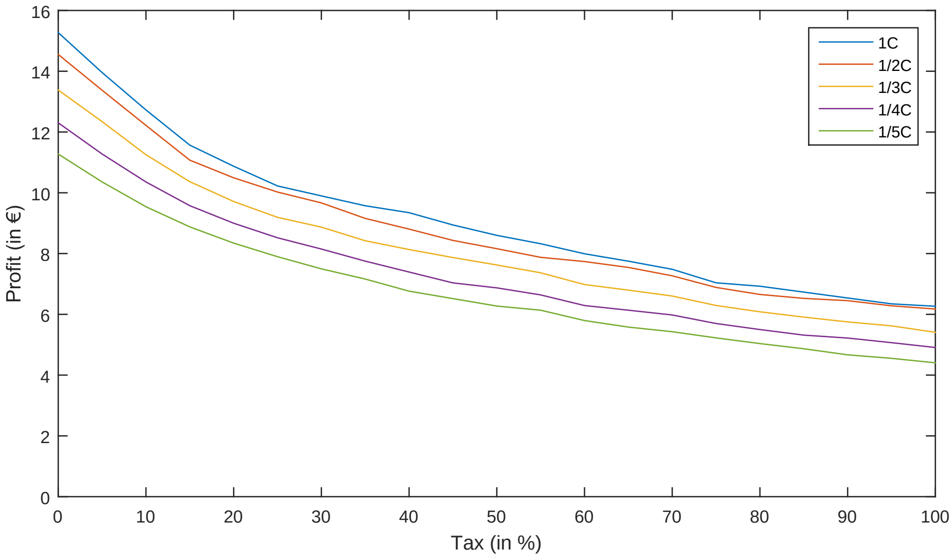


Figure 3.17: The influence of variable tax on the overall profit using the stand-alone model

3.4.5 Boundary Comparison

In order to show the added value of our methods we will compare the previously achieved results with both the lower bounds described in Section 3.2.1, and the upper bound found by solving Equation (3.2). Recall that both lower bounds do not make use of forecasts and therefore the comparison between our solution and the lower bound is mainly to show the added benefit of using forecasts. In order to show the power of our solution, we compare it to the upper bound.

Figure 3.18 shows the difference in result between the two lower bounds, the solution of the Stochastic Dynamic Program, and the theoretical upper bound, for different charging speeds.

Daily pattern refers to the lower bound achieved using the average daily price patterns and *Threshold* to the method using thresholds. For the faster charging batteries the *Daily pattern* method performs better. However, for slower charging methods this method actually starts to do worse than not having a battery at all. This is caused by the lack of profitable charging/discharging hour pairs in a day. When we are forced to charge and discharge five times a day it is impossible to turn a profit. A better tactic for these lower speeds may be to only charge once or twice a day. This would mean that the battery will never be fully charged and because the charging speed is lower than for the batteries of 1C and 1/2C, the profits will be lower proportional to the difference in charging speed.

The *Threshold* method will never lose money by design, we cannot buy and sell for prices that are not profitable. And, as can be seen from the graph, it actually makes more money for slower charging speeds, with the exception of $C = \frac{1}{5}$. This is caused by the structure of the electricity prices. The fastest charging battery fully charges the battery the first time the price drops below the buy threshold and fully discharges the first time it rises above the sell threshold. The price, however, is likely to drop even lower (or rise even higher) after the first time it crossed a threshold, because valleys and peaks generally last longer than one hour.

Ultimately though, neither method gets close to the theoretical upper bound or the solution achieved using Stochastic Dynamic Programming. This shows the added value of using forecasts in the planning of the battery strategy.

Comparing our solution to the upper bound, we can see that our solution performs quite well for the fast charging battery by getting within 89% of the optimal strategy. But where our solution performs best is for the slower charging batteries, even getting to within 92% of the optimum.

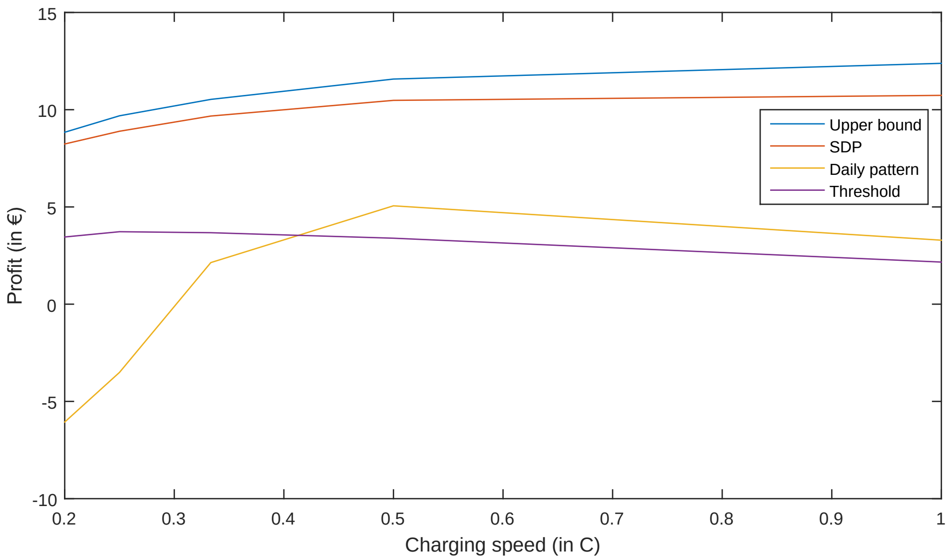


Figure 3.18: The difference in profit between two different lower bound methods, the SDP of the stand-alone model and an upper bound, for different charging speeds

3.5 Conclusion

In this section we sought to answer the question: *How do different characteristics affect the profitability of a stand-alone battery?* The results can be summarized as follows:

Variable	Influence
Forecast Accuracy	The profits of faster charging batteries are more susceptible to forecasting errors than those of slower charging batteries.
Window Size	The added value of increasing the window size diminishes rapidly. And for faster charging batteries it diminishes faster than for slower charging batteries.
Charging Speed	The profit decreases linearly with respect to the number of hours it takes for the battery to be fully charged.
Efficiency	The added value of a more efficient battery increases as the efficiency gets closer to 100%.
Cycle Costs	The higher the cycle costs, the slower the decline in profit, until all profits cease at a cycle cost of 0.05€/kWh.
Constant Tax	The same thing applies here as for the cycle costs, only the profits here cease at 0.04€/kWh.
Variable Tax	The higher these tax rates, the slower the decline in profit. However, even at 100% tax, the battery is still able to make a profit.

Additionally we have compared the results of our SDP solution, which makes use of forecasts, to two methods which do not take forecasts into account. Our results outperformed the other two methods and came within 92% of the theoretical upper bound.

Chapter 4

Home Battery Model

4.1 System Description

In the stand-alone model described in the previous section, the battery could be anywhere in the electricity network. In this section we want to focus on the added benefit of using a battery in a home setting with household electricity demand and an Electric Vehicle (EV). Figure 4.1 illustrates this situation.

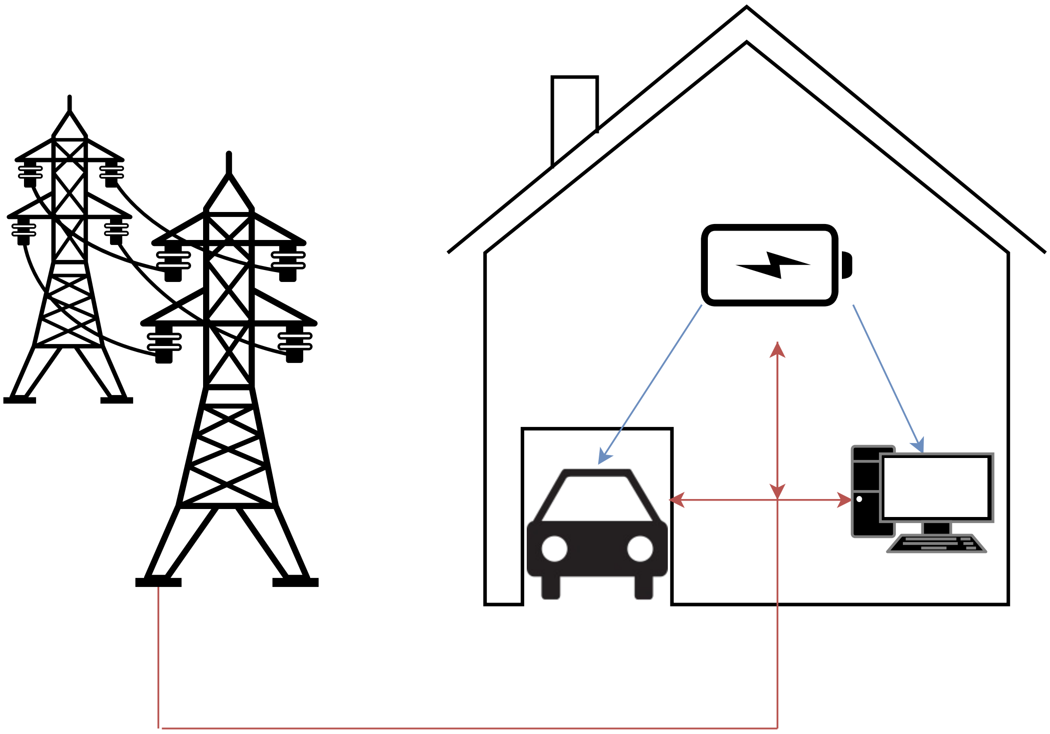


Figure 4.1: A schematic representation of the home battery model with a home battery, fixed household demand and an EV. The blue lines represent internal connections and the red lines are connections to the grid

This system consists of a household that is able to buy and sell energy from an electricity market. The market is the same market that was described in Section 3.1.1.
The household consists of three main components: the general household appliances, the home battery, and the Electric Vehicle (EV). The demand of the household appliances is the sum of all demand within the household, except for the battery and the EV. We assume this demand is fixed,

so there are no smart appliances in this household. In addition to the market, we will also be using the same battery model as in Section 3.1.2.

4.2 Methods

In this section we will be looking at the models used to describe the system above. In Section 4.2.1 we consider smart charging of the EV and we consider the future household demand to be uncertain. In Section 4.2.2 we fix both the future household demand and the demand of the EV in order to get computationally feasible results.

4.2.1 Smart EV Charging

The new situation is different in two ways, household demand and an EV battery. In order to take the uncertainty of household demand into account, we introduce a new parameter, d_t, to the state space. And in order to take the current charge of the EV battery into account, another parameter, x_t^E, will be added. This also results in a new action, u_t^E, representing the charging strategy at time t for the EV. In order to make a clear distinction between the battery and the EV, we will change the parameter of the current charge of the battery to x_t^B, and we will change the charging strategy at time t for the battery to u_t^B.
These additional dimensions of the state space result in a new reward and value function. Let us first define u^+ as the quantity of energy traded with the grid if the net amount is positive and u^- as the quantity traded with the grid if the net amount is negative, given by:

$$\begin{aligned} u^+ &= \max\left[\left(1-(1-\eta)\,\mathbb{1}_{\{u_t^B<0\}}\right)u_t^B + u_t^E + d_t, 0\right], \\ u^- &= \min\left[\eta u_t^B + u_t^E + d_t, 0\right]. \end{aligned}$$

The reward function is then given by:

$$R\left(u_t^B, u_t^E, d_t, p_t\right) = -u^+\left(p_t\left(1+(\gamma-1)\,\mathbb{1}_{\{p_t>0\}}\right)+\tau\right) - \max\left[u_t^B, 0\right]\sqrt{\eta}\kappa - u^- p_t, \tag{4.1}$$

with u_t^B the (dis)charging of the battery and u_t^E the charging of the electric vehicle at time t. Note that we do not take the cost for the degradation of the EV battery into account, because we assume these costs to be linear in the quantity charged by the EV. This means that, since we can only charge and not discharge the EV, these costs have no impact on the decision of either the EV, or the battery.
With the introduction of a new stochastic variable $D = d_t$ and a new action, the value function becomes:

$$V_t\left(x_t^B, x_t^E, d_t, p_t\right) = \max_{u_t^B, u_t^E}\left[R\left(u_t^B, u_t^E, d_t, p_t\right) + \mathbb{E}_{D_{t+1}}\left(\mathbb{E}_{P_{t+1}}\left(V_{t+1}\left(x_t^B + u_t^B, x_t^E + u_t^E, D_{t+1}, P_{t+1}\right)\right)\right)\right], \tag{4.2}$$

with x_t^B the current charge of the battery and x_t^E the current charge of the EV. In order to solve this, additional restrictions have to be set on the solution space, such that there will be enough energy in the battery of the EV when it departs.

4.2.2 Fixed EV Charging

However, with the two dimensional maximization and the double expectation, the problem described in (4.2) becomes so computationally intensive that no exact solution could be computed within a reasonable time-frame. In order to solve this computational issue, the following two assumptions were made:

- The demand of the EV is assumed to be fixed and thus no longer requires finding an optimal action. The charging is fixed by charging the required energy for the day uniformly between the last arrival of the previous day and the first departure of the current day.

- The household demand is assumed to be known and will no longer add uncertainty to the system.

Even though both the household demand and the EV charging are now fixed and deterministic, the positive and negative quantities of electricity traded can still be expressed by:

$$\begin{aligned} u^+ &= \max\left[\left(1-(1-\eta)\,\mathbb{1}_{\{u_t^B<0\}}\right)u_t^B + u_t^E + d_t, 0\right], \\ u^- &= \min\left[\eta u_t^B + u_t^E + d_t, 0\right]. \end{aligned}$$

And the reward function is also still given by:

$$R\left(u_t^B, u_t^E, d_t, p_t\right) = -u^+\left(p_t\left(1+(\gamma-1)\,\mathbb{1}_{\{p_t>0\}}\right)+\tau\right) - \max\left[u_t^B, 0\right]\sqrt{\eta}\kappa - u^- p_t. \tag{4.3}$$

However, because the charging of the EV is no longer a choice, and because the household demand is no longer stochastic, they are no longer part of the state space and the value function becomes:

$$V_t\left(x_t^B, p_t\right) = \max_{u_t^B \in U_t^B}\left[R\left(u_t^B, u_t^E, d_t, p_t\right) + \mathbb{E}_{P_{t+1}}\left(V_{t+1}\left(x_t^B + u_t^B, P_{t+1}\right)\right)\right]. \tag{4.4}$$

This equation, combined with the end of planning window Equation (3.3), and the end of the total time window Equation (3.4), still gives the highest expected value at time t and is very similar to the stand-alone model. However, the big difference between this model and the previous model is that the piecewise linearity of the reward function R no longer holds. Because of this, the proof of Appendix A no longer holds and the possible actions at time t become $u_t^B \in U_t^B = \left[\max\left(-1, -x_t^B\right), \min\left(1, x_{max}^B - x_t^B\right)\right]$.

Just as in the stand-alone model, we would like to solve this problem by backwards induction over the planning window, as is displayed in Figure 3.2. However, just as in the stand-alone model, we run into the problem of having to compute the expectation of a maximum, given by:

$$\begin{aligned} \hat{V}_t\left(x_t^B\right) &= \mathbb{E}_{P_t}\left(V_t\left(x_t^B, P_t\right)\right) \\ &= \mathbb{E}_{P_t}\left(\max_{u_t^B}\left[R\left(u_t^B, u_t^E, d_t, P_t\right) + \hat{V}_{t+1}\left(x_t + u_t^B\right)\right]\right). \end{aligned}$$

In the stand-alone model we solved this by splitting the expectation into three different parts. One part contained prices for which the battery was charged, another where nothing was done, and the last one contained the prices for which the battery was discharged. This was possible because we only had to consider maximum charging, discharging, or doing nothing, and not something in between those options. However, now that Appendix A no longer holds, it is not possible to split the expectation for each different action, because the action space is continuous. In order to solve this problem the state-space and action-space are discretized with step size u_{step}. This means that u_{step} either has to divide both u_{max}^B and x_{max}^B, or u_{max}^B and x_{max}^B have to be floored to the nearest multiple of u_{step}.
Using this discretization we can again begin to split the expectation with different intervals for each possible action. Take $U_t^B = \{u_t^{B,1}, u_t^{B,2}, \ldots, u_t^{B,N}\}$ the set of possible charging actions with $u_t^{B,1} < u_t^{B,2} < \cdots < u_t^{B,N}$.
If the best option is to charge $u^{B,k}$ for price p^1, we expect the best option for a price $p^2 > p^1$ to be to charge at most $u^{B,k}$. And for prices $p^2 < p^1$ we expect the best option to be to charge at least $u^{B,k}$. Therefore we expect u^B to be monotonically decreasing with respect to p.
Now we would like to know for each consecutive pair of actions $\left(u_t^{1,i}, u_t^{1,i+1}\right)$, the price for which the actions have equal pay-off. In other words, the price for which we have:

$$R\left(u_t^{1,i}, u_t^E, d_t, p\right) + \hat{V}_{t+1}\left(x_t^B + u_t^{1,i}\right) = R\left(u_t^{1,i+1}, u_t^E, d_t, p\right) + \hat{V}_{t+1}\left(x_t^B + u_t^{1,i+1}\right).$$

Let p_i be the solution to this equation with respect to p. Because of the monotonicity and the fact that $u_t^{1,i} < u_t^{1,i+1}$, we know that $p_i > p_{i+1}$. If we now define $p_0 = -\infty$ and $p_N = \infty$, we can rewrite the SDP into:

$$\hat{V}_{t+L+1}(x,p) = cx, \text{ for some price } c, \tag{4.5}$$

$$\hat{V}_{T+\alpha}(x) = 0, \forall \alpha \in \mathbb{N}, \tag{4.6}$$

$$\hat{U}_t\left(x_t^B, p_t\right) = \underset{u_t^B \in U_t}{\arg\max} \left[R\left(u_t^B, u_t^E, d_t, p_t\right) + \hat{V}_{t+1}\left(x_t^B + u_t^B\right)\right], \text{ with} \tag{4.7}$$

$$\hat{V}_t\left(x_t^B\right) = \sum_{i=1}^{N} \int_{p_{i-1}}^{p_i} f_P(p) \left(R\left(u_t^{1,i}, u_t^E, d_t, p\right) + \hat{V}_{t+1}\left(x_t^B + u_t^{1,i}\right)\right) \mathrm{d}p. \tag{4.8}$$

Inserting the current SOC of the battery, x_t^B, and the current price, p_t, in $\hat{U}_t$ gives the action with the highest expected reward. If we use a sliding window to calculate this for all $t \in 1, \ldots, T$, we get the set of actions with the highest expected reward. This set of actions has a total realised value given by:

$$V_{total} = \sum_{t=1}^{T} R\left(\hat{U}_t\left(x_0 + \sum_{i=1}^{t-1} \hat{U}_i, p_t\right), u_t^E, d_t, p_t\right). \tag{4.9}$$

4.3 Data

For the home battery model we will be using the same electricity market data as we used in the stand-alone model. Additionally we will be using data to represent household demand. This data will be described in the following section.

4.3.1 Household Demand

Household demand is composed of a base demand and an optional demand for an electric vehicle, as not all households have an EV. Later on in the simulations some households will also have demand from a battery, however since this demand is generated by our algorithms, it will not be described in the data chapter. First we will describe the data used as base household demand and after that we will describe the data used as EV demand.

Base Demand

The household demand data used for this project comes from a simulator created in [21]. This simulator is based on data from the Welfare, Prosperity and Quality of the Living Environment (WLO) study on several different aspects of Dutch society up to 2040, performed by the CPB and the PBL. This study was centered around four different scenarios, of which the “Strong Europe” scenario was the basis for the simulator. The size of the originally generated data was 3000 households, which was scaled down to 200 households and aggregated for this study.
Figure 4.2 shows an example of this aggregated demand over week 24. From this figure we can see that the nature of domestic electricity demand is extremely peaky.

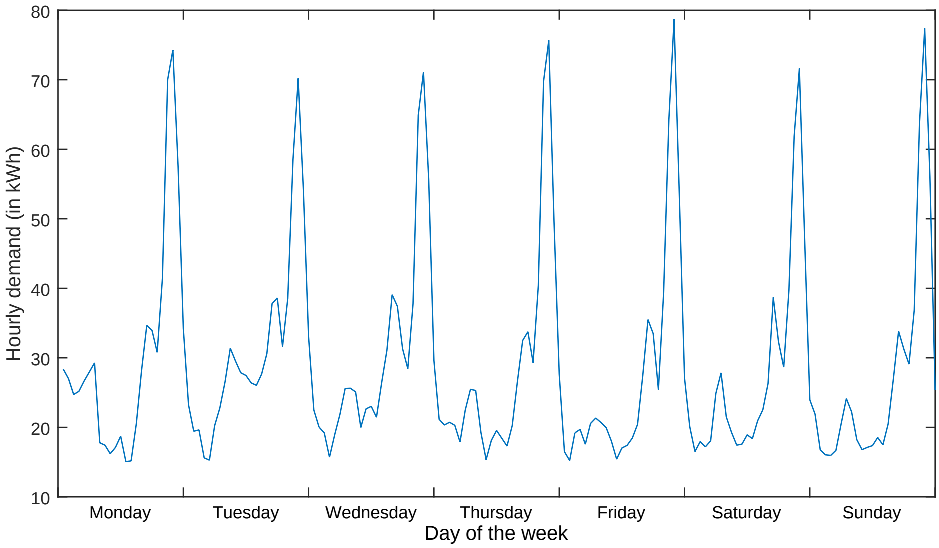


Figure 4.2: The hourly household demand aggregated over 200 households in week 24 of 2015, taken from [21]

EV Demand

The EV data used in this research comes from a model described in [27]. This model is based on a German survey among drivers and was used to generate a data set. In this data set the driving patterns of 10 different EV users are expressed by the departure time, arrival time, and consumed energy of every trip made by each vehicle. Because we cannot know if the vehicle has the opportunity to charge in between trips during the day, we assume that this is not possible. We assume that charging starts when an EV arrives home after its last trip of the day and is finished when it departs the next day. The energy required each day is the aggregate of the total energy consumed during each trip that day.

This data could be used to create smart charging schemes for the electric vehicle. However, this falls outside the scope of this research and therefore the data is converted to fit the scope. We assume that every time an electric vehicle departs, it contains the precise energy requirements for that day. And that this energy is charged uniformly from the previous day's last arrival until the first departure the next day.

Figure 4.3 shows an example of what this would look like in week 24. An important thing to note is that the average EV demand is about equal to the average base demand of a household. So adding an electric vehicle to a household practically doubles the total electricity demand.

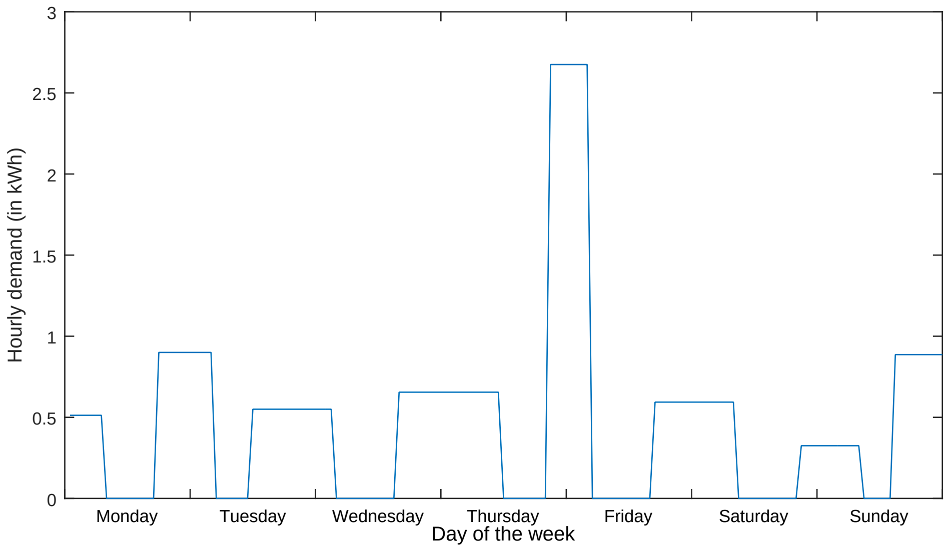


Figure 4.3: The hourly demand associated with EV charging in week 24 of 2015, taken from [27]

4.4 Results

In this section the results for the home battery model, as described above in Section 4.2.2, will be presented and analyzed. Many of the results obtained using this function are similar to the results obtained in Section 3.4. These similar results will not be discussed here, but can be found in Appendix B. Below we will first explain the experimental setup. After this, we will discuss the parameters whose influence changed most notably. And finally we will compare the results to both the lower bounds as well as to the results of the previous model.

4.4.1 Experimental Setup

The results are, just like in the stand-alone battery model, achieved through simulation. The data for these simulations is discussed in Sections 3.3 and 4.3.
The default values of the parameters for these simulations in Table 3.2 are still in effect in this section and can be seen again in Table 4.1. Two additional parameters have to be added for this model, because in order to be able to calculate any solution for this model, the set of possible actions has to be discretized.
In order to give an upper bound to this problem, we have to give the best actions at each time given perfect information. These actions influence the SOC of the battery and depend on the price and the demand in the system. To be able to find an upper bound we have to consider every possible action and state at every time, this requires the states and actions to have the same precision as the demand. In order to keep the computation of the upper bound feasible, the choice was made to discretize the state space and the action space in steps of 10 Watt and to round the household demand to the nearest multiple of 10 Watt as well. So the first additional parameter is the precision in the household demand, and the state space and the action space.
The second parameter that has to be added is the step size of the actions in the SDP. Using a step size of 10W is computationally infeasible. However, using the maximum charging capacity as step size, as was done in the stand-alone model, is too big. Because the chosen step size has to divide the maximum charging capacity, it cannot be the same across all charging capacities. Therefore, the choice was made to choose the step size equal to $u_{step} = \max\left(N | \frac{u_{max}}{N} < 2.5\text{kW}, N \in \mathbb{N}\right)$.

Table 4.1: The default values for the model parameters

Name	Symbol	Value
Round-trip efficiency	η	90%
Window length	L	36 hours
Length test window	T	50 days
Battery capacity	x_{max}	20 kWh
Starting charge	x_0	0 kWh
Price added tax	γ	1.21
Fixed tax	τ	0
Battery operation costs	κ	0.01€/kWh
Window leftover value	c	$\frac{1}{8}\sum_{l=t+L+1}^{t+L+8}\hat{p}_l$
Demand rounded to	$-$	10 W
Action step size	u_{step}	$\max\left(N \vert \frac{u_{max}}{N} < 2.5\text{kW}, N \in \mathbb{N}\right)$

The value of the best actions, given by Equation (4.9), gives the total profit of the system. This includes the reward for buying and selling energy by the battery, as well as the costs of buying energy for the household demand and EV. In order to make these results comparable to the results discussed in Section 3.4, we would like to find the added value of the battery in this system. In order to do this, we compare the results from our model with the situation where there is no battery. This situation can also be considered a lower bound on the performance, since our results should not be worse than having no battery at all. The value in this situation is given by:

$$V_{base} = \sum_{t=1}^{T} R\left(0, u_t^E, d_t, p_t\right). \tag{4.10}$$

The results presented in this section are then calculated as follows:

$$V = V_{total} - V_{base},$$

which represents the added value of using a battery in a home setting with taxes.

4.4.2 Impact of Tax Parameters

Constant Rate

The interesting differences between the stand-alone model and the home battery model begin with the effect of constant tax on the profits, as shown in Figure 4.4. As apposed to the stand-alone model, where there were no profits beyond four cents per kWh, here there is still a profit to be made at five cents per kWh. The relationship between the profit and the tax appears to be nearly linear.

In this figure we can also see the impact of the step size on the outcome. Charging speeds 1, 1/2, and 1/4 C all have the step size 2.5. While the charging speeds 1/3 and 1/5 C have step sizes smaller than 2.5. From the stand-alone model we know that trading no longer gives any profit at 4.5 and 5 cent per kWh. So all the profit at these tax rates originate from serving the fixed household and EV demand, for which a smaller step size is preferable.

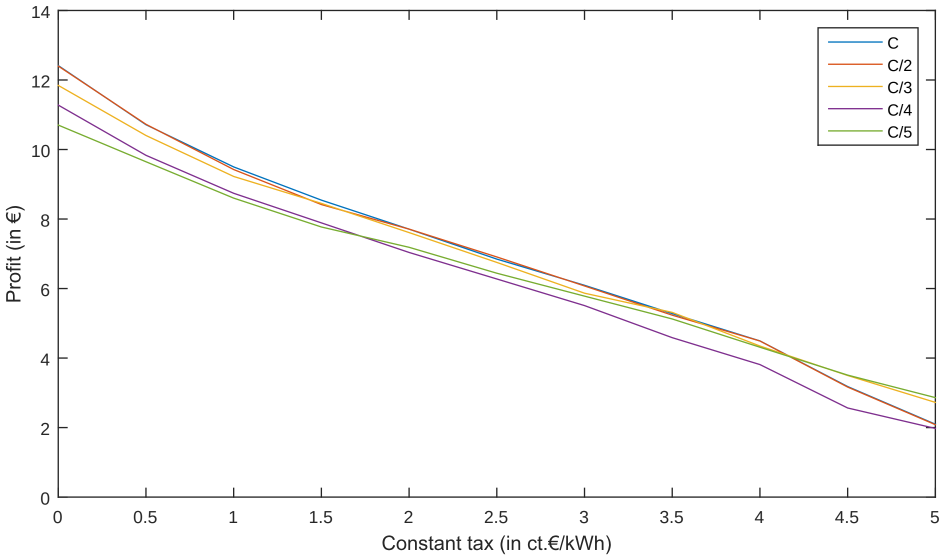


Figure 4.4: The influence of constant tax on the profit in the home battery model

Variable Rate

The biggest difference between the stand-alone model and the home battery model can be seen in Figure 4.5. In this figure, the influence of the percentage based tax is shown. It is clear that trading can provide very big profits when the tax rates are very low. However, this profit rapidly drops as tax rates get higher. Until, at around 15%, the drop levels out and starts to rise almost linearly. This rise is caused by the principle of buying low and consuming high.
Let us take the 100% tax rate as an example. If we buy and store electricity worth 1 cent, we have to pay 2 cents. If we use this energy later on when energy prices are at 5 cents, we would normally have to pay 10 cents. But since we already paid 2 cents, we just saved 8 cents. These savings are twice as high as the 4 cents we would have saved in the same situation with 0% taxes.

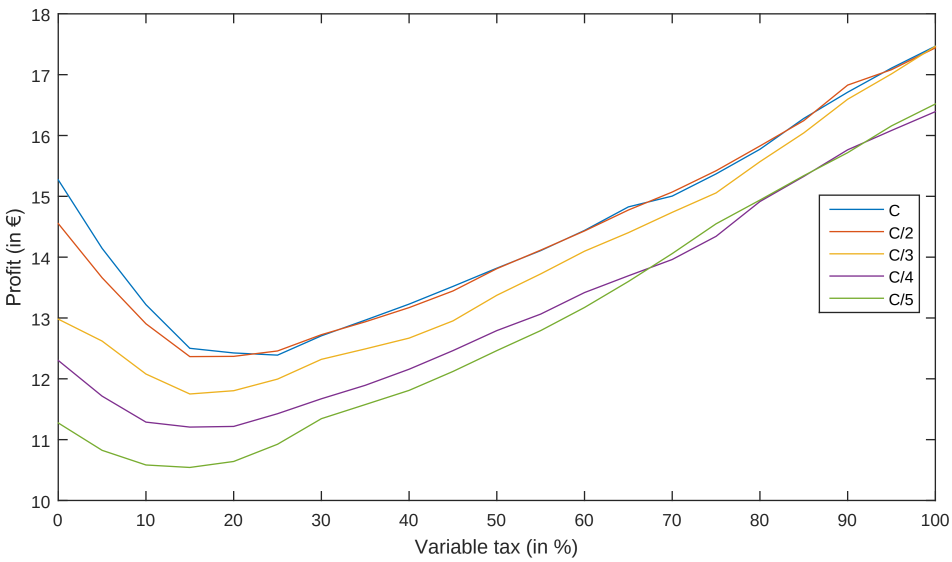


Figure 4.5: The influence of the variable tax rate on the profit in the home battery model

4.4.3 Comparisons

Boundary Comparison

In order to show the added value of our methods we will compare the previously achieved results with slightly altered versions of both the lower bounds described in Section 3.2.1, and the upper bound of our SDP. For the lower bounds, we determine how much energy to buy or sell in exactly the same way as we did before. However, when we decide to sell energy, we first use this energy to satisfy the household demand. And only if there is any energy leftover, do we sell it to the grid. This is guaranteed to increase the value of the lower bound, as using energy in house is more profitable than selling it to the grid because of taxes.
Figure 4.6 shows this comparison. Just as in the stand-alone model, we can see that our SDP solution outperforms both lower bounds considerably. Another similarity between the home battery model and the stand-alone model, is the fact that the SDP is capable of getting closer to the upper bound for slower charging batteries. The SDP gets to within 93% of the upper bound.

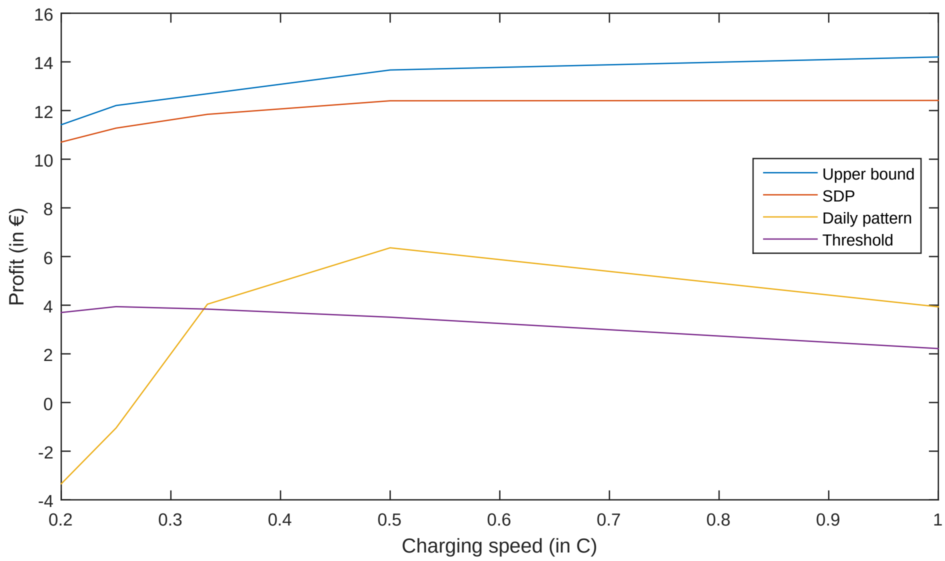


Figure 4.6: The comparison between the SDP solution and its upper and two lower bounds

Comparison to the Stand-Alone Model

Figure 4.7 shows a comparison between the profits in the stand-alone model and the profits in the home battery model. Taking the household demand into account when creating a battery strategy clearly has added benefits. However, the faster a battery charges, the closer the profits of the different models seem to get together. This is probably because of the fact that the fastest charging battery is very close to the second fastest charging battery, as was the case for every figure in this section.
But even though the results for the different models get closer together for higher charging speeds, the solution of the Stochastic Dynamic Program of the home battery model still outperforms even the theoretical upper bound for the stand-alone battery model. And the solution of the SDP for the home battery comes within 93% of its own upper bound.

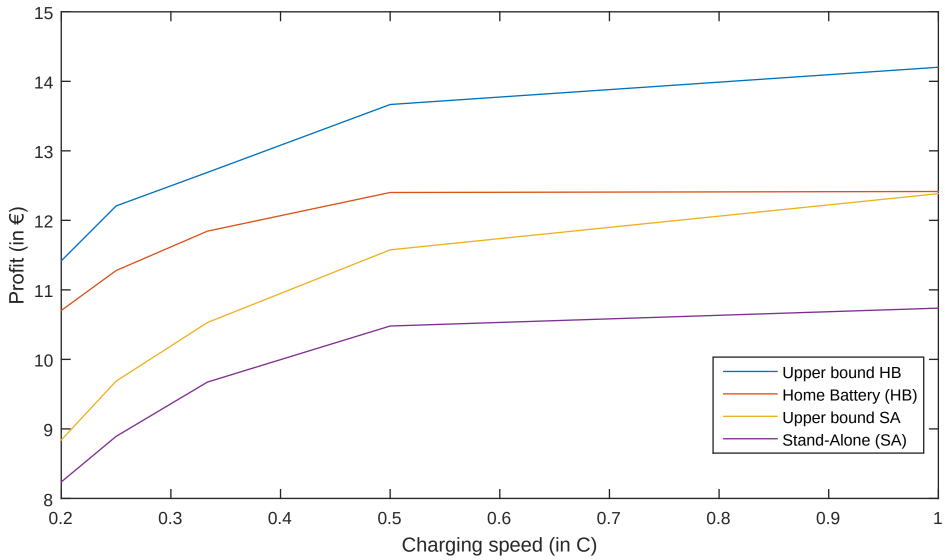


Figure 4.7: The comparison between the upper-bound and solution of the SDP of the home battery model, and the SDP solution of the stand-alone model

4.5 Conclusion

The goal of this chapter was to answer the question: *What are the main characteristics for the profitability of a home battery?* Most of the characteristics were very similar to the results obtained for the stand-alone model, as discussed in Section 3.5. The main differences are the characteristics related to tax.

Constant Tax. The negative influence of the constant tax on the overall profit is reduced for the home battery model when compared with the stand-alone model. However, constant tax will still result in zero opportunity for profit for the battery, only this time this happens for a tax of 0.06€/kWh instead of 0.04€/kWh.

Variable Tax. Because of the coupling of the battery with household demand, the influence of variable tax completely changes. Whereas the profits kept decreasing for increasing variable tax in the stand-alone model, the added value keeps increasing, after an initial slight decrease, for the home battery model. Our results have shown that the worst results for the business case for the home battery are reached around 15-20% tax.

Just as in the stand-alone model, the results for the home battery model outperformed the two lower bounds. Additionally, when compared to the theoretical upper bound, the results improved marginally from 92% in the stand-alone model, to 93% for the home battery model, and the home battery model outperformed the theoretical upper bound of the stand-alone model.

Chapter 5

Price Maker Model

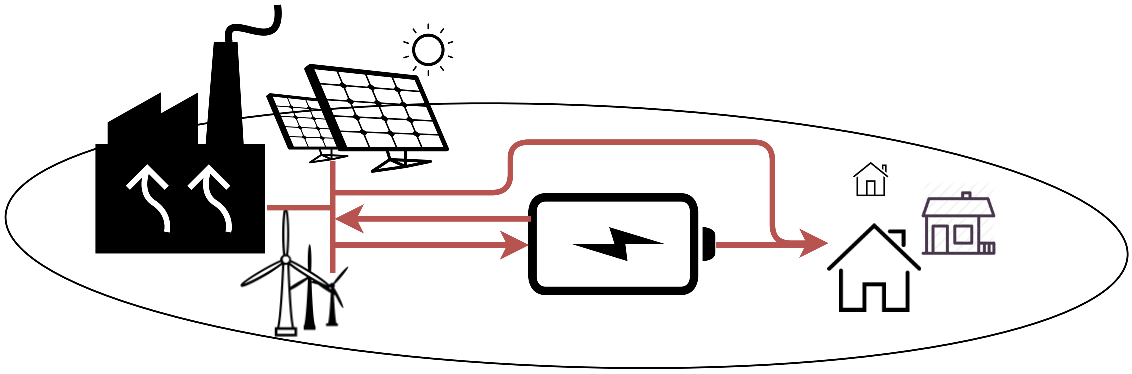
Up to this point we have always assumed the battery to be a price taker. This means that the actions of the battery were assumed to be insignificant compared to the combined actions of the rest of the system. In this model we will be looking at what happens when we assume the battery to be price maker. In Section 5.1 we will give a description of the system we will be looking at, in Section 5.2 we will explain the methods used to get to a solution, and finally in Section 5.3 we will show the results obtained for this model.

5.1 System Description

A situation where a battery is a price maker could happen when a huge battery is operating in the electricity market, when a lot of small batteries all use the same strategy and are therefore a combined price maker, or when the system is small enough for individual home batteries to have an influence on the overall outcome of the system.
For this model we will be looking at a system where all three situations are a possibility. The system is small enough for individual households to make a difference, but in order to see the full potential of the price maker we will have to either look at one much bigger battery, or several smaller batteries all using the same strategy.

Figure 5.1 depicts the system we will be considering in this chapter. The demand of the system is generated by 200 households. In addition to their regular electricity demand, every household is also equipped with an electric vehicle which is charged uniformly from the last moment of arrival to the first moment of departure, just as in the home battery system.
The supply of the system consists of some renewable energy in the form of wind and solar energy, and any remaining supply is generated by a diesel generator.

In the results associated with this system, we will be referring to the terms individual and social welfare. With individual welfare we mean the profit of the battery itself. And with social welfare we refer to the combined welfare of the battery and the demand in the system. So individual welfare is what we looked at in the stand-alone battery model and social welfare is what we examined in the home battery model.



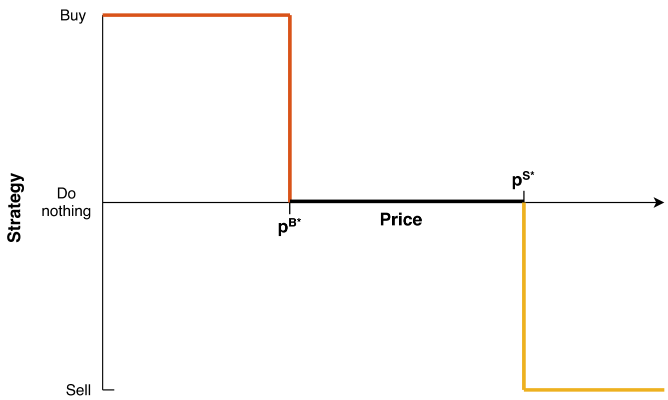


Figure 5.2: An example of a bid curve sent by the battery to the auctioneer

When the bid curve is submitted, it is combined with the bid curve of the demand of the households and the EVs to create an aggregated demand curve. Because the demand of the households and the EVs is assumed to be fixed, they will buy the required energy for any price and their bid curve is a flat line.
After this the market is cleared by finding the price, p^*, where the aggregated bid curve and price function intersect. At this point electricity demand and supply are equal. The price for which the market is cleared determines the action of the battery. If $p^* < p^{B*}$, or $p^* > p^{S*}$ the battery will respectively charge or discharge at maximum speed. When $p^{B*} < p^* < p^{S*}$ the battery will not charge and in case the intersection happens at either p^{B*} or p^{S*}, the quantity (dis)charged will be determined by the height of the intersection.

5.2.2 Known Price Influence

When the battery knows the influence of its actions on the price, an important choice has to be made. Namely, does the battery try to maximize its own profits, or does the battery focus on minimizing the cost of the system as a whole. And from this choice the question naturally arises what impact the decision has on the overall outcome of the system. From this point on we will refer to the battery that tries to maximize its own profits as the *selfish* battery and the battery that works for the benefit of the system as the *selfless* battery.

The main difference between the model for the selfish and selfless batteries is the reward function. Recall from Section 4.2.1 that the quantity bought from or sold to the market is respectively given by:

$$\begin{aligned}
u^+ &= \max\left[\left(1-(1-\eta)\,\mathbb{1}_{\{u_t^B<0\}}\right)u_t^B + u_t^E + d_t, 0\right],\\
u^- &= \min\left[\eta u_t^B + u_t^E + d_t, 0\right].
\end{aligned}$$

Take $\pi_t\left(x_t\right)$ the price function at time t, with its argument x_t the total combined demand of the system at time t. Now the reward function for the selfless battery is given by:

$$R\left(u_t^B, u_t^E, d_t, \pi_t\right) = -\left(u^+ + u^-\right)\pi_t\left(u^+ + u^-\right) - \max\left[u_t^B, 0\right]\sqrt{\eta}\kappa.$$

While the reward of the selfless battery is influenced by both its own actions as well as the charging of the EVs and the demand from the households, the selfish battery is only rewarded for its own actions. This results in a reward function given by:

$$\begin{aligned}
R\left(u_t^B, u_t^E, d_t, \pi_t\right) = &- u_t^B\left(\pi_t\left(u^+ + u^-\right) + \sqrt{\eta}\kappa\right)\mathbb{1}_{\{u_t^B>0\}}\\
&- \eta u_t^B \pi_t\left(u^+ + u^-\right)\mathbb{1}_{\{u_t^B<0\}}.
\end{aligned}$$

Suppose all future values are known, then, regardless of the reward function, the upper bound to the profit of the stochastic charging problem can be found by solving the set of equations given by:

$$V^*_{T+1}\left(x_{T+1}\right) = 0, \tag{5.1}$$
$$V^*_t\left(x_t\right) = \max_{u^B_t \in U^B_t}\left[R\left(u^B_t, u^E_t, d_t, \pi_t\right) + V^*_{t+1}\left(x_t + u^B_t\right)\right], \tag{5.2}$$

with $U^B_t = \left[\max\left(-1, -x_t\right), \min\left(1, x_{max} - x_t\right)\right]$.

In reality however, not everything is known in advance. The value function is based on the wind and solar energy generated in future time periods as well as on future household demand. In order to take this into account, we will be using forecasts, denoted by '^'.
As in the case of the home battery model (Chapter 4), we will not be looking at the uncertainty in the household demand. Instead we will only focus on the uncertainty in the price. This time however, there is not simply one probability distribution for the price with a single location and scale. Instead, the location of the distribution changes as the actions of the battery change. In order to still take the distribution of the uncertainty into account, the location and scale of the distribution are separated. Therefore let us introduce a stochastic variable Z_t to capture the uncertainty, with $\mathbb{E}Z_t = 0$ and probability density function $f_{Z_t}(z)$. In order to keep the notation clear, define: $R^*\left(u^B_t, u^E_t, d_t, \hat{\pi}_t, Z_t\right) = R\left(u^B_t, u^E_t, d_t, g_t\right)$, with $g_t\left(x_t\right) = \hat{\pi}_t\left(x_t\right) + Z_t$.
The solution to the stochastic problem can now be found by solving the set of equations given by:

$$\begin{aligned} V_{t+L+1}\left(x, z\right) &= cx, \text{ for some price } c, \\ V_{T+\alpha}\left(x_{T+\alpha}, z\right) &= 0, \forall \alpha \in \mathbb{N}, \\ V_t\left(x_t, z_t\right) &= \max_{u^B_t \in U^B_t}\left[R^*\left(u^B_t, u^E_t, d_t, \hat{\pi}_t, z_t\right) + \mathbb{E}_{Z_{t+1}}\left(V_{t+1}\left(x_t + u^B_t, Z_{t+1}\right)\right)\right], \end{aligned}$$

with $U^B_t = \left[\max\left(-1, -x_t\right), \min\left(1, x_{max} - x_t\right)\right]$.
However, solving this set of equations requires the same expectation of a maximum as we have seen in both the home battery model and the stand-alone model. This expectation is given by:

$$\begin{aligned} \hat{V}_t\left(x_t\right) &= \mathbb{E}_{Z_t}\left(V_t\left(x_t, Z_t\right)\right) \\ &= \mathbb{E}_{Z_t}\left(\max_{u^B_t}\left[R^*\left(u^B_t, u^E_t, d_t, \hat{\pi}_t, Z_t\right) + \hat{V}_{t+1}\left(x_t + u^B_t\right)\right]\right). \end{aligned}$$

Just as for the home battery, we will solve this by discretizing the state-space and action-space by steps of u_{step}. Take $U^B_t = \{u^{B,1}_t, u^{B,2}_t, \ldots, u^{B,N}_t\}$ the set of possible charging actions with $u^{B,1}_t < u^{B,2}_t < \cdots < u^{B,N}_t$. We assume the price function is monotonically increasing with respect to the demand and the optimal charging action is monotonically decreasing with respect to the price. Under these assumptions we can calculate the interval of Z_t for which action $u^{B,i}_t$ is at least as good as action $u^{B,i-1}_t$ and action $u^{B,i+1}_t$. We compute this interval by solving the following equation for z:

$$R^*\left(u^{B,i}_t, u^E_t, d_t, \hat{\pi}_t, z\right) + \hat{V}_{t+1}\left(x_t + u^{B,i}_t\right) = R^*\left(u^{B,i+1}_t, u^E_t, d_t, \hat{\pi}_t, z\right) + \hat{V}_{t+1}\left(x_t + u^{B,i+1}_t\right).$$

Let z_i be the solution to this equation. Because of the monotonicity and the fact that $u^{B,i}_t < u^{B,i+1}_t$, we know that $z_i > z_{i+1}$. If we now define $z_0 = -\infty$ and $z_N = \infty$, we can rewrite $\hat{V}_t\left(x_t\right)$ into:

$$\hat{V}_t\left(x_t\right) = \sum_{i=1}^{N}\int_{z_{i-1}}^{z_i} f_{Z_t}\left(z_t\right)\left(R^*\left(u^{B,i}_t, u^E_t, d_t, \hat{\pi}_t, z_t\right) + \hat{V}_{t+1}\left(x_t + u^{B,i}_t\right)\right) \mathrm{d}z_t. \tag{5.3}$$

Using this equation in the iterative scheme displayed in Figure 3.2, we can find the best possible action at time t by solving:

$$\hat{U}_t\left(x_t, \pi_t\right) = \arg\max_{u^B_t \in U_t}\left[R\left(u^B_t, u^E_t, d_t, \pi_t\right) + \hat{V}_{t+1}\left(x_t + u^B_t\right)\right].$$

Inserting the current SOC of the battery, x_t, and the current price function, π_t, in $\hat{U}_t$ gives the action with the highest expected reward. If we use a sliding window to calculate this for all $t \in 1, \ldots, T$, we get the set of actions with the highest expected reward. This set of actions has a total realised value given by:

$$V_{total} = \sum_{t=1}^{T} R\left(\hat{U}_t\left(x_0 + \sum_{i=1}^{t-1} \hat{U}_i, \pi_t\right), u_t^E, d_t, \pi_t\right). \tag{5.4}$$

5.2.3 Price Functions

In this section we will discuss and show the price functions used in the price maker model. It is important to note that these price functions have not been scaled. So the results obtained from one of the price functions do not represent realistic prices, nor can they be compared to the results from other price functions. They can however be used to compare different strategies and parameter settings evaluated using the same price function.

Average Pricing

The first price function is based on the principle of paying a weighted average price with the weights dependent on the source of the electricity. In our system, described in Section 5.1, we have three sources of electricity, wind, solar and diesel. The price per unit of energy is p_w for wind, p_s for solar, and p_d for diesel. The price function $\pi_t(x)$ is then given by:

$$\pi_t(x) = p_w \min\left(x, E_t^w\right) + p_s \min\left(\max\left(x - E_t^w, 0\right), E_t^s\right) + p_d \max\left(x - E_t^w - E_t^s, 0\right),$$

with E_t^w the generated wind energy at time t and E_t^s the generated solar energy at time t.
An example of this price function for the 8th of February 2015 at 15:00 ($t = 928$), $p_w = 1$, $p_s = 2$, and $p_d = 10$ can be found in Figure 5.3. In this example the first 30 kWh are wind, the second 30 kWh are solar, and the remainder is energy from the diesel generator. Of note are the flat line for the wind energy and the steep slopes near the beginning of the graph every time a new energy source has to be called upon. Also note the flattening of the slope for higher demands. This indicates that a change in demand when the demand is low will result in a big change in the unit price, while a similar change for higher demand will barely result in any change in price.
This price function represents a smaller system with few modes of generation, like an island. Another example is a microgrid, where there is some local generation, which has to be used locally, and where any excess electricity is drawn from a connection to a bigger grid, where the microgrid is a price taker. In this system there will most likely be a central entity responsible for buying electricity from the different sources and setting the price to the average of the costs of buying for the households.

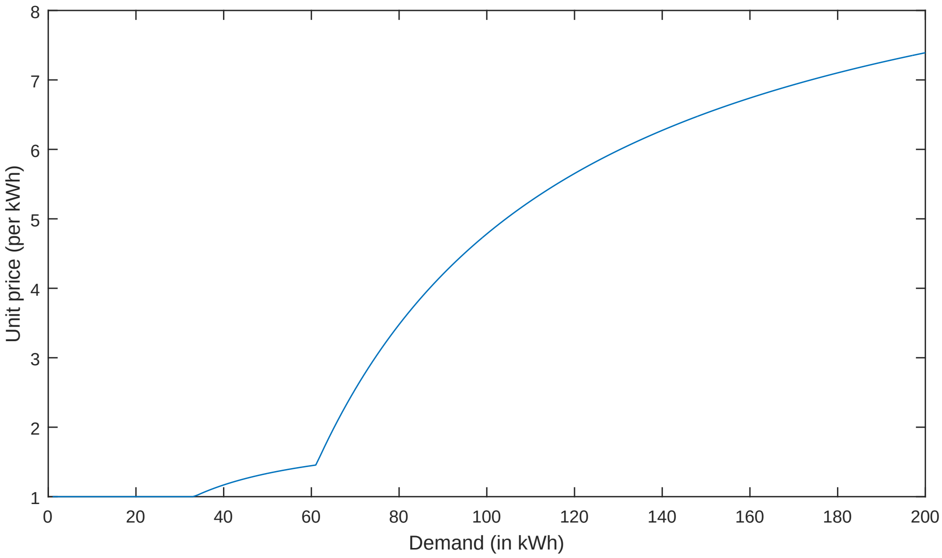


Figure 5.3: An example of the unit price at 15:00 on February 8 using the average price model

Merit Order Curve

The merit order is a method of organizing electricity suppliers based on their unit cost of production. The curve is created by making a bar graph, with each method of production represented by a bar with a height corresponding to the unit cost and the width corresponding to the current maximum production capacity. Figure 5.4 gives a fictional example of such a curve.

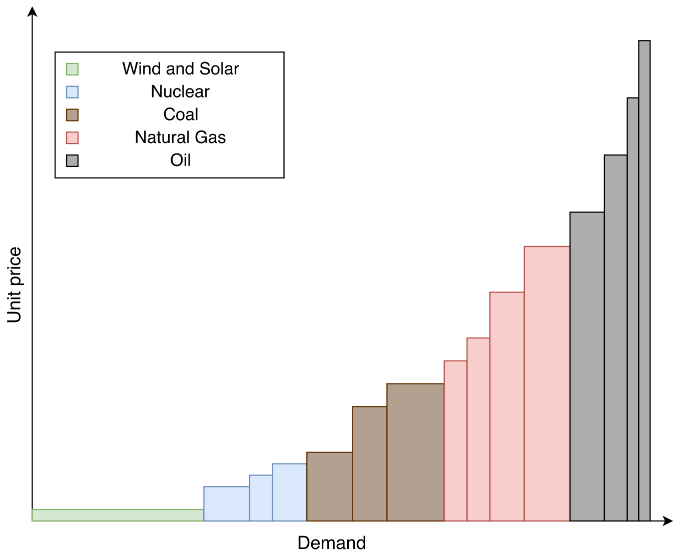


Figure 5.4: An illustrative example of the merit order curve

The authors of [16] have attempted to model this merit order curve. We will be using their exponential model, given by:

$$\pi_t(x) = e^{\frac{l'_t(x)-a}{b}} + z_t,$$

with l'_t the normalized load, a the horizontal shift, and b the scaling factor. For a and b we will be using the values as were given in [16], namely $a = -0.860$ and $b = 0.421$. In order to take the specific wind and solar energy for each time period into account, another horizontal shift was added in calculating the normalized load l'_t. The normalized load is given by:

$$l'_t(x) = \frac{l_t(x)}{l_{max}}, \text{ with}$$
$$l_t(x) = x - E^w_t - E^s_t, \text{ and}$$
$$l_{max} = \max_{t \in \{1,\ldots,T\}} (l_t(d_t)).$$

The value taken for the maximum load is $l_{\max} = 383$ kWh and is reached on the first of November 2015 at 22:00. Figure 5.5 shows an example of this price function on the 8th of February at 15:00. This price function is based on a bigger grid with a lot of different generation sources each with their own capacity and unit price. In this bigger grid there is a universal price. So, unlike the system for the average price, every generator receives the same price per unit of energy and therefore in order to get more supply, the price has to be raised to the minimum unit price of the next cheapest generator.

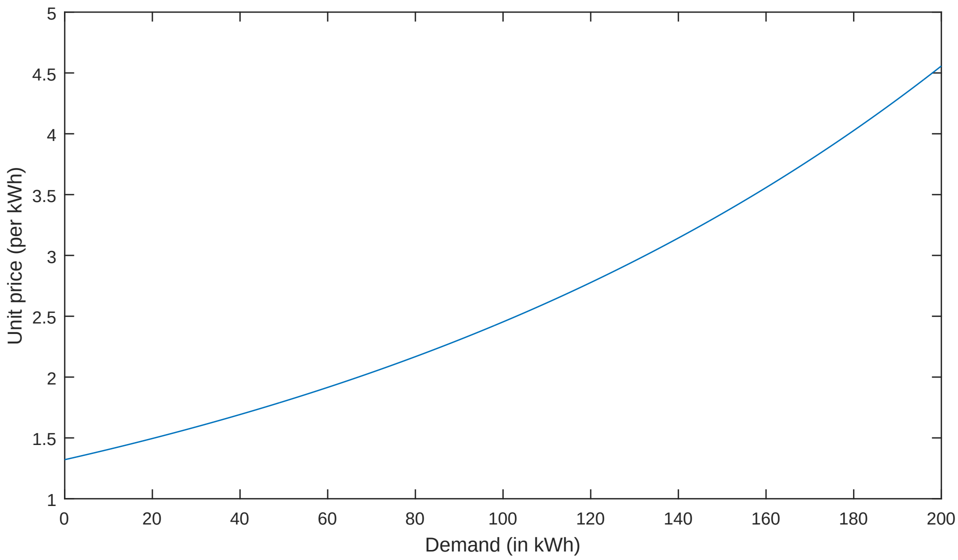


Figure 5.5: An example of the unit price at 15:00 on February 8 using the merit order model

5.3 Results

In this section the results of the models described above will be discussed. First we will give results for the model where the battery controller does not know its influence on the price. After this the results for the model with a known price influence will be discussed. And finally a comparison will be made between the two methods.

5.3.1 Experimental Setup

The results are, just like in the stand-alone and home battery model, achieved through simulation. The data for these simulations is discussed in Sections 3.3 and 4.3.
The default values of the parameters for these simulations in Table 4.1 are mostly still in effect in this section and can be seen again in Table 5.1. Some parameters have been slightly altered, in order to accommodate the bigger scale of this problem (200 households against 1 household for the home battery). We have also added one new parameter which represents the default number of iterations for the model with an unknown influence on the price, the function of this parameter

will become clear in the next section.

Table 5.1: The default values for the model parameters

Name	Symbol	Value
Round-trip efficiency	η	90%
Window length	L	36 hours
Length test window	T	50 days
Max charging speed	u_{max}	x_{max}
Battery capacity	x_{max}	250 kWh
Starting charge	x_0	0 kWh
Price added tax	γ	1
Fixed tax	τ	0
Battery operation costs	κ	0.01 kWh^{-1}
Window leftover value	c	$\frac{1}{8}\sum_{l=t+L+1}^{t+L+8} \hat{p}_l$
Demand rounded to	$-$	100 W
Action step size	u_{step}	50 kW
Number of iterations	$-$	20

5.3.2 Unknown Price Influence

The data we are using is the same data as was described in Sections 3.3 and 4.3. The big downside of using this data set is the fact that the forecasts do not take the influence of the battery into account. When the battery expects to buy energy in the future, it expects to buy this for the unchanged price, which is lower than the actual price will be due to its influence. Similarly when the battery expects to sell energy it overvalues this action because the price will drop when the battery sells energy. Due to this undervaluation of the costs associated to buying and the overvaluation of the revenues from selling energy, the battery will be buying and selling more often than it should.
In order to solve this problem we simulate the same period of 50 days multiple times. Between each iteration the forecasts will be updated using the following function:

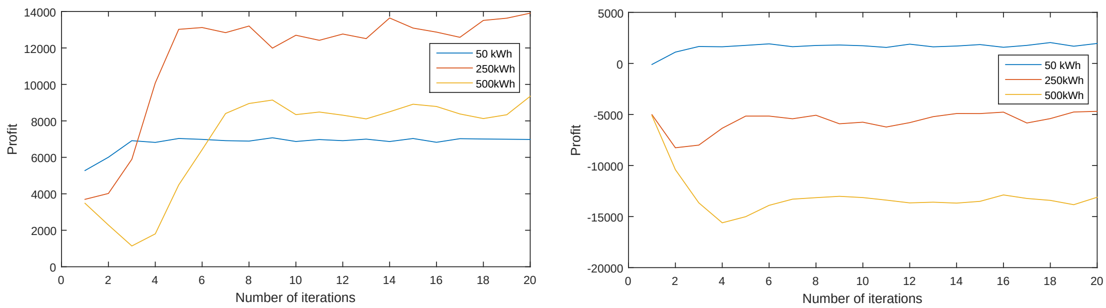
$$\hat{p}^k_{t,j} = \epsilon_{t,j} + p^{k-1}_t, \tag{5.5}$$

with t the point in time we are looking at, j the number of hours we look into the future, $\epsilon_{t,j}$ the error in the original forecast, and p^{k-1}_t the realized price at time t in the previous iteration. This way we adjust the price forecasts and realizations to a situation with an influential battery, while keeping the forecasting error as is.
We recognize that the forecasting of the actions of a battery or a group of batteries on the price influences the error in the system. It could be that the error is reduced by the dampening effect the battery can have on the price, or it may increase the error due to the introduction of a new error. However, since we did not create the forecasts ourselves, we choose to ignore the effect on the error.
In the remainder of this section we will be looking at the results of the model described above. This battery does not know its influence on the price and can therefore not be optimized on the social welfare. The only option is to use the profits of the battery itself as the optimization function of the model. However, since the battery inadvertently has an influence on the system as a whole, we will present the results for both the individual welfare of the battery, as well as for the social welfare of the system.

Average Price Model

Iterations
Figure 5.6a shows the impact of the number of iterations on the profits of the battery after simulating for 50 days in each iteration. The results are shown for three different battery sizes. We can see that for all battery sizes the profits eventually improve, level out and become stable. The smallest battery improves after the first iteration and becomes stable after the third iteration. The

second largest battery follows a similar path and stabilizes after roughly 5 iterations. However, the largest battery starts with declining profits after the first few iterations, which eventually increase and stabilize after approximately 8 iterations.
Most notably in this figure though, is the best performing battery size for each number of iterations. For the first couple of iterations, the smaller the battery, the higher its profits. However, from three iterations onwards, the battery of 250 kWh outperforms all other batteries. Interestingly the battery of 500 kWh is eventually able to outperform the battery which is ten times smaller, but it never catches up to the battery of 250 kWh.
The most likely cause for this, is the fact that the battery is only capable of offering its full capacity or nothing at all for charging and discharging, as was explained in Section 5.2.1. This means that a battery of 500 kWh may actually charge more than it should, because it can only offer its full capacity, while in some of these situations the 250 kWh battery may be limited by its size.
When we compare the results of the profits of the battery to the profits of the entire system, shown in Figure 5.6b, we see an entirely different story. The only battery size that improves the system performance is the smallest battery of 50 kWh. The other two batteries contribute negatively to the system profits from the moment they are introduced and do not improve over more iterations. Interestingly for the smallest battery, the profits of the entire system are always lower than the profits of the battery on its own. This means that the profits of the system without batteries are negative after the introduction of the battery. So households without batteries suffer from households that do buy batteries, once the batteries are collectively big enough to influence prices.



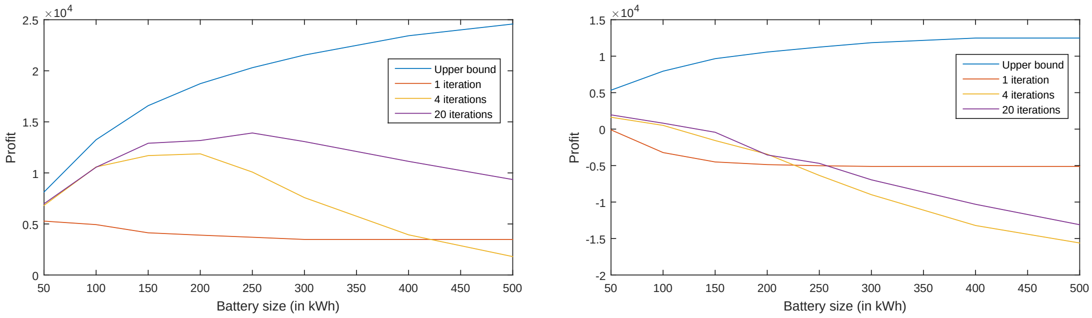
(a) Individual welfare

(b) Social welfare

Figure 5.6: The influence of the number of iterations on the individual and social welfare for different battery sizes under the average price model

Battery Size

In Figure 5.7 we compare the individual and social profits for different battery sizes and iterations to each other and to the upper bound. These profits are NOT per kWh, but for the entire battery in case of individual profits, and for the entire system when we are talking about the social profits. The upper bound is achieved by solving Equations (5.1) and (5.2).
If we look at the individual profits in Figure 5.7a we can see that, just as in Figure 5.6, medium sized batteries benefit the most from an increased number of iterations. It also shows that the bigger the influence of the battery on the price, the further away the profits are from the upper bound. This means that for this price function, the model does not perform well when its influence on the price is large.
Looking at the social welfare in Figure 5.7b we can see that batteries that are not aware of their influence on the price do not get close to the upper bound on the social welfare. Batteries below 250 kWh still improve their social welfare with improved iterations and get relatively close to positive. But the social welfare of batteries larger than 250 kWh only worsens with an increased number of iterations and gets further away from the upper bound.



(a) Individual welfare (b) Social welfare

Figure 5.7: The influence of the size of the battery on the individual and social welfare for different numbers of iterations and the upper bound under the average price model

Peak Shaving

An important reason for the introduction of batteries is the reduction of fluctuations in the system. One of the main fluctuations is the daily pattern in energy demand and in order to show the effectiveness of batteries, we will compare the demand with batteries to the demand without batteries. Note that these batteries are not optimized to create a demand pattern that is as flat as possible, but they are maximizing the profit of the batteries, the flattening of the demand is just a byproduct of this. Also note that because 10% of the energy is lost in the battery, the total demand increases due to the actions of the battery. However, the removal of fluctuations is more important for the reliability of the system than the increase of the total demand.
Figure 5.8 shows the base demand in the system as well as the demand in several systems with differently sized batteries. Combined with Table 5.2, this figure shows the impact of the battery on the peaks in the demand. We can see that even the smallest battery is capable of shaving the biggest peaks in demand, and the bigger the battery, the larger the reduction in the peaks. This is also expressed in the reduction in standard deviation amongst the different scenario's. However, while most batteries improve the minimum demand, the largest battery actually makes the lowest valley worse.

Table 5.2: Measures of spread in demand for the base case and several battery sizes in kWh for the average price model

	min	**max**	**mean**	**std**
Base	39	373	149	75
50 kWh	44	339	149	71
250 kWh	48	320	150	61
500 kWh	36	320	150	57

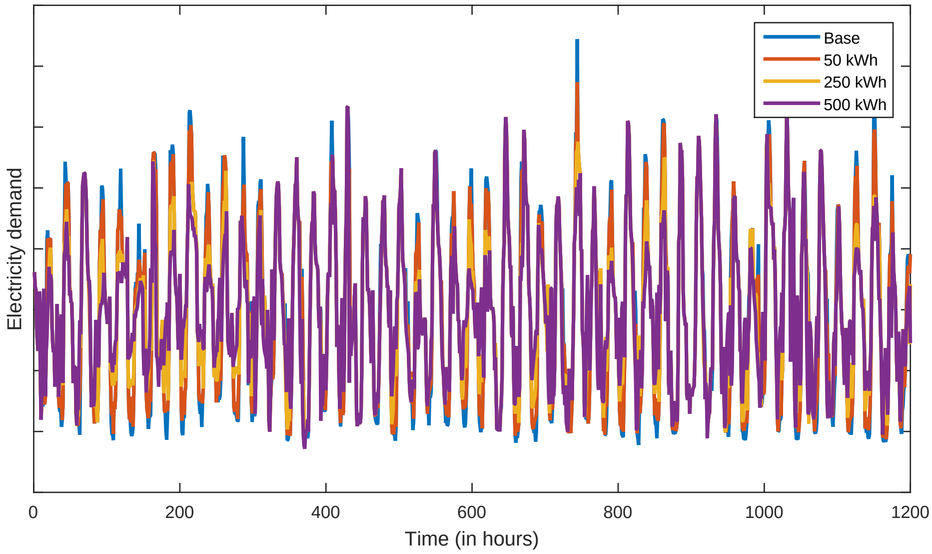


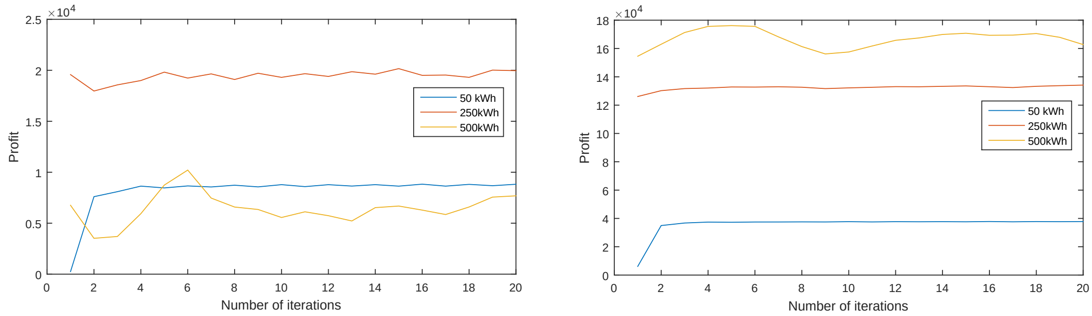
Figure 5.8: Demand peak shaving by several different battery sizes with an unknown price influence for the average price model

Merit Order Price Model

Please note that the results for the average price model in the previous section and the merit order price model in this section are not scaled to be comparable. The only thing which can be compared is the relative shape of the graph of the results and, more importantly, the implications they carry.

Iterations

Figure 5.9a shows the influence of an increase in the number of iterations on the profits of the battery for three different battery sizes. We can see that the battery of 50 kWh does not perform well in the first iteration, but improves significantly the following iteration. While both the 250 and 500 kWh battery are starting already in a relatively stable position. The instability of the solution increases as the size of the battery increases. And just as in Figure 5.6a the battery of 250 kWh outperforms the 500 kWh battery. However, in the case of the merit order, the 50 kWh battery almost consistently outperforms the 500 kWh battery. Another difference is the fact that stability in the merit order model is achieved much faster than in the average price model.
The results show an entirely different story when we look at influence of the number of iterations on the social welfare of the battery in Figure 5.9b. The bigger the battery, the bigger the social welfare in the system. Whereas the battery was a burden on the system for the average price model, the battery improves the system performance here. This can be seen again by comparing the individual welfare to the social welfare. We can see that for the 50 kWh battery the social welfare is roughly four times the individual welfare. In other words, the demand in the system benefits three times as much from the battery as the battery itself. For the 250 kWh battery this is roughly five times as much and for the 500 kWh battery approximately even eighteen times as much. This means that not only you profit from having a battery in your house, but your entire neighbourhood benefits from it.



(a) Individual welfare

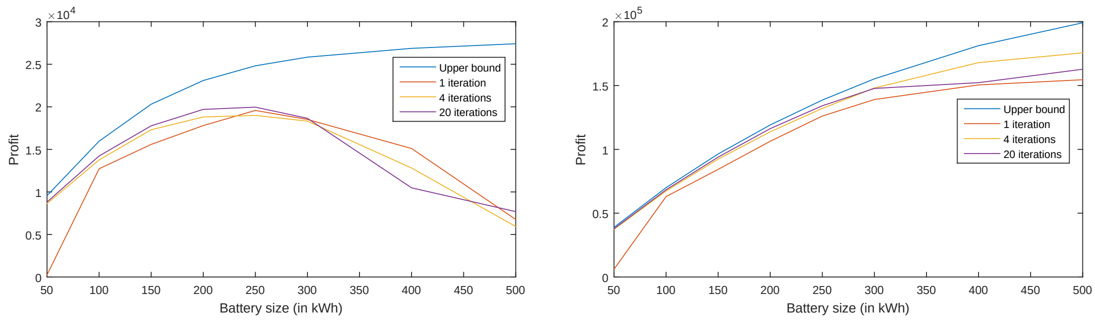
(b) Social welfare

Figure 5.9: The influence of the number of iterations on the individual and social welfare for different battery sizes under the merit order price model

Battery Size

Looking at the effects of the battery size on the individual profits of the battery in Figure 5.10a, we can conclude for the merit order price model, just as for the average price model, that it becomes increasingly difficult for larger batteries to get close to the upper bound.

Similar effects can be found in the social welfare in Figure 5.10b. However, the effects in this figure are much less pronounced and the approximation is capable of getting much closer to the upper bound. Also of note is the fact that beyond 300 kWh four iterations appears to be better than twenty iterations. This could be caused by the instability between iterations, or it could be something more permanent. Looking at Figure 5.9b suggests that it may be caused by randomness, but it is not clear if the solution will get as good again as it was between iterations four to six.



(a) Individual welfare

(b) Social welfare

Figure 5.10: The influence of the size of the battery on the individual and social welfare for different numbers of iterations and the upper bound under the merit order price model

Peak Shaving

Figure 5.11 shows the demand peak shaving that occurs when the battery does not know its influence on the price and the price is set using the merit order price model. When we compare this figure to the demand peak shaving using the average price model from Figure 5.8, we see that using the merit order price function results in more peak shaving. Additionally, comparing Table 5.2 to Table 5.3, we see that the results are very comparable for the small battery, but for the larger batteries, the merit order price model clearly outperforms the average price model in terms of the flattening of the demand curve, with a standard deviation of 44 for the largest battery in the merit order and 57 for the largest battery in the average price model.

Table 5.3: Measures of spread in demand for the base case and several battery sizes in kWh for the merit order price model

	min	**max**	**mean**	**std**
Base	39	373	149	75
50 kWh	44	328	149	70
250 kWh	50	273	150	52
500 kWh	51	272	150	44

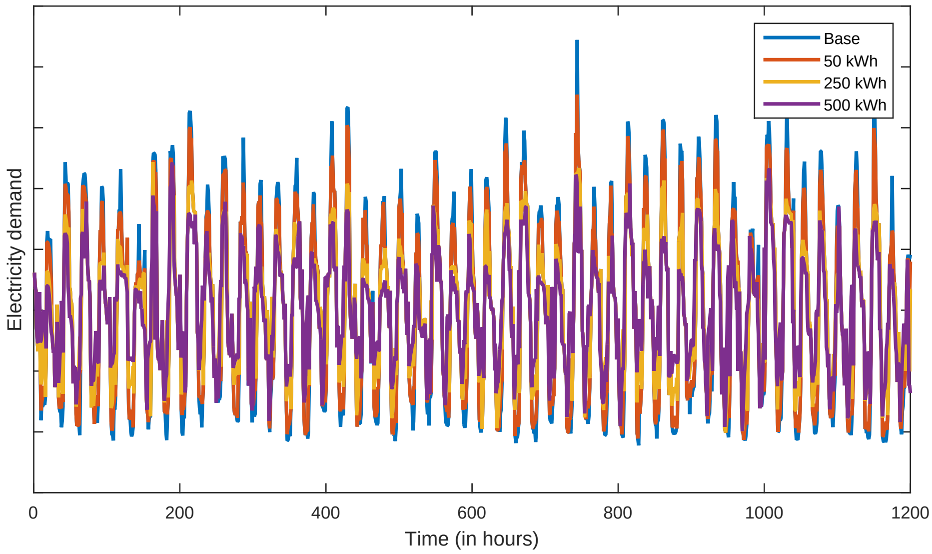


Figure 5.11: Demand peak shaving by several different battery sizes with an unknown price influence for the merit order price model

5.3.3 Known Price Influence

In this section we will discuss the results for the case where the influence of the actions on the price are known to the battery. The models used to obtain these results can be found in Section 5.2.2. Recall that in the case where the influence is known, a distinction has to be made between the battery that attempts to optimize the social welfare (selfless), and the battery that maximizes its own welfare (selfish). Whereas in the case of an unknown influence, there was just one battery that tried to optimize its individual welfare, but also inadvertently influenced the social welfare.

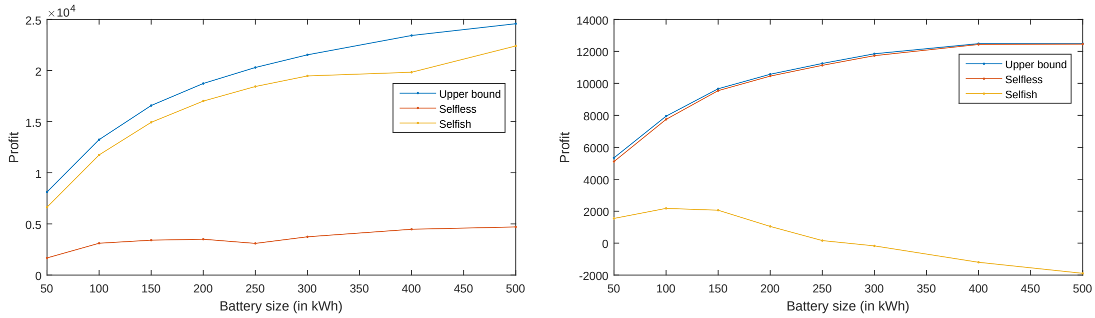
Average Price Model

Battery Size

Figure 5.12 shows the relation between the battery size on the individual and social welfare for the average price model. We can clearly see the difference between the selfless and selfish battery, with the selfish battery performing best for the individual welfare and the selfless battery performing best for the social welfare.

Noteworthy is the fact that the selfless battery is capable of getting very close to the upper bound, getting closer with increasing battery size. Deviation from the upper bound is caused by an error in the price forecast. Because the battery is now capable of influencing the price, it seems to be able to smooth out these errors in forecast by adjusting the price a bit more when the forecast is not correct. This is easier for larger batteries, as they have more reserve capacity to do this.

If we look what happens for household without a battery, we have to subtract Figure 5.12a from Figure 5.12b. This shows us that, as expected, the selfless battery is beneficial to households without a battery. Additionally it shows that the profits made by the battery in the selfish case are being paid for by an increase in the price of household demand.



(a) Individual welfare

(b) Social welfare

Figure 5.12: The influence of the size of the battery on the individual and social welfare for the selfless battery, the selfish battery, and the upper bound under the average price model

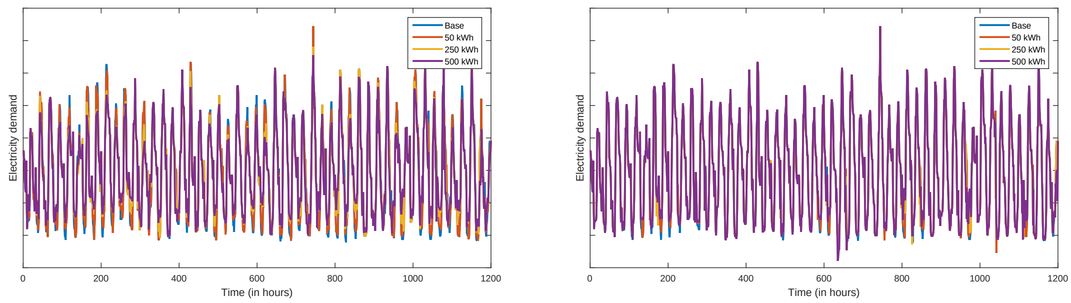
Peak Shaving

In Figure 5.13 we can see the demand peak shaving of the selfish battery (5.13a) and selfless battery (5.13b). Under the average price model, the selfish battery has more opportunities for profit than the selfless battery. The peak shaving is therefore more pronounced for the selfish battery than for the selfless battery. However, the peak shaving for both batteries is still less than it was when the battery did not know its influence in Figure 5.8. This is because the battery with an unknown influence buys energy up to the point where it is no longer profitable to buy energy. This point is higher than the point where the profits for the battery are maximized. And therefore, in order to reach this point of maximized profits, the selfish battery with a known influence buys less and subsequently has less influence on the price and total demand.

This is also expressed in Table 5.4. The selfish battery lowers the standard deviation more than the selfless battery, however, the lowest standard deviation for the average price model is reached by the battery with an unknown influence in Table 5.2. Of note as well is the fact that the selfless battery actually increases the gap between the minimum and the maximum demand, the opposite of what we would like to see. This increase is due to the fact that the biggest influence of the price can be achieved by selling when the price is already quite low. This results in a negative profit for the battery, but this is compensated by a larger decrease in the cost for the households.

Table 5.4: Variation in demand for the base case and several battery sizes for the selfish and selfless battery in kWh for the average price model

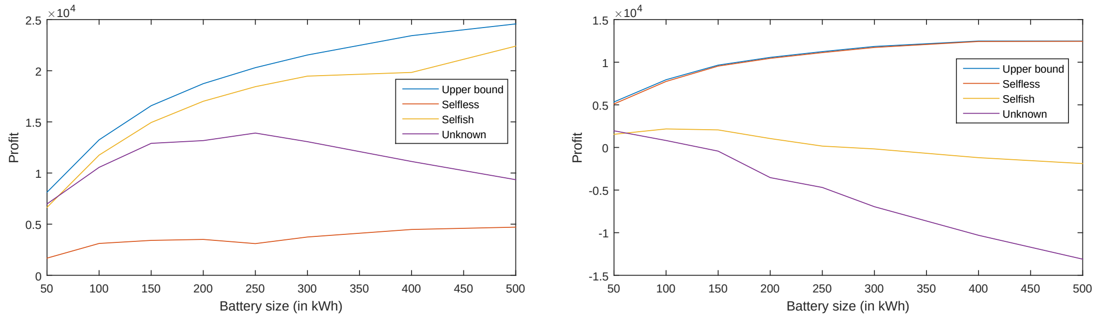
	Selfish battery				*Selfless battery*			
	min	**max**	**mean**	**std**	**min**	**max**	**mean**	**std**
Base	39	373	149	75	39	373	149	75
50 kWh	42	374	149	72	23	374	149	75
250 kWh	47	342	149	64	14	374	149	74
500 kWh	48	366	150	62	11	374	149	74



(a) Selfish battery (b) Selfless battery

Figure 5.13: Demand peak shaving by several different battery sizes with a known price influence for the average price model for batteries optimizing the individual, or social welfare

<u>Comparison between known and unknown influence</u>
Comparing the figures of known and unknown influence in Figure 5.14, we can see that for the smallest battery sizes, where the influence is smallest, the added information of knowing the influence does not seem to matter. Compared to these small batteries, not only is the influence on the price for the larger batteries much bigger, but also the added value of knowing the influence on the price. We can also see that not knowing the influence on the price is disastrous for the social welfare, as not knowing the influence is even worse than being selfish.

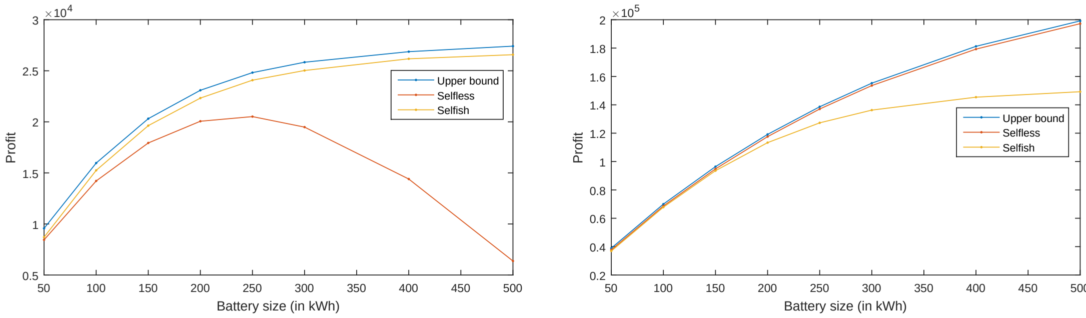


(a) Individual welfare (b) Social welfare

Figure 5.14: The influence of the size of the battery on the individual and social welfare for the selfless battery, the selfish battery, the unknown influence, and the upper bound under the average price model

Merit Order Price Model

<u>Battery Size</u>
In Figure 5.15 we can see the relationship between the battery size and the individual and social profits of the merit order price model when the influence on the price is known. Contrary to the average price model, small selfish and selfless batteries are very comparable in both individual and social welfare. Only for bigger batteries does the difference between the two batteries become significant. When we look at the selfless and selfish battery in their own domain, we see that both solutions get very close to their respective upper bound.
When we again look at what happens to the cost of household demand by subtracting Figure 5.15a from Figure 5.15b, we see that the social welfare is always bigger than the individual welfare. This means that adding a battery to the system is always better for an individual than not having a battery in the system, regardless of whether the individual himself owns this battery or not.



(a) Individual welfare (b) Social welfare

Figure 5.15: The influence of the size of the battery on the individual and social welfare for the selfless battery, the selfish battery, and the upper bound under the merit order price model

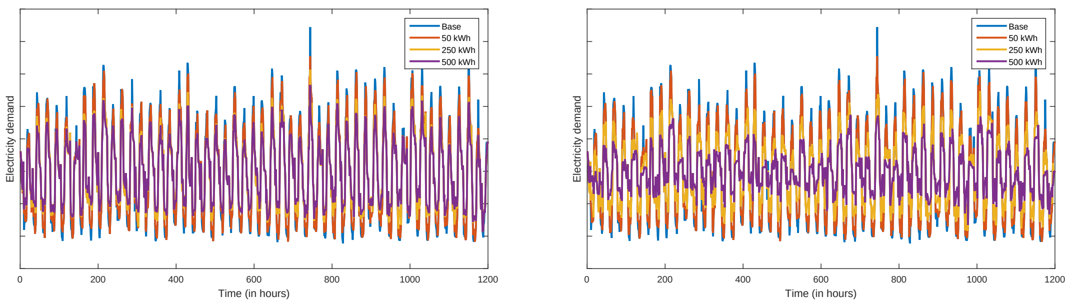
Peak Shaving

Figure 5.16 shows the peak shaving of the selfish (5.16a) and the selfless (5.16b) battery under the merit order price model. For this price model the selfless battery shaves the peaks significantly more than the selfish battery, this is the opposite of what happened for the average price model from Figure 5.13. The peak shaving of the selfless battery under the merit order price model is the best of all the methods researched.

This becomes especially clear when we look at the standard deviation in Table 5.5. The demand for the biggest selfless battery has a standard deviation of 32, where the second lowest is a standard deviation of 44 for the battery with unknown influence also for the merit order price model. It is also interesting to see that the 50 kWh battery has the same results for the selfish and selfless battery, indicating that a pronounced penetration of storage capacity is required to notice the difference between the two strategies.

Table 5.5: Variation in demand for the base case and several battery sizes for the selfish and selfless battery in kWh for the merit order price model

	Selfish battery				*Selfless battery*			
	min	**max**	**mean**	**std**	**min**	**max**	**mean**	**std**
Base	39	373	149	75	39	373	149	75
50 kWh	42	328	149	70	42	328	149	70
250 kWh	57	308	149	54	58	275	150	51
500 kWh	57	284	150	49	68	237	151	32

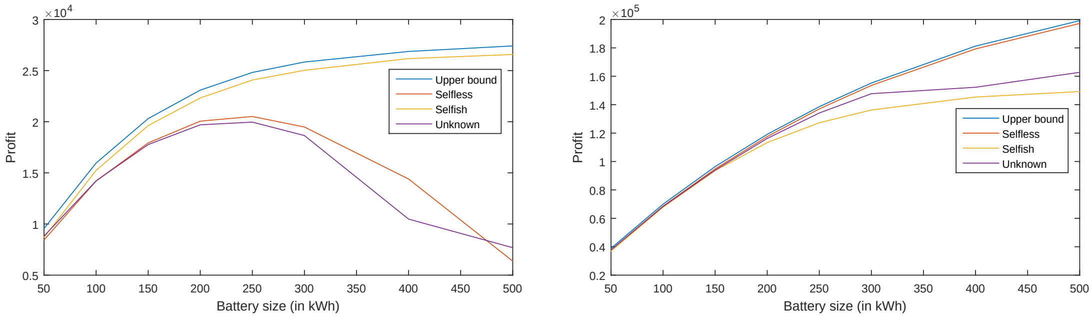


(a) Selfish battery (b) Selfless battery

Figure 5.16: Demand peak shaving by several different battery sizes with a known price influence for the merit order price model for batteries optimizing the individual, or social welfare

Comparison between known and unknown influence

Looking at the comparison between the known and unknown influence in Figure 5.17, we see that optimizing without knowing the influence is very similar, when it comes to individual welfare, to being selfless with a known influence. When we look at the social welfare, we initially see the same thing with the battery with an unknown influence being close to the selfless battery. However, for sizes above 300 kWh, the lack of knowledge of its influence makes the battery with an unknown influence behave only slightly better than the selfish battery. So we can see that not knowing the influence makes the battery which does not know its influence perform like the selfless battery up to 300 kWh. After which the influence becomes too big to ignore and the performance of the battery with unknown influence drops.



(a) Individual welfare

(b) Social welfare

Figure 5.17: The influence of the size of the battery on the individual and social welfare for the selfless battery, the selfish battery, the unknown influence, and the upper bound under the merit order price model

5.4 Conclusion

The question we were looking to answer in this section was: *What are the main characteristics for the applicability of batteries that are price makers?* Even though, knowing or not knowing the influence on the price, and being selfless or selfish had an influence on the outcome, the most influential characteristic is the price function. However, it is important to note here that the merit order price model is made to represent the German electricity market, while the average price model is made to represent a small system with only a few means of production. This means that the merit order price model can be used in a small system as is done in this chapter, but it may not be able to represent the true costs of such a small system.

Having a price function based on the merit order curve means that the actions of the battery reduce the costs on the demand side of the system, regardless of whether you know your influence on the price, or whether you are trying to be selfish or selfless. This means that the actions of the battery not only result in a profit for the battery itself, but also lower the cost of all other demand in the system. If we compare this to the price based on the average price model, only the selfless battery, working to benefit the welfare of the system, is capable of reducing the costs for the demand side. When the battery knows its influence and is acting selfishly, the supply side of the system is better off and the demand side has to pay for it. This effect is amplified when we look at the situation where the battery does not know its influence on the price.

When we look at the reduction of peaks in demand, the merit order price model is generally better at reducing peaks than the average price model. For the merit order price model, the selfless battery shaves the peaks the most, followed by the battery that does not know its influence, and the selfish battery reduces the peaks the least. In the average price model, the battery that does not know its influence is the best at peak shaving, while the selfish battery is second best, and the selfless battery is the worst.

This chapter also shows that using the price taker algorithm to control a large amount of batteries with an unknown influence does not destroy the system, it remains relatively stable. However, the solutions get further from the optimum when the influence, and thus the combined size of the batteries, increases. This effect is more pronounced for the average price model than it is for the

merit price model, however, regardless of the price model, this is a problem worth looking into.
So looking at the price function we can conclude that the choice of price function strongly influences the actions of the battery. And whenever an aggregator or electricity provider gets to choose the price function that is being used in the system, it has to take this influence into account. A price model like the merit order price model is better at reducing the costs for the demand side, while the average price model increases the profits of the suppliers. The difference between the selfish and selfless batteries, especially for the average price model, also suggests that relying on the free market to solve the storage problem may be bad for the population. This means that it may be beneficial for governments to preemptively look into measures and regulations that can be used to prevent these problems.

Chapter 6

Conclusion and Further Research

6.1 Conclusion

Before we began work on the main research questions, we looked at several different decentralized coordinated planning methods proposed in the literature. We found that nearly all planning methods are market based and cleared frequently, usually ever hour or every 15 minutes.
Most non-market alternatives still require participants to respond to a price signal. However, the difference is that this price signal may be changed in an iterative scheme, or a more direct signal may be given in order to achieve a balance in the system.
If we look at this literature, we see that whatever design the future smart grid may have, it is likely to be based on reacting to a price signal that changes in fixed time intervals of 15 minutes or an hour. For this reason the market system we chose to use in this thesis, a market based system which clears hourly, is justifiable.

In order to answer the main research questions of this thesis, we developed and evaluated three control algorithms for electrical energy storages for three different settings. The main questions we addressed in this thesis were:

- How to optimally control an electrical energy storage in the smart grid context given forecasts with uncertainty?
- What are the main characteristics that determine the profitability and applicability of electrical energy storages in the smart grid context given forecasts with uncertainty?

We started by looking at a setting where a stand-alone battery trades on the electricity market as a price taker. In this setting the battery is able to buy and sell electricity on the market in order to maximize its profits. We proposed a Stochastic Dynamic Programming (SDP) algorithm to control the battery. We compared this solution to two lower bounds which do not use forecasts and to the theoretical upper bound using data based on realistic prices. From this we found that our proposed algorithm significantly outperforms both lower bounds and gets within 92% of the upper bound.
We used this algorithm to investigate the influence of certain parameters on the profitability of the battery. From this we found that the current Dutch tax system of having a fixed electricity tax per kWh is detrimental to the viability of batteries. This can be solved by switching to a variable tax rate that will generate a similar tax revenue. Additionally, we found that the added value of increasing the planning window decreases rapidly, the higher the charging speed of the battery, the faster the added value is reduced. A battery that takes one time period to fully charge requires 16 future prices to reach its maximum, while a battery that takes five time periods to fully charge requires 36 future prices to reach its maximum.

Next, we expanded the previous setting to represent a household with a home battery by introducing household demand and an Electric Vehicle (EV). We proposed an SDP formulation with

uncertainty in household demand, electricity prices, and controllable EV charging. However, in order to keep our algorithm computationally feasible, we converted our SDP formulation such that it no longer takes the uncertainty in household demand into account and it considers the EV charging to be fixed. We again compared this solution to two lower bounds and to its theoretical upper bound and found that our algorithm outperforms the two lower bounds and gets within 93% of the theoretical upper bound.
Using this algorithm to examine the influence on the profit of the battery for the same parameters as before, we found that most influences were similar to the setting with the stand-alone battery. The most important difference was the influence of the tax parameters. Constant tax becomes more viable when we consider a home battery. However, the 10 cent per kWh currently imposed by the Dutch government is still infeasible. In contrast, high variable tax rates make home batteries extremely profitable.

In order to show what would happen if our algorithm is adapted by home batteries on a scale big enough to collectively become a price maker, we considered a setting with the supply and demand of 200 households where some households have a home battery. In this setting the batteries are not aware of their influence on the price and therefore still use the same SDP algorithm. However, in order to model the influence on the price, we compared two different price functions.
The results show that bigger batteries are better for the individual welfare up to a point, after which the individual welfare starts to decrease. However, depending on the price function, bigger batteries are always better for either the welfare of the supply side, or the welfare of the demand side. Additionally, bigger batteries always have a bigger influence on the price and therefore the demand in the system.

Lastly we considered a setting with one high capacity battery that is a price maker and aware of its influence on the price. In this setting the battery is either attempting to maximize its own welfare (e.g. a battery owned by a storage company), or its trying to maximize the welfare of itself combined with the system demand (e.g. a battery used by the government or network operator). In this situation we used the same two price functions as we had previously.
The results show that the maximization criterion that influences the price and demand the most, depends on the price function. The price function also determines whether optimizing the battery's own welfare benefits the supply side or the demand side of the system.

6.2 Further Research

6.2.1 Electric Vehicle

In this research the Electric Vehicle (EV) is considered to have a known and fixed demand. A possible extension would be to introduce uncertainty to the demand of the EV. This could be uncertainty in the arrival and departure times of the EV, or uncertainty in the amount of used energy, or a combination of both. By introducing more stochastic variables to the system, it becomes harder to compute the expectation of the value function for future time periods.
Another possible extension could consider combining the optimization of the battery with the optimization of the EV. In this case the household will be capable of controlling both the charging of the battery and the EV. This change will drastically increase the state space, and with it the required computation time. An EV battery is typically large enough to hold enough energy for multiple days or even a week. This means that it will likely be beneficial to consider a longer planning window, because we can charge the battery of the EV with just enough energy for the next trip, or we could fill it up if we expect prices to rise the coming days.

6.2.2 Household Demand

In this research we derived the demand of a single household by dividing the average demand of 200 households by 200. In addition, we considered this demand to be known and fixed. Possible extensions include taking the uncertainty of the demand into account, being able to shift or shed

a part of the demand, and making the demand of a single household more realistic.
Taking the uncertainty into account will again increase the complexity of calculating the expectation of the value function for future time periods.
Adding the option to shift or shed demand is comparable the addition of control for the EV. However, the planning window in this case will most likely stay similar to the current window.
The impact of increasing the accuracy of the household demand will depend on the data and chosen scenario. If data is available for households with, for example, schedules and preferences for a dishwasher, a highly accurate planning model can be considered. In case only the total demand at each timestep is known for a single household, the same approach as in this report can be used.

6.2.3 Prosumers

In this report we confined ourselves to looking at consumer households only. In the future this could be expanded to include prosumer households with, for example, solar panels or a micro Combined Heat and Power (micro-CHP) unit. The implications of this change on the approach will depend on the chosen scenario. In the current Dutch tax system with net metering (i.e., instead of being paid for energy exported, the total amount of energy consumed is reduced accordingly), it is always better to supply generated energy to the grid than it is to store it in a battery. This is because of the loss of energy due to the round-trip efficiency of the battery.

6.2.4 Taxation for the Price Maker

In this research we do not consider taxation in the scenario of the price maker. We expect the impact of taxation to depend on the scenario considered. A scenario with a high capacity battery owned by an independent investor maximizing its own profit will most likely have the same results as the stand-alone battery. So, with increasing tax rates the profits of the battery will decrease and the battery will be trading less. However, in a scenario with multiple home batteries we expect to see the same as in the home battery model. With increasing tax rates the profits will initially decrease, but eventually they will increase. However, further research is required to see if our expectations are correct.

6.2.5 Price Function

Since this research has shown that the influence of the price function on the actions of the battery is significant, additional research into price functions is needed to understand the relationship between the price function and flexibility in the system. This could become especially important when we want to consider a smart grid where prices are set regionally depending on the local supply, demand, and network infrastructure.

6.2.6 Unknown Influence

In order to see what would happen if our algorithm was applied on a large scale, we applied our price taker algorithm to a price maker scenario. We found that the quality of the solution decreased as the scale (and influence) increased. This means that further research is required to see how this effect can be reduced. Ideally the results are improved by changing the algorithm. However more drastic changes may be required. Some possibilities are: a steering signal from a centralized entity, learning the effect of the collective influence, changing the market system to include future planning.

Appendices

Appendix A

Charging Strategy Proof

Claim: In Equation (3.5) $\max_{u_t \in [-1,1]}$ is the same as $\max_{u_t \in \{-1,0,1\}}$
Proof: Recall that in order to find the best charging strategy at time t, the following stochastic dynamic programming equations have to be solved:

$$V_{t+L+1}(x,p) = cx, \text{ for some price } c.$$

Next, for times $\theta = t, \ldots, t+L$ and state (x_θ, p_θ) we have:

$$V_\theta(x_\theta, p_\theta) = \max_{u_\theta} \left[R(u_\theta, p_\theta) + \mathbb{E}_{P_{\theta+1}} \left(V_{\theta+1}(x_\theta + u_\theta, P_{\theta+1}) \right) \right],$$

with:

$$R(u_t, p_t) = -u_t \left(p_t \left(1 + (\gamma - 1) \mathbb{1}_{\{p_t > 0\}} \right) + \sqrt{\eta}\kappa + \tau \right) \mathbb{1}_{\{u_t > 0\}} - \eta u_t p_t \mathbb{1}_{\{u_t < 0\}}.$$

And let us define:

$$\hat{V}_t(x) = \mathbb{E}_{P_t} \left(V_t(x, P_t) \right).$$

Iteratively we have to make decisions on how much we should charge, u, at times $\theta = t, \ldots, t+L$ by solving:

$$V_t(x,p) = \max_u \left[R(u,p) + \hat{V}_{t+1}(x+u) \right].$$

We assume that the current battery level, x, is an integer. We know that $R(u,p)$ for fixed p is piecewise linear for $u \in [0,1]$ and for $u \in [-1,0]$. If we can prove that $\hat{V}_{t+1}(x+u)$, with $x \in \mathbb{N}^0$, is always piecewise linear for $u \in [0,1]$ and for $u \in [-1,0]$, we know that the maximum can always be reached in one of the extreme points, so $u \in \{-1,0,1\}$.

In order to prove the piecewise linearity we will use induction.
Base: By definition, $\hat{V}_{t+L+1}(x+u) = c(x+u)$ is piecewise linear in the aforementioned intervals and thus can serve as a base for proof by induction.
Induction: Let us now assume the piecewise linearity holds for $\hat{V}_{t+1}(x+u)$ with $x \in \mathbb{N}^0$. We will now prove that this also means that the piecewise linearity holds for $\hat{V}_t(x+u)$.
If the charge $x+u$ is outside the range of the battery capacity, the expected value is always $-\infty$ and therefore linear. So let us assume that $x+u$ lies within the range of the battery capacity. We want to prove that it is piecewise linear with x an integer and $u \in [-1,0]$, or $u \in [0,1]$, so we have to prove that:

$$\hat{V}_t(a) = \mathbb{E}_P \left(\max_u \left[R(u,P) + \hat{V}_{t+1}(a+u) \right] \right) \tag{A.1}$$

is linear for some $a \in [\lfloor a \rfloor, \lfloor a \rfloor + 1)$.
Let us define:

$$\delta_0 = \hat{V}_{t+1}\left(\lfloor a \rfloor\right) - \hat{V}_{t+1}\left(\lfloor a \rfloor - 1\right),$$
$$\delta_1 = \hat{V}_{t+1}\left(\lfloor a \rfloor + 1\right) - \hat{V}_{t+1}\left(\lfloor a \rfloor\right),$$
$$\delta_2 = \hat{V}_{t+1}\left(\lfloor a \rfloor + 2\right) - \hat{V}_{t+1}\left(\lfloor a \rfloor + 1\right).$$

Since $\hat{V}_{t+1}(x)$ is linear between integer values for x, δ_0, δ_1, and δ_2 are the slopes of the corresponding intervals. Now in order to write out the expectation of the maximum, we will make a case distinction. Since both $R(u,p)$ and $\hat{V}_{t+1}(a+u)$ are piecewise linear, the maximum is reached in one of the extreme points. In figure A.1 we can see that the extreme points are $U = \{-1, \lfloor a \rfloor - a, 0, \lfloor a \rfloor + 1 - a, 1\}$.

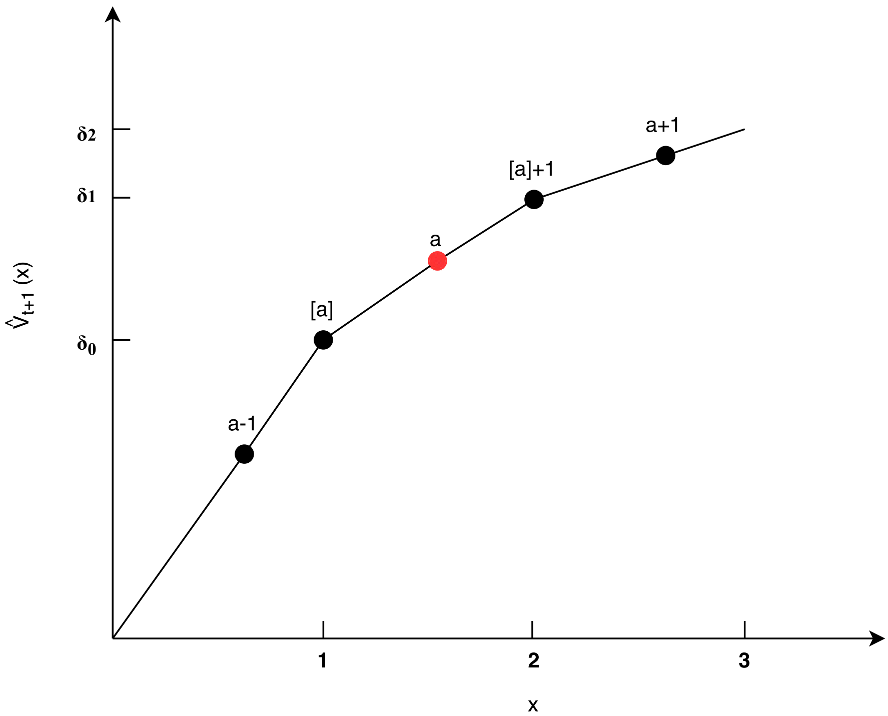


Figure A.1: A graphical representation of a

Case 1: $u = -1$, happens when $R(-1,p) > \delta_0$ and has a corresponding value:

$$R(-1,p) + \hat{V}_{t+1}(a-1) = R(-1,p) + (a - \lfloor a \rfloor)\,\delta_0.$$

Case 2: $u = \lfloor a \rfloor - a$, happens when $\delta_0 \geq R(-1,p) > \delta_1$ and has a corresponding value:

$$R(\lfloor a \rfloor - a, p) + \hat{V}_{t+1}(a + \lfloor a \rfloor - a) = R(\lfloor a \rfloor - a, p) + \delta_0.$$

Case 5: $u = 1$, happens when $R(1,p) < \delta_2$ and has a corresponding value:

$$R(1,p) + \hat{V}_{t+1}(a+1) = R(1,p) + \delta_0 + \delta_1 + (a - \lfloor a \rfloor)\,\delta_2.$$

Case 4: $u = \lfloor a \rfloor + 1 - a$, happens when $\delta_2 \leq R(1,p) < \delta_1$ and has a corresponding value:

$$R(\lfloor a \rfloor + 1 - a, p) + \hat{V}_{t+1}(\lfloor a \rfloor + 1) = R(\lfloor a \rfloor + 1 - a, p) + \delta_0 + \delta_1.$$

Case 3: $u = 0$, happens when $R(1,p) \leq \delta_1 \wedge R(-1,p) \geq \delta_1$ and has a corresponding value:

$$R(0,p) + \hat{V}_{t+1}(a) = \delta_0 + (a - \lfloor a \rfloor)\,\delta_1.$$

In order to calculate the expectation by combining these cases, we first have to define $R^{-1}(u,\delta)$ as the preimage of $R(u,p)$ with respect to the price p, such that $R^{-1}(u,R(u,p)) = p$. Now we can rewrite equation A.1:

$$\hat{V}_t(a) = \int_{-\infty}^{R^{-1}(1,\delta_2)} f_{P_t}(p)\left(R(1,p) + \delta_0 + \delta_1 + (a - \lfloor a \rfloor)\delta_2\right)\mathrm{d}p \tag{A.2}$$

$$+ \int_{R^{-1}(1,\delta_2)}^{R^{-1}(1,\delta_1)} f_{P_t}(p)\left(R(\lfloor a \rfloor + 1 - a, p) + \delta_0 + \delta_1\right)\mathrm{d}p \tag{A.3}$$

$$+ \int_{R^{-1}(1,\delta_1)}^{R^{-1}(-1,-\delta_1)} f_{P_t}(p)\left(\delta_0 + (a - \lfloor a \rfloor)\delta_1\right)\mathrm{d}p \tag{A.4}$$

$$+ \int_{R^{-1}(-1,-\delta_1)}^{R^{-1}(-1,\delta_0)} f_{P_t}(p)\left(R(\lfloor a \rfloor - a, p) + \delta_0\right)\mathrm{d}p \tag{A.5}$$

$$+ \int_{R^{-1}(-1,\delta_0)}^{\infty} f_{P_t}(p)\left(R(-1,p) + (a - \lfloor a \rfloor)\delta_0\right)\mathrm{d}p, \tag{A.6}$$

with $f_{P_t}(p)$ the probability density function for the price at time t and $a \in [\lfloor a \rfloor, \lfloor a \rfloor + 1)$. If the battery is nearly empty, i.e. $\lfloor a \rfloor = 0$, we cannot sell more energy than we have. Therefore the value of maximum discharging becomes $\hat{V}_{t+1}(\lfloor a \rfloor - 1) = -\infty$, so $\delta_0 = \infty$. In order for the reward to be infinite when selling electricity, the price has to be infinite as well, in other words $R^{-1}(-1,\delta_0) = \infty$. For this reason Summand (A.6) disappears and the upper-bound of the integral of Summand (A.5) becomes ∞.
In case the battery is nearly full, we cannot buy more energy than fits in the battery. In this case the value of maximum charging becomes $\hat{V}_{t+1}(\lfloor a \rfloor + 1) = -\infty$, and $\delta_2 = \infty$. The price for which buying energy results in an infinite reward is minus infinity, i.e. $R^{-1}(-1,\delta_2) = -\infty$. Therefore Summand (A.2) disappears and the lower-bound of the integral of Summand (A.3) becomes $-\infty$.

Regardless of the state of the battery, $\hat{V}_t(a)$ is linear with respect to a in the interval $[\lfloor a \rfloor, \lfloor a \rfloor + 1)$. Thus $\hat{V}_t(x)$ is linear between x and $x+1$ and between x and $x-1$ given that x is an integer. And therefore we have proven that $\hat{V}_t(x+u)$ is piecewise linear for $u \in [0,1]$ and for $u \in [-1,0]$, given that $\hat{V}_{t+1}(x+u)$ is piecewise linear. And we know that $\hat{V}_{t+L+1}(x+u)$ is piecewise linear. And therefore we have proved that it is optimal to only consider $U = \{-1,0,1\}$ as possible charging strategies for the stand-alone battery model.

Appendix B

Additional Results Home Battery Model

B.1 General Parameters

B.1.1 Window Size

Figure B.1 shows the influence of the size of the planning window on the profit of the home battery model. The results are similar to the results obtained in the stand-alone model. Lower charging speeds need longer time windows to level out and the slower the battery the lower it seems to level out.

The difference in the results from the home battery model is the fact that the two fastest charging speeds converge to the same value. This is a recurring event throughout the results for this model. The reason for this might be that most of the stored energy is used to supply household demand and that both of the fastest charging batteries are capable of both providing this demand, and buying when energy is cheapest.

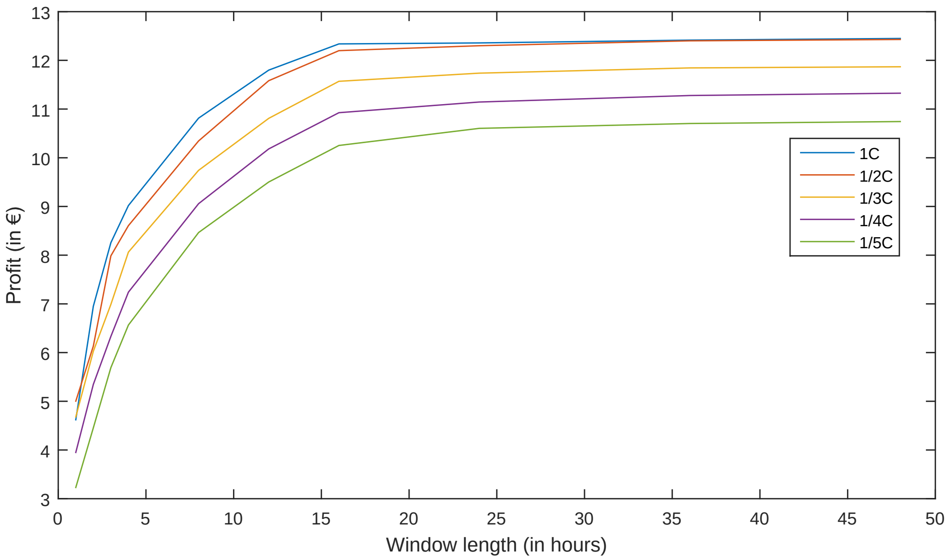


Figure B.1: The influence of the window-size on the profit in the home battery model

B.2 Battery Parameters

B.2.1 Size

Figure B.2 shows the nearly linear relationship between the battery size and the profit. It appears that the slower the battery charges, the more the line curves downwards.

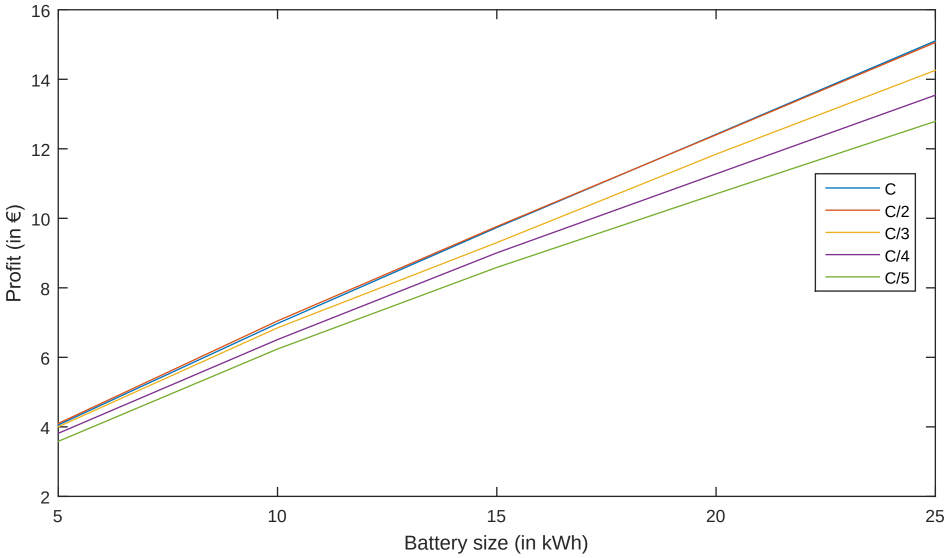


Figure B.2: The influence of the battery size on the profit in the home battery model

B.2.2 Efficiency

Figure B.3 shows the results for the effects of the round-trip efficiency on the total profit. These results are very similar to the results obtained for the stand-alone model.

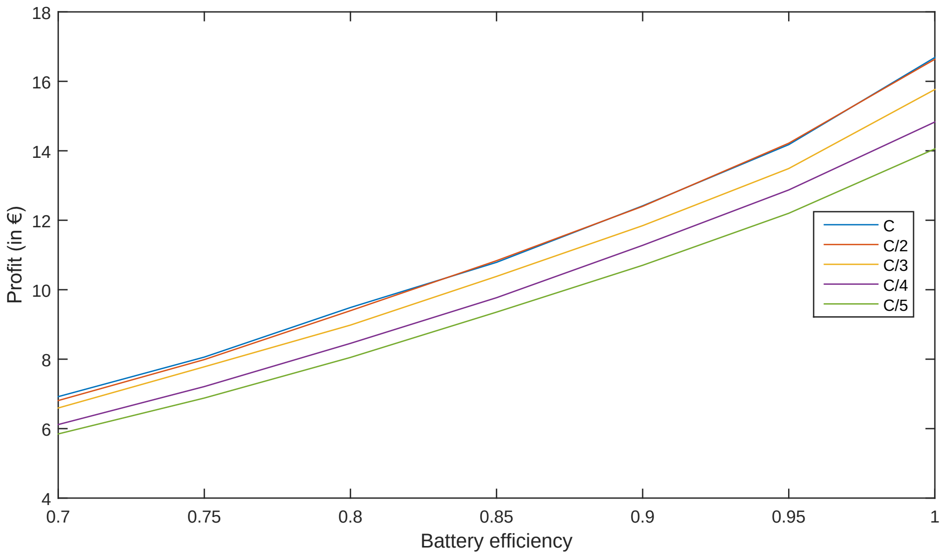


Figure B.3: The influence of the round-trip efficiency on the profit in the home battery model

B.2.3 Cycle Costs

Figure B.4 shows that the relationship between the cycle costs and the profit are comparable between the home battery model and the stand-alone model. This time, however, there is still a slight profit to be made at cycle costs of five cents per kWh, but they are also gone from 5.5 cents and above.

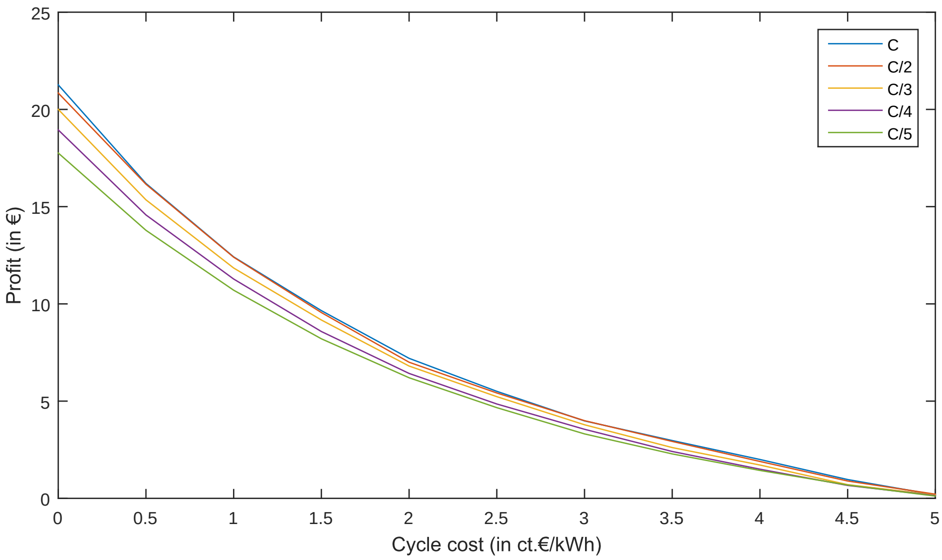


Figure B.4: The influence of cycle costs on the profit in the home battery model

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